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COMPONENTS OF THE MECHANICAL DISPLACEMENTS OF THE PIEZOCERAMIC RING POINTS

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Abstract. *In this paper the components of the mechanical displacements of the piezoceramic ring points in the radial and thickness direction of oscillation are determined. Mechanical displacements are obtained as the function of ring thickness and the internal-external radius ratio. The influence of any dimension changes of the ring on the front form of its displacements is discussed. Theoretical analyses show that the proposed method is simple, easy to calculate and especially suitable for the study of coupled vibrations.*

Key Words: *Mechanical displacements, Piezoceramic ring, Resonance frequency, Vibrational modes*

1. INTRODUCTION

The initial condition for designing a distinguishable ultrasonic sandwich transducer is full understanding and recognition of its resonant frequencies as well as the waveguide mode [1]. One of all the resonant modes is always predominant in the working resonant frequency. The presence of the unwanted (parasite) modes, which can exist at some frequency close to working frequency, can significantly reduce the quality of ultrasonic cleaning and (or) welding.

In powerful industrial ultrasound applications, for the realization of high-frequency oscillations in thickness (extension or longitudinal) mode, piezoelectric ceramics (piezoceramics) of different shapes as excitation parts of complex and composite ultrasonic transducers are used. It is necessary to be acquainted with their complete piezoelectric characteristics, as well as elasticity properties. These ceramic elements are metalized on their main surfaces in the thickness direction (which is at the same time the direction of their polarization). Metalized surfaces also serve as electric excitation access points. The thickness and radial oscillations of the ring, which are mutually coupled, are provoked.

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The existence of both a thickness and radial resonant mode causes the surface of the piezoceramic transducer usually to oscillate nonuniformly. Therefore it is necessary to perform a comparative analysis of every piezoceramic transducer of a specific geometric shape with other transducers of a similar shape and for analogous application, in order to optimize the emitting surface and make it more efficient [2].

Using the suggested theoretical model, axisymmetrical vibrating modes of a piezoceramic ring as the ultrasonic transducer are analyzed in this paper, and the effects of all its dimension variations [3]. The spatial distribution of the mechanical displacements of the ring is determined by a three-dimensional approach based on wave front spreading along a coordinate axis in the radial and thickness direction, making it possible to predict dynamic behavior of the ring [4].

2. Model of External Behavior of THE Piezoceramic Ring

Piezoceramic elements which are the subject of the analysis in this paper are piezoceramic rings polarized across thickness (that is polarized parallel to the *z*-axis), with an outer radius *a*, inner radius *b*, and thickness 2*h*, and with completely metalized circular-ringed (plane) surfaces supplied with alternating excitation voltage. Dimensions of the ring and polar-cylindrical coordinate system with its origin in the ring centre, are defined in Figure 1(a). Every ring surface is loaded with acoustic impedance Z_i, where v_i and F_i are velocities and forces on those contour surfaces P_i (i=1, 2, 3, 4).

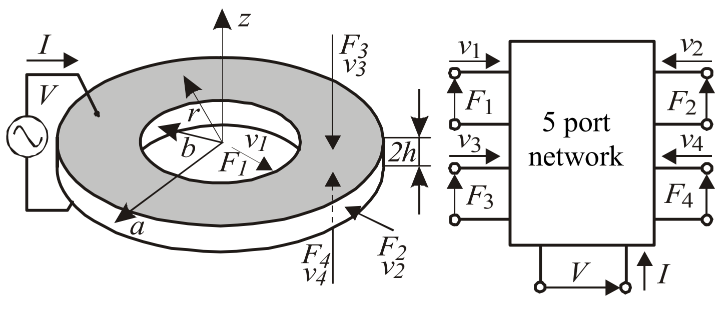

Fig. 1. Loaded piezoceramic ring:
(a) geometry and dimensions; (b) ring as a 5-access network

