FACTA UNIVERSITATIS
Series: **Working and Living Environmental Protection** Vol. 2, N° 4, 2004, pp. 293 - 300

DESIGN OF ULTRASONIC TRANSDUCERS BY MEANS OF THE APPARENT ELASTICITY METHOD

UDC 539.4.011

Dragan D. Mančić, Milan Dj. Radmanović

Faculty of Electronic Engineering, University of Niš, Serbia and Montenegro
E-mail: dmancic@elfak.ni.ac.yu; radmanovic@elfak.ni.ac.yu

Abstract. *In this paper, we applied the equivalent elastic constants derived by the apparent elasticity method. We studied the coupled vibrations of sandwich transducers used in high power ultrasonic, obtaining the frequency equation for the transducer design and calculation of its resonance frequency. The transducer consists of metal head and metal tail with circular cross-section, two piezoelectric ceramic rings and metal central bolt. Theoretical analyses show that the apparent elasticity method was simple, easy to calculate and especially suitable for the study of coupled vibrations. It is shown that the calculated resonant frequencies according to the apparent elasticity method provide better approximation of the measured results than those from one-dimensional theory.*

Key Words: *Ultrasonic Transducers, Resonance Frequency, Frequency Equations, Vibrational Modes, Method*

1. INTRODUCTION

Bolt-clamped Langevin-type (sandwich) transducers are widely used as efficient vibratory sources in various fields of industrial application of high-power ultrasonics. The transducer of this type can steadily generate high-amplitude ultrasonic vibrations. Fig. 1 illustrates typical constructions of the sandwich transducer, in which piezoelectric elements are sandwiched by two metal end-blocks (parts) and clamped with a screw bolt. In one-dimensional sandwich transducer design theory it is assumed that transducer vibrates in longitudinal mode [1], [2], and radial vibration is negligible. That means that the lateral dimensions of a transducer need to be much less compared with its longitudinal dimension. Generally, when the lateral dimensions are much less than a quarter of the longitudinal wavelength, then the one-dimensional design theory can be applicable, and the errors between the measured and designed frequencies are negligible. However, with develop-

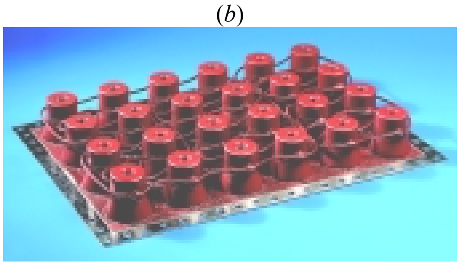
Received October 20, 2003

ment of the high power ultrasonic techniques, ultrasonic transducers are used in many ultrasonic applications, such as ultrasonic plastic welding and cleaning, where the large ultrasonic power is needed.

In these applications, the lateral dimensions are usually greater than a quarter of a wave-length of the longitudinal vibration, and a one-dimensional vibrating theory of sandwich transducers will bring appreciable errors. Additionally, when the resonant frequency of the transducer is increased, the longitudinal wavelength and dimension will decrease accordingly. According to the assumptions introduced in one-dimensional theory, the lateral dimensions of the transducer must also be decreased, where if the cross-section of the transducer is small, the mechanical strength will be lowered. Therefore, lateral dimensions of transducer cannot be too small, so that lateral vibration of such transducer must be taken into account, in order to avoid appreciable frequency error.

(*a*)

(*b*)



(*c*)

Fig. 1. The most frequently sandwich transducers patterns for:
(a) ultrasonic welding, (b, c) ultrasonic cleaning

For the coupled vibration of piezoelectric sandwich ultrasonic transducers, various numerical methods have been used to study the frequency characteristics and vibrational modes. Numerical models of high power ultrasonic systems are usually based on finite element or boundary element methods [3]. Though algorithms based on the finite element method are able to solve the steady state problem, analytical methods are often preferred because the numerical approaches do not give sufficient insight into the physical parameters that should be kept under control in the transducer design.

