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Editor of series: Ljiljana Rašković, e-mail: ral@kalca.junis.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš,YU, Tel. +381 18 547-095, Fax: +381 18 547-950
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CRITERION OF DIFFUSNESS OF SOUND FIELD IN A RECTANGULAR ROOM

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Miroslava A. Milošević, Nenad D. Milošević, Milan S. Milošević

Faculty of Electronic Engineering, Beogradska 14, Niš, Yugoslavia

Abstract. *The determining diffusness of the sound field in rectangular room on the basis of correlation function and spatial distribution of the sound pressure levels is considered in this paper. The main objective is to provide acousticians criterion of diffuseness of sound field at low frequencies in a room. The results are compared to those of the ideal diffuse field. Comparisons and analysis of theoretical results and measured values are conducted for the given empty reverberant room in the form of the parallelepiped. One of the conclusions is to extend the previous calculations to lower frequencies range, in terms of the least permissible number of room modes to achieve an adequate diffusion..*

1. INTRODUCTION

The reflected sound at any point in room includes contributions from reflections from all boundaries of the room - it does not include the sound, which travels to the point directly from the source, so that it is relatively independent on the distance from the source. It is sometimes referred to as the reverberant sound, but the term reverberant sound implies sound, which is decaying, reflected sound may either be decaying or steady state. The level of reflected sound depends only on the characteristics of the room and on the characteristics of the sound source if: the room has fairly regular proportion of its dimensions; the average absorption coefficient of room is small, i.e., less than 0,3; the boundaries of the room are irregular; and the source contains no strong discrete frequency components. Under these conditions the sound pressure level of the reflected sound is everywhere equal in room except close to the source and the boundaries.

The reflected field in room is called a diffuse field if a great many reflections cross from all possible direction and the sound energy density is very nearly uniform throughout the field. In a large, irregular room it is possible, in principle, to have a diffuse sound

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field, one that consists of a superposition of sound waves travelling in all direction with equal probability. This characteristic ensues that the average energy density is the same at all points. If this was actually so, there would be no net flow of power in any direction. It is possible to build such a field by superposition of an infinite number of freely propagating plane waves, such that all direction of propagation are equally probable and the phase relations of the waves are random. This "construction" is usually called the Plane-Wave Model (PWM). From a conceptual point of view this definition is quite adequate but it gives no practical help or criterion to determine the degree of diffuseness of an acoustical field in room at low frequencies [1-3].

Hence, a diffuse sound field never actually exists because there is always a net flow of power away from the places where the energy is ultimately absorbed. If a source with a narrow band of many frequencies excites a large irregular room, the fluctuations in the sound pressure as a microphone is moved through the room can be observed, just as it would for the simple case of a small regular room. In this case the sound field is not diffuse [1-4].

Many experimental and theoretical methods have been proposed for evaluating diffuseness of the acoustical field in a reverberant room. This characterisation of a sound field has been widely investigated in the last five decades. Even if the concept of diffuse field is well understood, there are few practical and meaningful theoretical tools to characterise the sound field in a room. Many definitions of diffuseness and descriptors used to quantify the diffuseness are often misused, if not misunderstood. There are many questions: What are the frequency limits or what is the spatial extent of such a field? What is the "quality" of the diffuseness? What will be the effect of the measurement system and the other objection in room on the diffuseness?

Most of the studies have been dedicated to high frequencies and there is no precise indication on how to characterise sound field at lower frequencies. Generally speaking, the well-known "Shchroeder frequency" limit is given but it gives no clear insight on number of modes per band required to achieve an adequate diffusion [1-4].

This paper tries to give some answers to these questions for an empty rectangular room. The main objective is to provide to acousticians concrete criterion of diffuseness using the cross-correlation and the spatial uniformity of the pressure field, as the most commonly used techniques, which are suggested by Nelisse and Nicolas [4]. Its have been chosen because its can be easily measured and, for simple rooms, easily computed. An analytical modal approach is presented to compute these descriptors. The results are compared with the "perfect" diffuse field model. Comparisons and analysis between theoretical predictions and measurements are also presented.

