ANALYSIS OF BEAM AND CRANK MECHANISM’S ACTUATOR BRAKING IN SAFE POSITION

UDC: 621.979.1

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Abstract. This paper deals with braking process and stopping beam and crank press actuator in safe position under unexpected double travel of presser. Movement of beam and crank mechanism is described by Lagrange equation of the second order. In order to resolve the problem and to define braking angle, all the parameters that influence change of system's moment of inertia have been taken into account.

Key words: Beam and crank press, safety operating, braking angle, beam and crank mechanism

1. INTRODUCTION

Under single action regime of mechanical crank presses, there is possibility to occur "double travel" of presser that frequently causes injuries of operator's hands. Various protection systems have been developed to find solution for this problem, and one of those is the concept of two frictional breaks - one principal and another additional safety brake. Principal brake stops and holds presser in upper dead center after every single work cycle until operator gives command for another action [3].

Additional, safety brake is used to stop press in safe position in case of principal brake failure. It could be concluded that safe performance of mechanical crank presses (eccentric and similar) mainly depend on synchronized performance of clutch, principal brake and safety brake.

Presses of this kind are usually equipped with friction clutch and brake in single chassis and this solution is correct since their synchronization is needed to stop actuator in safe position. The problem is to define braking time and angle of beam and crank mechanism. Under regular performance, presser stops in upper dead center. This performance is realized through one single, double-hand actuation regime. Synchronized operation of clutch and brake via micro-switch enables that presser stops at safe position.
after each working cycle.

However, if long time operation and wear affect synchronically, brake does not activate in due course after clutch action, and unexpected double feed of presser occur. Operator does not expect this dangerous movement, and following usual working rhythm he move hands toward dangerous space below presser in order to put new peace of material in die (figure 1).

![Fig. 1. Hand position under the presser](image)

Designer should take in account this problem while making concept of the machine itself. The problem is how to chose optimal brake that is capable to stop press actuator in unexpected position. This paper aims to highlight the problem which was rather neglected among designers until now. Analysis disposed in the paper regards calculation of time and braking angle of beam and crank mechanism for any crank position. It is necessary to take into account all moving masses that are of importance for possible positions of the presser. Discussed problem concerns occurrence of double feed of the presser and possibilities to stop it in safe position, thus protecting operator.

In case of "double travel" occurrence, it is necessary that brake activates automatically, and stop the actuator in safe position (figure 1).

2. BRAKING PROCESS OF BEAM AND CRANK MECHANISM OF ECCENTRIC PRESS

Beam and crank mechanism of the press possesses single degree of freedom. Eccenter transforms rotation of the main shaft to linear movement of the presser, as shown in the figure 2.

Braking process of the mechanism shown in the figure 2 is non-linear problem. Angle and time to brake mechanism depend on moment of inertia, moving masses and achieved braking moment. The process itself could be divided in two phases: in the first phase the control mechanism is being activated (system response), and in second phase begins effective braking.

Mechanism activation time is constant for each particular press and it depends on response of control elements (system response) - for some presses, response time is up to
120 ms. In order to shorten travel of the presser and to stop it in safe position, it is necessary both phases of braking to last as short as possible. It could be achieved if the press is equipped with fast-response switching devices, and brake possesses proportionally large braking moment.

![Diagram of beam and crank mechanism](image)

**Fig. 2. Scheme of beam and crank mechanism of eccentric press**

Time response of the system switching devices should be diminished up to a point, but too large braking moment could provoke inertial forces that jeopardize stability of the press and installation. This fact one should keep in mind while selecting principal and safety brake that should have optimal size, in accordance with other press elements.

To find solution for braking at occurrence of "double travel" of the presser of a mechanical crank press, typical positions of crank mechanism should be defined as shown in the figure 3.

Shown analyses reveals three theoretical situations:
- when angle $\phi_d > \phi_k$, safety brake is stronger than principal brake, since it possesses stronger braking moment
- when $\phi_d > \phi_b$, both brakes have the same moments $M_{DG} = M_{DK}$
- when $\phi_d > \phi_k$, safety brake has inferior characteristics than principal brake, i.e. $M_{DG} < M_{DK}$

Taking into account that safety brake activates later than principal one, it should have as large braking moment as possible in order to decouple moving masses before the angle reaches value $\phi_k$.  

**Symbol Key:**
- **UMT** - upper dead center
- **SMT** - lower dead center
- $\omega_p$ - angular speed
- $\phi$ - angle between crank and direction of presser movement
- $R$ - radius of beam and crank mechanism
- $L$ - length of piston rod
- $\lambda = R/L$ - characteristic of beam and crank mechanism
- $m_0$ - mass of the main shaft
- $m_1$ - piston rod mass
- $m_2$ - mass of the presser
There are two opposite requirements considering selection of suitable safety brake. One states that brake should have largest possible braking moment, and another approach suggests that the value should be as small as possible. Therefore, selection of safety brake is in essence search for compromise.