Piezoceramic materials, and therefore the piezoceramic rings too, are characterized by tensors of their elastic, piezoelectric, and dielectric constants. The most frequently used set of constitutive piezoelectric equations (of the four existing ones) presents tensors of mechanical stresses ***T*** and an electric field ***E*** inside the material in function of the tensor of relative mechanical deformations (dilatations) ***S*** and dielectric displacement ***D***. In case of the piezoceramic ring from Figure 1, which will be analyzed in this paper, components of the electric field E_r and E_θ are equal to zero on two plane surfaces, because they are metalized, and it is also assumed that they are negligible (equal to zero) everywhere inside the material. Besides that, due to axial symmetry, only symmetric (radial and thickness) modes of oscillation are excited. Accordingly, all values are independent of the angle θ, so the displacement u_θ is equal to zero. The proposed model of a piezoceramic ring is obtained with the assumption that coordinate axes r and z are directions of pure (uncoupled) modes of wave propagation, with mechanical displacements in the radial and thickness direction $u_r=u_r(r, t)$ and $u_z=u_z(z, t)$. With these assumptions, the most frequently used set of constitutive equations which describe the oscillation of a piezoceramic ring, is reduced to the following system of equations in a polar-cylindrical coordinate system [1]:

$$\begin{aligned} T_{rr} &= c_{11}^D S_{rr} + c_{12}^D S_{\theta\theta} + c_{13}^D S_{zz} - h_{31} D_z \\ T_{\theta\theta} &= c_{12}^D S_{rr} + c_{11}^D S_{\theta\theta} + c_{13}^D S_{zz} - h_{31} D_z \\ T_{zz} &= c_{13}^D S_{rr} + c_{13}^D S_{\theta\theta} + c_{33}^D S_{zz} - h_{33} D_z \\ E_z &= -h_{31} S_{rr} - h_{31} S_{\theta\theta} - h_{33} S_{zz} + D_z / \varepsilon_{33}^S \end{aligned} \tag{1}$$

where c_{ij}^D are the coefficients of the elastic constant tensor; ε_{33}^S is the dielectric constant of the ring in a compressed state; h_{ij} are elements of the piezoelectric constant tensor (i,j=1, 2, 3).

Relations between components of the relative deformations tensor $\boldsymbol{S}$ and mechanical displacement vector $\boldsymbol{u}$ are the following (S_{pq}=0, if $p\neq q$): $S_{rr} = \partial u_r / \partial r$, $S_{\theta\theta} = u_r / r$, $S_{zz} = \partial u_z / \partial z$.

Partial differential equations that describe the oscillations of a piezoceramic ring in the radial and thickness direction are [1]:

$$\begin{aligned} \frac{\partial T_{rr}}{\partial r} + \frac{T_{rr} - T_{\theta\theta}}{r} &= \rho \frac{\partial^2 u_r}{\partial t^2} \\ \frac{\partial T_{zz}}{\partial z} &= \rho \frac{\partial^2 u_z}{\partial t^2} \end{aligned} \tag{2}$$

By By substituting (1) in (2), one gets partial differential equations of oscillation in the radial and thickness direction in the following form

$$\begin{aligned} c_{11}^D \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) &= \rho \frac{\partial^2 u_r}{\partial t^2} \\ c_{33}^D \frac{\partial^2 u_z}{\partial z^2} &= \rho \frac{\partial^2 u_z}{\partial t^2} \end{aligned} \tag{3}$$

Assuming that the waves are harmonic ($D_z = D_0 e^{j\omega t}$), components of the mechanical displacement in the radial and thickness direction are solutions of previous equations and they are presented through two orthogonal wave functions:

$$u_r = [A\,J_1(k_r r) + B\,Y_1(k_r r)]\,e^{j\omega t}$$
$$u_z = [C\sin(k_z z) + D\cos(k_z z)]\,e^{j\omega t} \tag{4}$$

where $k_r = \omega / v_r$, $k_z = \omega / v_z$, $v_r = \sqrt{c_{11}^D / \rho}$ and $v_z = \sqrt{c_{33}^D / \rho}$ are wave (characteristic, eigenvalue) numbers and phase velocities of two uncoupled waves in the radial and thickness direction, respectively; ω is angular frequency; ρ is density of the piezoceramic; J_1 and Y_1 are Bessel's functions of the first order, of the first and second kind, respectively. The consequence of choosing such orthogonal functions for displacement is that boundary conditions cannot be fulfilled at every point on external surfaces exactly, but only approximately.