2. CALCULATION OF THE SOUND PRESSURE LEVEL

The wave theory makes it possible to analyse the sound field in rooms in case when the approximate methods do not give satisfactory results. For instance, such a situation occurs when the sound field is excited in the room of regular shape at the low frequencies. In this case, a commonly used mean of predicting the sound field generated by a time-harmonic sound source is the normal-mode solution of the acoustic wave equation. The sound pressure expression is obtained in the form of a triple infinite series [1-5]. The expression shows that the steady state pressure at a point in room is the sum of waves corresponding to the different normal modes of standing waves. However, the total pressure distribution calculations are inconvenient because of the slow-convergent nature of the triple infinite series. A convenient transformation from the three-dimensional differential equation to the one-dimensional one is applied here for the calculation of the sound pressure. The final solution is obtained in the form of double infinite sum [5].

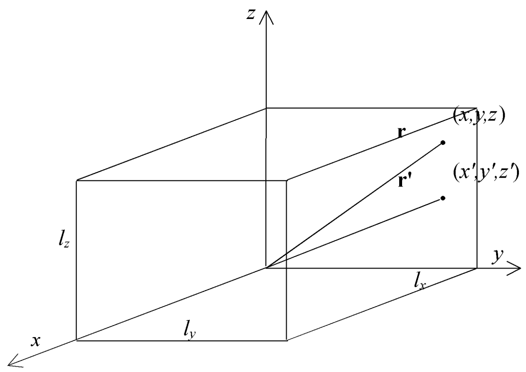

Fig. 1. Geometrical representation of an rectangular room

A rigid-walled rectangular room (dimensions l_x, l_y, l_z) is considered, as shown in Fig. 1. To compute the sound pressure generated in an empty reverberant room that exhibits strong modal behaviour, a previously mentioned efficient solution has been used. With this expansion the sound pressure reads

$$p(\mathbf{r}, f) = \frac{j\omega\rho \exp(j\omega t)}{l_x l_y} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \lambda_{mn} \frac{\Psi_x \Psi_y \Psi_z}{\gamma \sinh(\gamma l_z)}, \qquad (1)$$

where

$$\Psi_x = \cos(m\pi x / l_x) \cos(m\pi x' / l_x),$$

$$\Psi_y = \cos(n\pi y / l_y) \cos(n\pi y' / l_y),$$

$$\Psi_z = \begin{cases} \cosh[\gamma(l_z - z'] \cosh(\gamma z), & z < z' \\ \cosh[\gamma(l_z - z)] \cosh(\gamma z'), & z > z' \end{cases},$$

$$\gamma = \sqrt{\left(\frac{m\pi}{l_x}\right)^2 + \left(\frac{n\pi}{l_y}\right)^2 - \left(\frac{\omega}{c}\right)^2 \frac{1}{1 + j\overline{\alpha}}},$$

$$\lambda_{mn} = \begin{cases} 1, & m = 0, n = 0 \\ 2, & m = 0, n \neq 0 \text{ ili } m \neq 0, n = 0 \\ 4, & m \neq 0, n \neq 0 \end{cases},$$

c is velocity of the sound, ρ is density of the medium, $\overline{\alpha}$ mean absorption coefficient of the room and λ_{nm} corresponding different normal modes of standing waves.