For the analysis of the very effective high-power cylindrical ultrasonic radiators with large dimensions, the apparent elasticity method is developed and used [4]. This method is also used for designing directional converters for ultrasonic longitudinal mode vibration [5]. Application of this method for design ultrasonic sandwich transducers, which consists of metal end-parts with rectangular cross-section and two piezoceramic rings, without consideration of influence of central bolt, is presented in literature [6].

In this paper we studied the coupled vibration of circular sandwich transducers used in high power ultrasonic techniques by introducing the equivalent elastic constants. The disks structure with central hole (ring structure) of the metal head, metal tail and two piezoelectric ceramic plates is determined by the necessity to prestress this in composite Langevin transducers for power applications, while central metal hole is whole cylinder. Therefore, the transducers studied in this paper are the most similar to practical ultrasonic transducers. In the following analysis of the paper, the equivalent elastic constants of the transducer components are defined, and frequency equation of the sandwich transducer is derived. Finally, the computed and experimental results are compared.

2. APPARENT ELASTICITY METHOD

As it is well known, in large scaled resonators such as thick circular disk and thick walled cylinder, two kinds of vibration, which are longitudinal vibration and radial vibration, are coupled to each other orthogonally. Therefore, designing of those resonators is very difficult. This problem is simplified by appearance a new design method based on "apparent elasticity" derived by Mori et al. [4].

In this approach it is assumed that coupling of those vibrations causes a change of elasticity. Now, imagine a thick circular disk resonator. And the longitudinal vibration on the longitudinal end and the radial vibration on the lateral face are anti-phase to each other. In this case, an apparent elasticity in the resonator (E_Z) is smaller than Young's modulus (E). Because, longitudinal strain (ε_Z) includes not only strain caused by longitudinal stress (σ_Z), but also strain caused by Poisson's phenomenon generated by radial vibration. So the longitudinal strain with radial vibration is greater compared with that without radial vibration. As the result, the practical elasticity ($E_Z=\sigma_Z/\varepsilon_Z$) is smaller than Young's modulus (E). In the same way, one can derive the apparent elasticity of radial direction (E_r). Note that now the apparent elasticity is a function of the dimensions of resonator.

Fig. 2 illustrates the most simple piezoceramic sandwich transducer usually used for cleaning. In this figure l_i ($i = 1, 2, 3, 4$) are the lengths of the front metal horn (metal head), the piezoceramic ring, the middle metal cylinder (bolt) and the back metal cylinder (metal tail), respectively, each having a circular cross-section with external radius a_i

(i = 1, 2, 3, 4), and internal radius b_i (i = 1, 2, 4). In our analysis, lateral dimensions a_1 and a_4 are comparable with longitudinal dimensions l_1 and l_4. This means that the one-dimensional theory cannot be used directly for transducer design. The equivalent elastic constants differ from the Young's modulus and depend not only on the material parameters but also on the geometrical dimensions and the vibrational modes of the transducer.

The expressions for equivalent elastic constants of the piezoelectric ceramic circular plates with central hole, when the piezoelectric effect is ignored but the anisotropy is taken into account, are [6]:

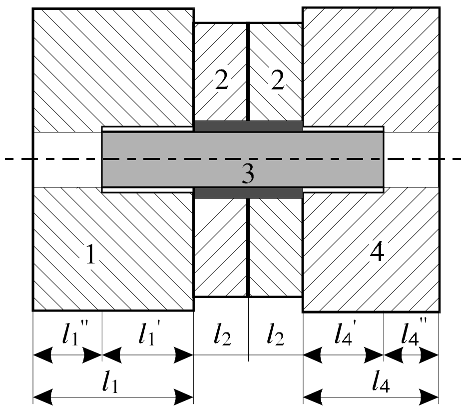


Fig. 2. Half-wave sandwich transducer:
(1) metal head, (2) PZT piezoceramic rings, (3) central bolt, (4) metal tail

$$E_{2r} = [s_{11}^E(1 - \nu_{12}^2 - \nu_{13} n_2 (1 + \nu_{12}))]^{-1} \quad (1)$$

$$E_{2Z} = [s_{33}^E(1 - 2\nu_{31} / n_2)]^{-1} \quad (2)$$

where $E_{2r} = \sigma_{2r}/\varepsilon_{2r}$, $E_{2Z} = \sigma_{2Z}/\varepsilon_{2Z}$, $\nu_{12} = -s_{12}^E/s_{11}^E$, $\nu_{13} = -s_{13}^E/s_{11}^E$, $\nu_{31} = -s_{13}^E/s_{33}^E$. σ_{2r}, σ_{2Z} and ε_{2r}, ε_{2Z} are the stresses and strains along the polar axes, s_{ij}^E are the elastic compliance constants. $n_2 = \sigma_{2Z}/\sigma_{2r}$ is the coupling coefficient between the longitudinal and radial vibrations of the piezoceramic rings.