3. Numerical Results and Discussion

In order to further analyze the nature of the resonant modes of the piezoceramic rings, it is necessary to determine in detail the components of the mechanical displacements of the piezoceramic ring points in the radial and thickness direction. Displacements u_r and u_z in the r and z direction are already presented through two orthogonal wave functions (4), where, if it is assumed that the piezoceramic ring is mechanically isolated, one should apply boundary conditions on the external surfaces that reflect this strain state without mechanical stress. Since the equations in (4) do not fulfill these boundary conditions, there applies the condition that the integral of the mechanical stress on every external surface is equal to zero, which corresponds to the condition that the resulting force on those contour surfaces is equal to zero, based on which the unknown constants A, B, C, and D can be determined:

$$\int_{P_1} T_{rr}(b)\,dP = 0\,, \qquad \int_{P_2} T_{rr}(a)\,dP = 0\,,$$
$$\int_{P_3} T_{zz}(h)\,dP = 0\,, \qquad \int_{P_4} T_{zz}(-h)\,dP = 0\,. \tag{5}$$

By substituting the constitutive equations in (1) into the boundary conditions defined by the equations in (5) and using the known relations in (2), an equation system is obtained with which unknown constants A, B, C, and D can be determined. By solving this system, with the condition of sinusoidal excitation ($D_z = D_0 e^{j\omega t}$), one gets:

$$A = \frac{k_2 - k_5}{k_4 - k_1} B\,, \quad B = \frac{k_9 - k_3}{k_b} C\,, \quad C = \frac{k_9 k_b - k_a k_{10}}{k_3 k_b - k_a k_8}\,, \quad D = 0\,, \tag{6}$$

where:

$$
\begin{gathered}
k_1 = 2h\,c_{11}^D k_r J_0(k_r b) + 2h\frac{J_1(k_r b)}{b}(c_{12}^D - c_{11}^D)\,,\\
k_2 = 2h\,c_{11}^D k_r Y_0(k_r b) + 2h\frac{Y_1(k_r b)}{b}(c_{12}^D - c_{11}^D)\,,\\
k_3 = 2\,c_{13}^D \sin(k_z h)\,,\\
k_4 = 2h\,c_{11}^D k_r J_0(k_r a) + 2h\frac{J_1(k_r a)}{a}(c_{12}^D - c_{11}^D)\,,\\
k_5 = 2h\,c_{11}^D k_r Y_0(k_r a) + 2h\frac{Y_1(k_r a)}{a}(c_{12}^D - c_{11}^D)\,,\\
k_6 = c_{13}^D[aJ_1(k_r a) - bJ_1(k_r b)]\,,\\
k_7 = c_{13}^D[aY_1(k_r a) - bY_1(k_r b)]\,,\\
k_8 = \frac{a^2 - b^2}{2}c_{33}^D k_z \cos(k_z h)\,,\\
k_9 = 2h\,h_{31} D_0\,,\quad k_{10} = \frac{a^2 - b^2}{2}h_{33} D_0\,,\\
k_a = \frac{k_2 - k_5}{k_4 - k_1}k_1 + k_2\,,\quad k_b = \frac{k_2 - k_5}{k_4 - k_1}k_6 + k_7\,.
\end{gathered}
\tag{7}
$$

Based on previous computations, the first to be determined are radial displacements of the ring points in the function of r/a for five rings with the same outer radius and different inner radii (b/a=0; 0.2; 0.4; 0.6; 0.8), and all that in the case of two different thickness values for the ring ($2h$=5mm and $2h$=20mm). Figure 2 presents these radial displacements of points between the inner and outer cylindrical contour ring surface in the function of r/a, where by comparing corresponding graphs one may see the influence of the increase of the inner ring opening and its thickness on the radial displacement shape for the first four resonant modes.