If one is interested in a frequency band response one has:

$$p_{\Delta f}(\mathbf{r}_i, f) = \sum_{f'=f_1}^{f_2} p(\mathbf{r}, f')\delta f', \qquad (2)$$

with $f = (f_2 + f_1)/2$, the center of the band. In this case one-third octave bands have been used. It has been verified that a sufficient number of frequency points have been chosen to ensure convergence. In the same way, for all room modes in each of the one third-octave band used for computation of $p(f)$ for considered point. The sound pressure level $L(\mathbf{r}_i, f)$ is defined by

$$L_{\Delta f}(\mathbf{r}_i, f) = 10\log\left(\sum_{f'=f_1}^{f_2} \left|p(\mathbf{r}, f')\right|^2 \delta f'\right) \qquad (3)$$

If one considers N points for which the sound pressure levels $L(\mathbf{r}_i, f)$ are computed in the central volume of the room, one defines the standard deviation σ to be given by

$$\sigma(f) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}[L(\mathbf{r}_i, f) - \overline{L}(f)]^2}\ , \qquad (4)$$

where $\overline{L}(f)$ is the mean sound-pressure level in this volume.

3. Correlation Function

The spatial cross-correlation function of the sound pressure field between two points in room, or simply the Correlation Function (CF), is defined by

$$C(\mathbf{r}, \mathbf{r}') = \frac{\overline{p(\mathbf{r})p(r')}}{\sqrt{\overline{p^2(\mathbf{r})p^2(\mathbf{r}')}}}\ , \qquad (5)$$

where the acoustic pressure at two points which position are defined by vectors $\mathbf{r} = \vec{r}\,(x,y,z)$ and $\mathbf{r}' = \vec{r}\,'(x',y',z')$, given by $p(\mathbf{r})$ and $p(\mathbf{r}')$, Fig. 1.,while the horizontal bar means time average [3].

If a harmonic state is assumed for the sound pressure field, the CF for plane wave model becomes

$$C(kR) = \mathrm{Re}\left(\frac{p(\mathbf{r})p^*(\mathbf{r}')}{\sqrt{|p(\mathbf{r})|^2|p(\mathbf{r}')|^2}}\right) = \mathrm{Re}\left(e^{-jkR\cos\theta}\right) = \cos(kR\cos\theta)\,. \qquad (6)$$

This function only depends on distance R between two observed points. For diffuse field an integration over all the possible directions of the plane waves has to be performed. It gives the following result for three dimensions in room

$$C(kR) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\cos(kR\cos\theta)\sin\varphi\; d\theta d\varphi = \frac{\sin(kR)}{kR}\,. \qquad (7)$$

If one is interested in a frequency-band response, one must integrate the correlation function over the frequency in the bandwidth $\Delta f = f_2 - f_1$, so that the CF is

$$C_{\Delta f}(kR) = \frac{1}{\Delta k}\int_{k_1}^{k_2} \frac{\sin(kR)}{kR} dk. \tag{8}$$

It can be shown after few manipulations for typical finite frequency bands [4] that it is given by

$$C_{\Delta f}(kR) \approx \frac{\sin(\Delta kR/2)}{\Delta kR/2}\,\frac{\sin(kR)}{kR}\,. \tag{9}$$

where k is defined by $k = (k_1 + k_2)/2$. It then gives a simple closed from for the influence of the $\Delta k/k$ correction.