In this case, apparent elasticity along z and r directions for metal end-parts and central bolt are represented as follows (i=1, 3, 4) [4]:

$$E_{ir} = E_i[1 - \nu_i^2 + \nu_i n_i (1 + \nu_i)]^{-1} \quad (3)$$

$$E_{iZ} = E_i (1 + 2\nu_i / n_i)^{-1} \quad (4)$$

where ν_i is the Poisson's ratio of material, E_i is the Young's modulus, and $n_i=\sigma_{iZ}/\sigma_{ir}$, like in the piezoceramic case, determining degree of wave coupling at an arbitrary point of the element.

3. FREQUENCY EQUATION FOR THE SANDWICH TRANSDUCER

From above analysis and the expressions for the equivalent elastic constants of the components of the transducer, when the radial dimensions of the transducer are large, the coupled vibration of the transducer can be regarded as the longitudinal vibration and the radial vibration. However, these two vibrations are not independent, but are related by coupling coefficients. Thus, with free boundary conditions, we can obtain the frequency equation as a combination of longitudinal and radial frequency equations of the transducer. For simplicity, we are discussed only a transducer with a fundamental vibrating mode.

The frequency equation for the radial vibration of the piezoceramic ring is [6]:

$$\frac{k_{2r}a_2N_0(k_{2r}a_2)-(1-\nu_{12})N_1(k_{2r}a_2)}{k_{2r}a_2J_0(k_{2r}a_2)-(1-\nu_{12})J_1(k_{2r}a_2)}=\frac{k_{2r}b_2N_0(k_{2r}b_2)-(1-\nu_{12})N_1(k_{2r}b_2)}{k_{2r}b_2J_0(k_{2r}b_2)-(1-\nu_{12})J_1(k_{2r}b_2)} \quad (5)$$

where a_2 and b_2 are the outer and inner radii of the piezoceramic ring. J_0, J_1 and N_0, N_1 are the first and the second kind of Bessel's functions of the zero and first order, $k_{2\mathrm{r}}=\omega/c_{2\mathrm{r}}$, $c_{2\mathrm{r}}=(E_{2\mathrm{r}}/\rho_2)^{1/2}$, ρ_2 is the density of the ceramics. For given values ν_{12}, a_2 and b_2, the solution of equation (5) is:

$$k_{2r}a_2=R_2 \quad (6)$$

R_2 is the root of frequency equation for the radial vibration and it is only function of ν_{12} and ratio a_2/b_2. Now n_2 can be defined:

$$n_2=\frac{1-\nu_{12}^2-c_{01}^2R_2^2/\omega^2a_2^2}{\nu_{13}(1+\nu_{12})} \quad (7)$$

where $c_{01}{}^2=1/(s_{11}{}^{\mathrm{E}}\rho_2)$.

Analogously frequency equation for the radial vibration matter for the metal end-parts, where ν_{12} and ν_{13} is necessary to replace with ν_i, as well as R_2 with R_i. a_i and b_i will be outer and inner radius of the metal end-parts, respectively ($i=1, 4$). In this case:

$$n_i=\frac{1-\nu_i^2-(c_{0i}^2R_i^2)/(\omega^2a_i^2)}{-\nu_i(1+\nu_i)} \quad (8)$$

where $c_{0i}{}^2=E_i/\rho_i$, E_i are the Young's modulus, and ρ_i are the densities of the metal end-parts materials ($i=1,4$).