First step in selection of proper safety brake would be defining safe travel of actuator \( h_b \) and corresponding angle \( \phi_B \). The value \( h_b \) denotes how long is tolerable movement of presser (from upper dead end of the mechanism) so that operator’s hands are not jeopardized.

The value of this clearance depends on entire feed of the press actuator \( (H = 2R) \), and on design of the tool that is mounted on the press and working platform.

For crank mechanism, travel of the presser is given by equation:

\[
h = R(1 - \cos \phi) + \lambda/4(1 - \cos 2\phi)
\]

Upper equation could be used to calculate angle \( \phi_B \), starting with maximal allowed travel that ensures safe performance, and using relations \( (h_b = H - h) \), i.e. \( (\phi_B = \pi - \phi) \).

To find solution for the problem analyzed in this paper (i.e. "double travel" occurrence), it is necessary to stop press actuator in safe position \( h_b \) and angle \( \phi_B \). Safety brake should have larger braking moment then principal brake \( M_{DG} > M_{OK} \), since moments of inertia of moving masses increase significantly for certain values of crank angles.
3. DYNAMIC EQUATION OF BEAM AND CRANK MECHANISM

Movement of beam and crank mechanism could be described by Larange’s equation of second order. This equation is formed by expressing entire kinetical energy of the system as a function of generalized coordinate of angle $\varphi$ (angle that crank forms with direction of presser movement).

$$\frac{d}{dt} \left( \frac{\partial E_K}{\partial \dot{\varphi}} \right) - \frac{\partial E_K}{\partial \varphi} = Q$$

(1)

where:
- kinetic energy of beam and crank mechanism
- entire work caused by change of potential energy

To find solution of equation (1), it is necessary to calculate moments of inertia of moving masses as well as entire work gain obtained through friction and moving masses of the mechanism. Taking those values in consideration, Lagrange’s equation could be written as follows:

$$J(\varphi) \frac{d\omega}{dt} + \frac{1}{2} \omega^2 \frac{dJ(\varphi)}{d\varphi} = Q(\varphi)$$

(2)

where:

$$Q(\varphi) = \pm M_k + m_1gR\left( \sin \varphi - \frac{\lambda}{4} \sin^2 \varphi \right) + m_2gR\left( \sin \varphi - \frac{\lambda}{2} \sin^2 \varphi \right)$$

(3)

Reduced moment of inertia for rotating shaft with all moving masses of crank mechanism included is represented by equation:

$$J(\varphi) = J_0 + m_1R^2 \left[ \sin \varphi - \frac{\lambda}{4} \sin 2\varphi \right]^2 + \frac{1}{4} \cos \varphi +$$

$$+ m_2R^2 \left[ \sin \varphi - \frac{\lambda}{2} \frac{\sin^2 \varphi}{\sqrt{1 - \lambda^2 \sin^2 \varphi}} \right]^2 + J_k \frac{\cos^2 \varphi}{\left( 1 - \frac{\lambda^2}{2} \sin^2 \varphi \right)^2}$$

(4)

where:
- $\varphi_0$ - mean value of shaft’s moment of inertia
- $\varphi_k$ - piston rod moment of inertia

Braking time and angle, and so position of press actuator could be calculated by resolving differential equation (2) with the following initial values:

$$\omega|_{t = t_p} = \omega_p \quad \varphi|_{t = t_p} = \varphi_p$$

(5)

and boundary values:

$$\omega|_{t = t_k} = 0 \quad \varphi|_{t = t_k} = \varphi_k$$

(6)

Differential equation (2) is Riccati-type equation, and in general case it’s solution can not be derived from quadratures, but numerically as it has been done in this paper.
4. APPROXIMATE DETERMINATION OF BEAM AND CRANK MECHANISM BRAKING ANGLE

When analyzing this problem in literature (1,2,6), authors presuppose that the moment of inertia is constant \( J = \text{const.} \), and independent of crank angle \( \varphi \). Based on this presumption, results of acceptable accuracy are obtained only around upper dead end of the presser \( (0^\circ < \varphi < 10^\circ) \).

For example, according to literature (6), braking angle is calculated by expression:

\[
\varphi_k = \frac{1}{2} \omega^2 \cdot \frac{J}{M_k}
\]

(7)

where it has been accepted that the moment of inertia is constant \( J = \text{const.} \) regardless position of the crank.