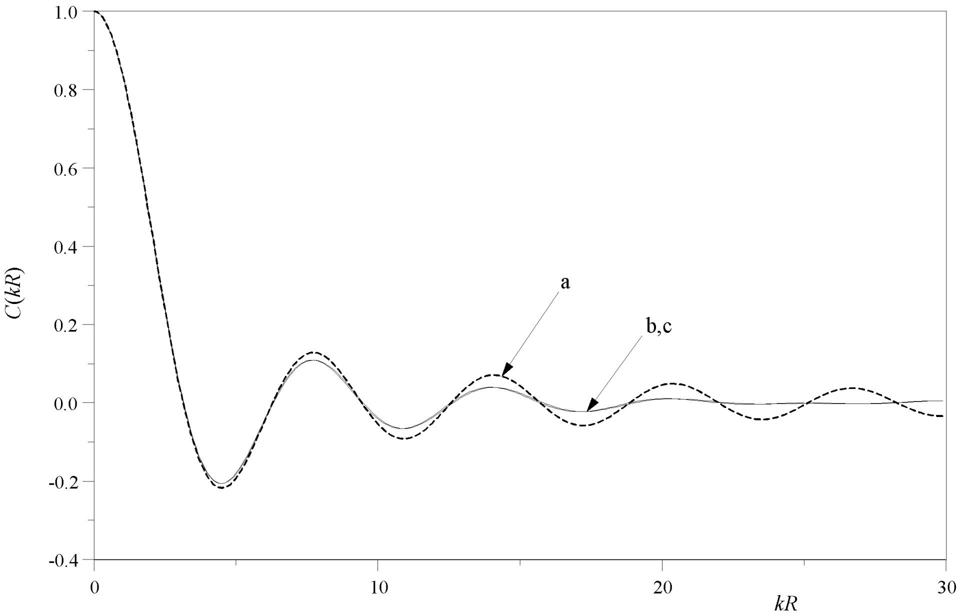

Fig. 2. Correlation function from Eq.(7) - a; Eq.(9) - b; and Eq.(10) - c

To validate the relation for $C\Delta f(kR)$ (Eq.9), the frequency integrated (7) has been computed numerically. Fig. 2. presents the results from the simple form $\sin(kR)/(kR)$, from (9), and from the numerical integration of (7) as a function of kR. The Figure shows that the PWM, which CF leads to

$$C(\mathbf{r},\mathbf{r}') = \frac{\sin(kR)}{kR}\,, \tag{10}$$

is a very good approximation for finite frequency band correlation function. The importance of the $\Delta k/k$ correction to the usual $\sin(kR)/(kR)$ from clearly appears for high values of kR. This function has been widely studied theoretically and experimentally.

Generally, the PWM prediction sin(kR)/kR is obtained if a narrowband containing enough room modes is used.

To compute the Correlation Function Prediction (CFP) in an empty reverberant room can be used

$$C_{\Delta f}(kR) = \frac{\mathrm{Re}\sum_{f'=f_1}^{f_2} p(\mathbf{r})p^*(\mathbf{r}')}{\sqrt{\sum_{f'=f_1}^{f_2}\left|p(\mathbf{r})\right|^2 \sum_{f'=f_1}^{f_2}\left|p(\mathbf{r}')\right|^2}}, \tag{11}$$

where the acoustic pressure at two points which position are defined by vectors $\mathbf{r} = \vec{\mathbf{r}}\ (x,y,z)$ and $\mathbf{r}' = \vec{\mathbf{r}}'\ (x',y',z')$, given by $p(\mathbf{r})$ and $p(\mathbf{r}')$, Figure 1., is given by Eq. (2). If there is only one room mode in the bandwidth or a monochromatic source is used, one can show that the CFP becomes 0 or ± 1. It can be seen that the prediction mode gives satisfactory results, in particular for low values of kR.

4. Experimental procedure

A 3.93x4.14x4.11m^3 empty room with a mean measured absorption coefficient 0,02 has been used to compute the CF [6]. Table 1. gives the number of room modes in each of the one third-octave band used for computation of $p(f)$.

Table 1. Number of modes in the one-third-octave bands for the 3.93×4.14×4.11 m^3 room

$f_1 - f_2$	Number of modes/band
90–112	9
112–140	17
140–180	27
180–224	51
224–280	95
280–355	188
355–450	371
450–560	663

The CFP by Eq.(11) for the 254.5 - 254.6 Hz frequency band as a function of kR is shown at Figure 3. For this room only one mode has its natural frequency in the frequency band of interest (f = 254.54 Hz). All the different points used to compute the correlation have been chosen in order to avoid the effect of the walls on the acoustic pressure. The PWM predictions are shown in this Figure, too. A value of ±1 is correctly obtained for the CF with this technique. Similar results have been obtained for higher frequencies. It suggests that diffuse field cannot be established in a room with a monochromatic source, a result not predicted by the plane-wave model.