The presented mechanical displacements are normalized with radial displacement of the points on the internal cylindrical surface ($r=b$) for the first mode of the PZT8 ring with dimensions of $2a$=38mm, $2b$=15mm, $2h$=5mm [5], while the distances r in the radial direction are normalized with an outer radius a. If, for example, one observes the first few lowest modes of this ring (b/a=0.4), one may notice that for the first and the third radial mode the internal and external cylindrical surfaces move in the same directions, while for the second mode, which is also radial, the observed surfaces move in opposite directions (which means that for this resonance case with a value r/a=0.7 there exists a circular line that represents the wave node, for which the displacement is u_r=0), etc. The directions of movement of the cited surfaces are presented in appropriate graphs. With the increase of the inner radius (that is the increase of b/a), there are linearized displacements of the first and the second radial mode of the ceramic element with a thickness of 5mm and all four modes of ceramic elements with a thickness of 20mm, while in other cases displacements mostly keep the classic theoretical form of Bessel’s functions.

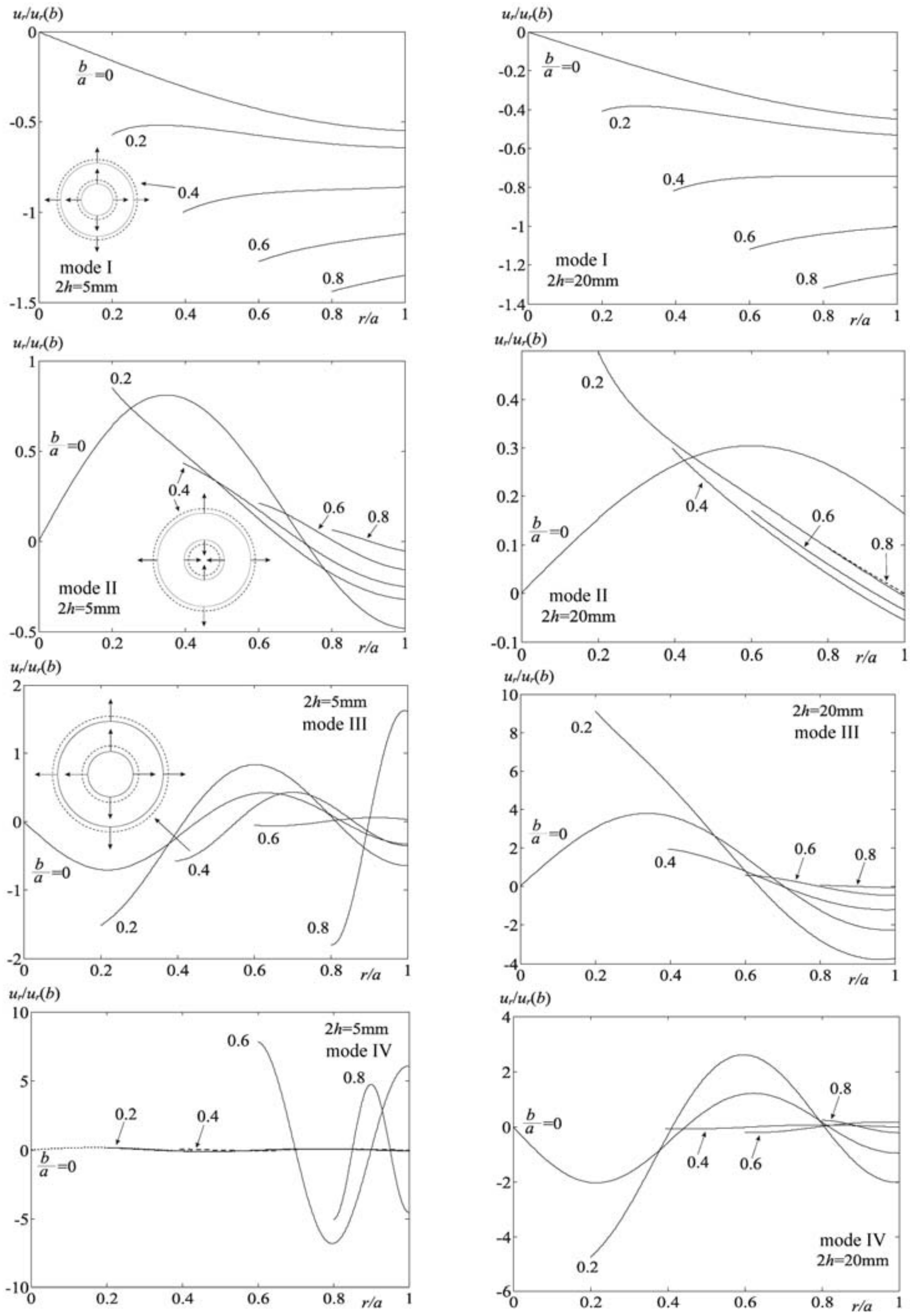

Fig. 2. Normalized radial displacements of the points between cylindrical surfaces