**First level approximation**

If we assume that moment of inertia \( J(\varphi) \), changes insignificantly for any angle of crank mechanism, first derivation could be neglected, so:

\[
J(\varphi) \frac{d\omega}{dt} = Q(\varphi)
\]

(8)

and with separated variables:

\[
\omega \, d\omega = \frac{Q(\varphi)}{J(\varphi)} \, d\varphi
\]

(9)

After integration, expression is:

\[
\frac{d\varphi}{dt} = \sqrt{2 \left( \frac{Q(\varphi)}{J(\varphi)} \right) \, d\varphi + C}
\]

(10)

Further procedure for angle determination depends on possibilities to solve integral from upper equation (10).

**Second level approximation**

Discussed problem becomes much more simple if assumption is that the moment of inertia is constant \( J = \text{const.} \) and overall work of friction that comprises braking moment \( M_k \) is also constant \( Q(\varphi) = -M_k = \text{const.} \), where:

\[
\frac{d^2\varphi}{dt^2} = -\frac{M_k}{J_0} = \text{const.}
\]

(11)

and where: \( J_0 = m_0 r^2 / 2 \) - mean value moment of inertia of the main shaft

\( m_0 \) - mass of the main shaft

\( r \) - mean value of the main shaft radius

After two integration, and taking in consideration initial values (5) and (6), very practical expression for angle calculation could be derived:

\[
\varphi_k = \varphi_0 + \omega_0 t_k - \frac{M_k}{2J_0} \cdot t_k^2
\]

(12)
Braking time could be calculated if initial angular velocity ($\omega_p$) is known, as well as relation between moment of inertia and braking moment ($J_0/M_k$):

$$t_k = \omega_p \frac{J_0}{M_k}$$

(13)

If braking time expression (13) applies in equation (12), then:

$$\varphi_k = \varphi_p + \frac{1}{2} \omega_p^2 \frac{J_0}{M_k}$$

(14)

Equation (14) provides braking angle that is accurate enough for usual engineering practice. It stands for position of the presser around upper dead center (UMT). Out from this position, equation (14) does not provide satisfactory results, and upper approximation is therefore not valid. According to this, further analysis would be done without above mentioned approximations.

5. DETERMINATION OF BRAKING ANGLE FOR ANY POSITION OF BEAM AND CRANK MECHANISM

Common praxis was to apply mentioned approximations in order to calculate braking angle of press crank mechanism. Those simplifications are allowed if presser stops in upper dead and after completion of working travel. In all other cases simplification does not provide satisfactory results.

In order to provide accurate results for discussed problem, comprehensive equation of beam and crank motion has been derived:

$$J(\varphi) \frac{d\omega}{dt} + \frac{1}{2} \omega^2 \frac{dJ(\varphi)}{d\varphi} = Q(\varphi, \omega, t)$$

(15)

Upper equation comprises all masses of moving parts (main shaft, piston rod, presser with the tool, etc.). The equation is non-linear, and numerical method has been applied to solve it’s values.

For the purpose of finding numerical solution, relationships between particular values of the press actuator have been established, so that it is not necessary to know their exact values, but just their dimensionless relations.

Using dimensionless values, i.e. dividing equation (15) by ($J_0$), expression becomes:

$$[j(\varphi) - 1] \frac{d\omega}{dt} + \frac{1}{2} \omega^2 \frac{dj(\varphi)}{d\varphi} = Q(\varphi, \omega, t)q$$

(16)

where: $j(\varphi)$ - relative deviation of reduced moment of inertia $J(\varphi)$ from constant value of main shaft moment of inertia ($J_0$).

$q$ - reciprocal value of main shaft’s moment of inertia

Relative deviation of reduced moment of inertia of the crank mechanism is:

$$j(\varphi) = \frac{J(\varphi)}{J_0} - 1$$

(17)
\[
\begin{align*}
j(\varphi) - 1 &= 2 \left( \frac{R}{r} \right)^2 \cdot \frac{m_1}{m_0} \left( \frac{1}{4} \cos^2 \varphi + \left( \sin \varphi - \frac{\lambda}{4} \sin 2\varphi \right)^2 \right) \\
&+ \frac{1}{12} \left[ 1 + \left( \frac{b}{L} \right)^2 \right] \cos^2 \varphi + \left( \frac{m_2}{m_1} \left( \sin \varphi - \frac{\lambda}{2} \sin 2\varphi \right)^2 \right] \\
\end{align*}
\]

where piston rod of rectangular cross section (width \(b\), length \(L\)) has been taken into account.

Introduction of moment of inertia’s relative deviation \(j(\varphi)\) has practical significance, since it enables analysis of reduced moment of inertia \(J(\varphi)\) knowing only relationship between masses and relationship of dimensions of press actuator, but not their exact values. This approach is important for theoretical research of disposed problem.