Fig. 4. shows the CFP for the same room, but a frequency band ranging from the 90-112 Hz, 112-140 Hz, 140-180 Hz, 180-225 Hz, 225-280 Hz, 280-355 Hz and 355-

450 Hz. For the test room used here, there exist 9, 17, 27, 51, 95, 188 and 371 modes in the above mention bandwidth, respectively. These results indicate that a diffuse field can be obtained for a measurement bandwidth containing enough room modes.

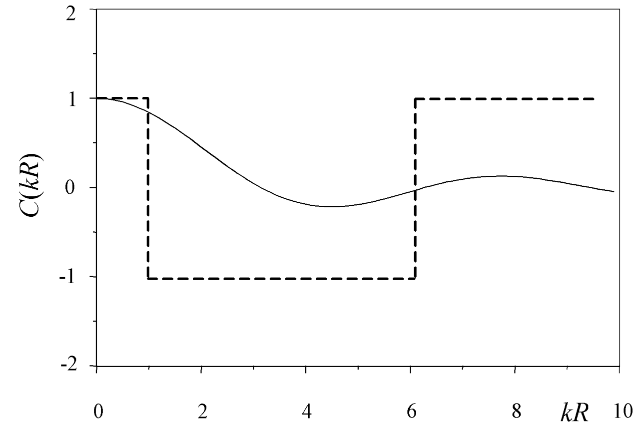

Fig. 3. Correlation function for the 254,5 - 254,6 Hz band from Eq.(11) (---) and Eq. (10) (—)

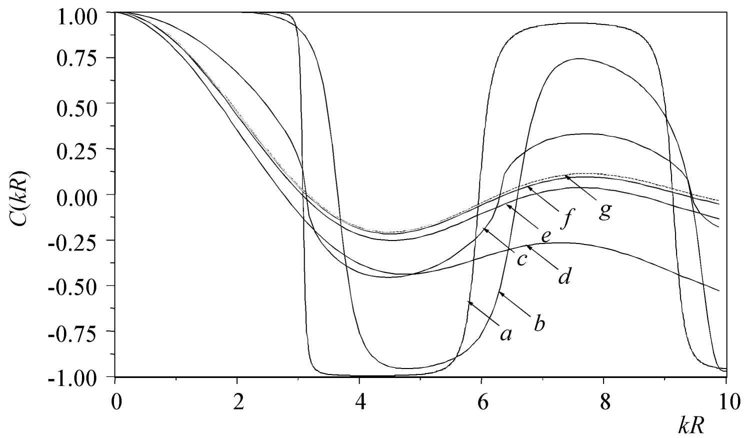

Fig. 4. Correlation function for the 90-112 Hz - *a*; 112-140 Hz - *b*; 140-180 Hz - *c*; 180-225 Hz - *d*; 225-280 Hz - *e*; 280-355 Hz - *f*; 355-450 Hz - *g* and for MWP (…)

This Figure suggests that the field is not perfectly diffuse (in the sense of the PWM) for frequencies below band 112-140 Hz. However, it can be seen that the CFP is less than 0.5 for band 140-180 Hz and 180-225 Hz, but less than 0.25 for band 225-280 Hz and at all. However, there was no indication about the least permissible number of room modes to achieve adequate diffuseness. To do so one can use the spatial uniformity as an indicator. Someone can notice that the CF is not very sensitive to deviations from perfect diffuseness.

Because of that, the descriptor standard deviation for measured sound pressure level for several points in this room and its dependance on frequency for one third octave bands is shown in Fig. 5. The (4) has been used for the computation of the standard deviation σ in the same room at the same points, and these values are also shown in this Figure. It has been accepted that an acoustic field in a qualified reverberation room exhibits adequate diffuseness if the standard deviation remains under 1.5 dB [4]. Naturally, the plane-wave model gives $\sigma = 0$ dB.