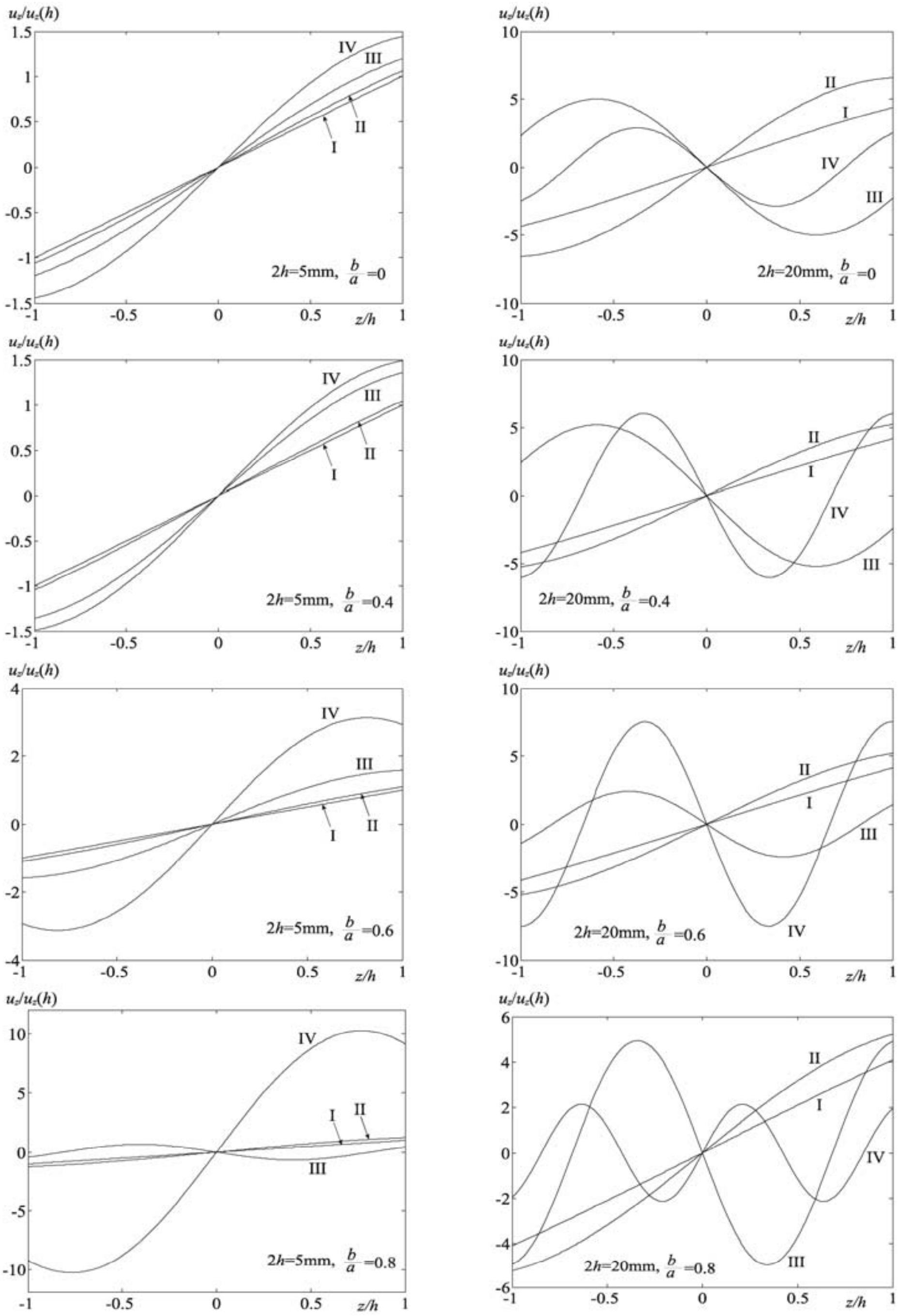

Fig. 3. Normalized thickness displacements of the points between circular-ringed surfaces

Figure 3 shows the thickness displacements of the points between circular-ringed boundary surfaces for the first four resonant modes, for the rings with the same outer radius and different inner radii (*b/a*=0; 0.4; 0.6; 0.8), and with the same length values as in the previous analysis of radial displacements (2*h*=5mm and 2*h*=20mm). Here too the mechanical displacements of the points in the thickness direction are normalized with the thickness displacement of the points on the circular-ringed surface (*z*=*h*) for the first mode of the PZT8 ring with dimensions 2*a*=38mm, 2*b*=15mm, 2*h*=5mm, while the distances *z* in the thickness direction are normalized with one half of the thickness (*h*). It is obvious in all the presented cases that the plain, metalized surfaces always move in opposite directions. As expected, ceramic thickness has the greatest influence on the thickness displacement appearance, while the influence of the inner opening exists, and it is greater with thicker ceramics, while with thinner ceramics it exists only for larger inner diameters. With thicker ceramics, cylindrical lateral boundary surfaces also increase, so the influence of the radial oscillations on thickness displacements due to the presence of a greater lateral load is also greater.

4. CONCLUSION

The oscillatory behavior of piezoceramic rings is essentially different in respect to piezoceramic discs with the same outer diameter and thickness. In this paper we determined the mechanical displacements of a piezoceramic ring in the thickness and radial directions. This method can serve for optimal selection (determining) of transducer shape depending on its dimensions, i.e. thickness, inner and outer diameter ratio, in order to design a transducer with