When the bolt is concerned, because bolt is whole cylinder, the frequency equation for the radial vibration is [4]:

$$k_{3r}a_3J_0(k_{3r}a_3)=(1-\nu_3)J_1(k_{3r}a_3) \quad (9)$$

with solution $k_{3r}a_3 = R_3$, where a_3 is radius of bolt.

In order to derive the frequency equation for the longitudinal vibration, it is assumed that the displacement nodal plane locates at the middle of the two piezoceramic rings. Based on the frequency equation obtained for same case by one-dimensional design theory, the frequency equations for the transducer parts before and after the displacement nodal plane can be obtained. Thus, for the part before the displacement nodal plane (Fig. 2), is [2]:

$$\begin{aligned}&\rho_1 c_{1Z} S_1 \operatorname{ctg}(k_{1Z} l_1' + \operatorname{arcctg} \frac{\rho_2 c_{2Z} S_2}{\rho_1 c_{1Z} S_1} \operatorname{ctg} k_{2Z} l_2) + \\ &+ \rho_3 c_{3Z} S_3 \operatorname{ctg} k_{3Z} (l_1' + l_2) - \rho_1 c_{1Z} S_1 \operatorname{tg} k_{1Z} l_1'' = 0\end{aligned} \tag{10}$$

where $k_{iZ} = \omega/c_{iZ}$, $c_{iZ} = (E_{iZ}/\rho_i)^{1/2}$, and S_i are the cross-sectional areas of the several transducer componential parts ($i = 1, 2, 3$).

Therefore, the frequency equation for the coupled vibration (10), whereas in equations (1) and (3) are included equations (7) and (8), depend not only on the material parameters, the longitudinal dimensions and the frequency, but also on the lateral dimensions, and this is different from the traditional one-dimensional theory.

4. EXPERIMENTAL RESULTS

Some piezoceramic sandwich transducers were designed and made by proposed method, where their resonant frequencies were determined (measured) with the HP 3042A network impedance analyzer. These transducers are symmetrical, that is tail and head duralumin metal part are equal length, with the following material parameters: $E_1 = E_4 = 7.4 \times 10^{10}$ N/m^2, $\rho_1 = \rho_4 = 2790$ kg/m^3, $\nu_1 = \nu_4 = 0.34$. The piezoceramic rings are two pieces of PZT4 ceramic, and its material parameters are as follows: $\rho_2 = 7500$ kg/m^3, $s_{11}^E = 12.3 \times 10^{-12}$ m^2/N, $s_{33}^E = 15.5 \times 10^{-12}$ m^2/N, $\nu_{12} = 0.33$, $\nu_{13} = 0.43$, $\nu_{31} = 0.34$. Table 1 gives dimensions of designed transducers used in the analysis.

Table 1. Transducer dimensions used in the analysis

dimension	transducer I	transducer II
l_1' [mm]	22.5	30
l_2 [mm]	6.35	6.35
$2a_1$ [mm]	40	51
$2b_1$ [mm]	10	16
$2a_2$ [mm]	38	50
$2b_2$ [mm]	13	20
$2a_3$ [mm]	10	16

Simultaneously abbreviation of the both metal end-parts is accomplished, and experimentally resonance frequencies versus duralumin length $l_1 = l_4$ are obtained. These results are compared with analogous dependencies of the frequency versus length, which are obtained according to frequency equation (10). These dependencies, together with analogous one-dimensional characteristics for the same transducers, are presented for both transducers in the Fig. 3 and Fig. 4.

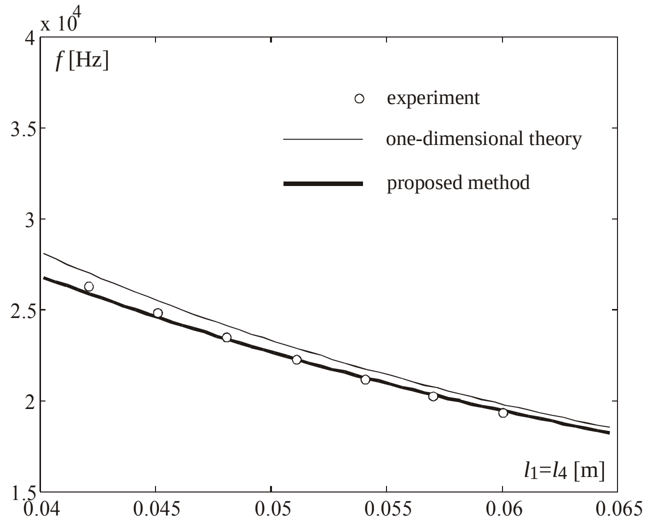


Fig. 3. Resonance frequency of the first mode versus metal part length for first transducer