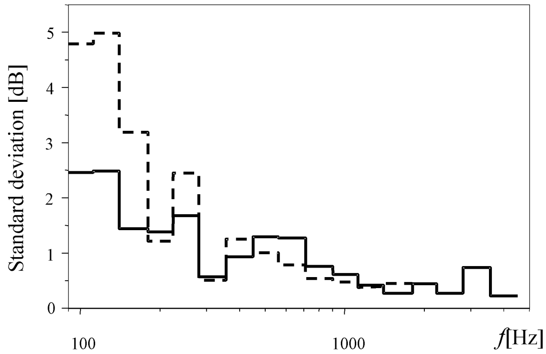

Fig. 5. Standard deviation for measured and computed (---) sound pressure level as a function of the frequency for one-third-octave bands

At the lowest frequencies there is significant discrepancy between shown results and permissible ones (1.5 dB). It is observed that this value becomes less than 1.5 dB at the 140 − 180 Hz band containing 27 modes. From these results, a tentative criterion can be stated: A diffuse field can be established in a rectangular room if there are at least 20 − 30 modes in the measurement bandwidth. For a bandwidth, which contains few modes, there is no adequate diffuseness, in particular at a given frequency (monochromatic source).

Similar simulations have been performed for this room, but for different frequency bands, to examine if a possible "critical" frequency band, above which the field is assumed to be diffuse, can be estimated. It can be shown that the "Schroeder frequency" is given by

$$f_s \approx \sqrt[3]{\frac{ac^3}{4\pi\bar{\alpha}V}}\ , \tag{10}$$

where a is a modal overlap. Schroeder has proposed a modal overlap $a = 3$. For the test room it obtains $f_s = 193$ Hz. This is in great accordance with the "critical" frequency f_c found above. However, it gives a good approximation for the "critical" frequency and shows how to present model can be related to the well-known "Schroeder frequency".

From the above results it becomes clear that a frequency limit for which the diffuse field exists can be estimated under certain conditions. In the limit where there are enough modes in the measurement bands the usual "Schroeder frequency" relation can be applied to estimate the diffuse field frequency limit. For very narrow bands the frequency limit will be higher than for broadbands and no clear relation can be given for this "critical frequency". This frequency depends on the room dimensions, the frequency, and the width of the band. With this information, the spatial uniformity is a practical tool to determine from which band the field can be considered as a diffuse, in terms of modal density. Moreover, the use of the CF assures whether or not the field is really adequately diffuse. It is thought that the two descriptors have to be used together to ensure the quality of the diffuseness. The plane-wave model gives a good example of a case where only the use of the spatial uniformity is not sufficient to characterize a sound field.

5. CONCLUSION

By the use of the correlation function, this paper has presented a simple and convenient model to characterize the sound field in a rectangular room. The advantage of the proposed approach is ability to simply implement it, both experimentally and analytically. Results have shown that a diffuse field can be established in a room if there is at least 20–30 room modes in the measurement band. For a bandwidth, which contains few modes, there is no adequate difusness.

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KRITERIJUM ZA DIFUZNOST ZVUČNOG POLJA U PROSTORIJI

Miroslava A. Milošević, Nenad D. Milošević, Milan S. Milošević

U ovom radu je prikazan način za određivanje difuznosti zvučnog polja na osnovu korelecione funkcije i prostorne raspodele nivoa zvučnog pritiska u pravilnoj prostoriji. Rezultati su najpre upoređeni sa onim koji bi trebalo da se dobiju za savršeno difuzno polje, a onda je izvršena analiza teorijskih i izmerenih vrednosti za konkretnu praznu reverberantnu prostoriju. Zaključeno je da postoji mogućnost da se na osnovu broja neophodnih pobuđenih sopstvenih frekvencija u prostoriji, kao osnovnog kriterijuma, odredi difuznost zvučnog polja.