For resolving differential equation (16), the method of finite graduations has been applied. According to this method, retardation at braking is in finite time intervals that are approximated by constant value and following equations are derived for the initial conditions of braking (5) and (6):

\[
\begin{align*}
\left( \frac{d\omega}{dt} \right)_{\varphi_{i+1}} &= \frac{Q_j(\varphi) - 1 - \frac{1}{2} \omega_i^2 - \frac{dj_j(\varphi) - 1}{d\varphi}}{1 + j_j(\varphi) - 1} \varphi = \varphi_i; \quad i = 0, 1, 2, \ldots \\
\omega_{i+1} &= \omega_i + \left( \frac{d\omega}{dt} \right)_i \Delta t; \quad i = 0, 1, 2, \ldots \\
\varphi_{i+1} &= \varphi_i + \omega_i \Delta t + \frac{1}{2} \left( \frac{d\omega}{dt} \right)_i \Delta t^2; \quad i = 0, 1, 2, \ldots 
\end{align*}
\]

Indexes in upper equations denote that analyzed values are monitored at the time \(t = i\Delta t\).

Presser braking angle is calculated at various crank starting angles \(\varphi \in [0, \pi]\), various rotation speed of the main shaft \(n \in [20, 100]\), various mass relations: \(m_1/m_0 \in [0;1]\); \(m_2/m_0 \in [0;2]\); and various relations between braking moments \(M_k\) and moments of inertia \((J_0)\) \(M_k/J_0 \in [10,100]\).

Table 1 shows braking angles of press actuator, for the following parameters:

- \(m_1/m_0 = 0.5\); \(m_2/m_0 = 2\); \(R/r = 1\); \(b/L = 0.5\); \(\lambda = R/L = 0.2\)

Table 1. Values of braking angle \((\varphi_k)\) of beam and crank mechanism as a function of starting braking angle and number of presser travel

| Starting braking angle \(\varphi_0\) | Number of travels of the presser per minute \(n[\text{min}^{-1}]\) |
|---|---|---|---|---|---|---|---|
| 0°  | 20 | 40 | 60 | 80 | 100 |
| 0.34 | 1.37 | 3.13 | 5.67 | 9.08 |
| 0.61 | 2.47 | 5.73 | 10.71 | 18.13 |
| 1.58 | 6.83 | 18.58 | slipping |
| 5.74 | slipping |

...
From obtained results it has been noticed that for chosen parameters under certain positions of the crank mechanism slipping occurs at disks of the brake, so that presser doesn’t stop at safe position.

Given analyses raises following question - do relations of masses and other chosen values correspond to real conditions - which is of particular importance for further analyses of the problem.

6. CONCLUSION

Operators of crank eccentric presses are particularly jeopardized if the press actuator can not stop in safe position due to unexpected travel.

Analyze described in this paper highlights safety problem that designers should take in consideration. Disposed problem is related to calculation of braking angle of the press actuator. Lagrange’s equation of second order was applied for the purpose.

Values for the braking angle of crank mechanism are based on choose relations between masses and dimensions of press mechanism elements so that solution of the problem is relatively generalized.

Previous method for calculating braking angle of crank mechanism is valid only for stopping of the presser under normal operation, i.e. around upper dead center (UMT, \( \varphi \in [0, 10^\circ] \)). Braking process in that case could be linearized, while for all other positions of crank and presser in working cycle that process is non-linear. In those cases, approximations applied in classical methods does not provide satisfactory results. Proper solution of this problem requires all the parameters that influence change of moment of inertia while crank angle changes to be taken in consideration.

Determination on rated braking moment of additional, safety brake depend on safe travel \((h_B)\) of the presser, and angle \((\varphi)\). It is possible to obtain maximal value of rated braking moment only by comprehensive analysis of design concept and working regime of the press.

REFERENCES

ANALIZA KOČENJA IZVRŠNOG DELA
KRIVAJNOG MEHANIZMA PRESE
U BEZBEDNOSNOM POLOŽAJU ZA OPSLUŽIOCE

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U radu se razmatra proces kočenja i zaustavljanja izvršnog dela krivajne prese u bezbednom položaju za opslužioce pri nepredviđenom - duplom hodu pritiskivača. Kretanje krivajnog mehanizma prese opisano je Lagrangeovom jednačinom druge vrste. Za rešavanje postavljenog problema i određivanje ugla kočenja uzeti su u obzir svi parametri koji utiču na promenu momenta inercije krivajnog mehanizma.

Ključne reči: krivajne prese, bezbednost opslužilaca, ugao kočenja, krivajni mehanizam