It is obvious from Fig. 3 and Fig. 4 that the frequencies obtained from our analysis are in better agreement with the measured frequencies than those from one-dimensional theory. Also, can be seen that one-dimensional theory for same metal parts length gives better results for the first transducer, who has a smaller cross-section area, since then transducer is closer to the ideal one-dimensional model. For small lengths of metal end-parts there exists a discrepancy between experimental results and results obtained by proposed method in both cases. This is a consequence of consideration of only first resonance (vibrating) mode of each componential transducer parts (considered only R_{imin} in expressions (6) and (9)).

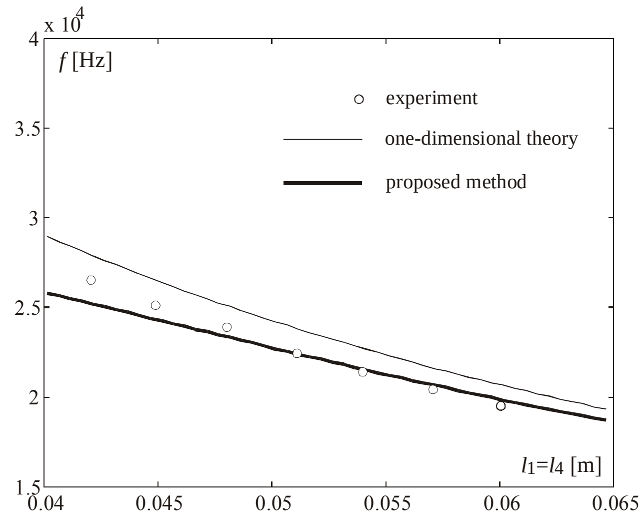


Fig. 4. Resonance frequency of the first mode versus metal part length for second transducer

5. Conclusion

In this paper, a method for calculating the fundamental resonant frequencies of sandwich transducer with various cross-section is presented. The method presented in this paper is simple compared with numerical methods. Compared with the results of the one-dimensional theory, the calculated resonant frequencies according to the apparent elasticity method are more closely approximate to the measured results. Design theory of the transducer is based on the assumption that the neutral plane is located at the middle of the piezoceramic rings, although a similar procedure may be realized in case of asymmetrical transducers. Based on the obtained results, the displacement distribution in the external surfaces of the transducer will be determined. Since the vibration is three-dimensional, it is obvious that the analysis will be complex.

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PROJEKTOVANJE ULTRAZVUČNIH PRETVARAČA POMOĆU METODA PRIVIDNIH MODULA ELASTIČNOSTI

Dragan D. Mančić, Milan Dj. Radmanović

U ovom radu primenjene su ekvivalentne konstante elastičnosti, koje su dobijene pomoću metoda prividnih modula elastičnosti. Analizirane su spregnute vibracije sendvič pretvarača korišćenih u oblasti snažnog ultrazvuka, i dobijena je frekventna jednačina za projektovanje pretvarača i izračunavanje rezonantne frekvencije. Pretvarač se sastoji od metalnog emitora i metalnog reflektora kružnog poprečnog preseka, dva piezoelektrična keramička prstena i metalnog centralnog zavrtnja. Teorijske analize pokazuju da je metod prividnih modula elastičnosti prost, lak za izračunavanja i posebno pogodan za analizu spregnutih vibracija. Pokazano je da se rezonantne frekvencije dobijene metodom prividnih modula elastičnosti bolje slažu sa merenim rezultatima od rezultata dobijenih jednodimenzionalnom teorijom.

Ključne reči: *ultrasonični / nadzvučni pretvarači, rezonantna frekvencija, izjednačavanje frekvence, način vibriranja, metod*