the best vibration characteristics. Spatial displacement distribution of the PZT8 ceramic ring is defined to determine the physical nature of every resonant mode, or in other words, to successfully identify all the radial and thickness resonant modes. The derived mechanical displacements are of the Bessel function shape in almost all the cases.

On the basis of this analysis, the optimal thickness and diameter ratio of the ring can be determined in order to achieve an adequate performance, i.e. it is possible to predict and achieve more complex displacements by varying the ring dimensions, making it possible to design more sophisticated ultrasonic sandwich transducers.

Unlike the complete ultrasonic sandwich transducer [4], only piezoceramic rings are analyzed in this case, and because of the electric excitation of any metalized surfaces with suitable electrodes, which represents the mechanical load for the ring, only the measurement of displacement in the radial direction can be done with sufficient accuracy. These measurements, derived using an analog sensor system with an optic fiber, will be the subject of the author's further interest in the future.

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KOMPONENTE MEHANIČKIH POMERAJA TAČAKA PIEZOKERAMIČKOG PRSTENA

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U ovom radu određene su komponente mehaničkih pomeraja tačaka piezokeramičkog prstena u radijalnom i debljinskom pravcu oscilovanja. Mehanički pomeraji su dobijeni u funkciji debljine prstena, kao i odnosa unutrašnjeg i spoljašnjeg prečnika. Diskutovan je uticaj promene dimenzija prstena na oblik fronta njegovih pomeraja. Teorijske analize pokazuju da je predloženi metod jednostavan, lak za proračune i posebno pogodan za analizu spregnutih vibracija.

Ključne reči: *Mehanički pomeraji, Piezokeramički prsten, Rezonantna frekvencija, Oscilatorni mod*