FACTA UNIVERSITATIS
Series: **Working and Living Environmental Protection** Vol. 2, N° 2, 2002, pp. 143 - 149

A NEW APROACH TO V-DIPOLE ANTENNA

UDC 621.396.674

Dejan M. Petković[1], Dejan D. Krstić[2]

[1]Faculty of Occupational Safety, University of Nish, Serbia, Yugoslavia
E-mail: dejan@znrfak.znrfak.ni.ac.yu
[2] Faculty of Occupational Safety, University of Nish, Serbia, Yugoslavia
E-mail: dejank@znrfak.znrfak.ni.ac.yu

Abstract. *In this paper a new integral equation for current distribution along the arms of symmetric center-driven V-dipole antenna is presented. In this case when the arms are co-linear, this equation reduces to Hallen`s one. A polynomial is employed as an approximation for the current distribution and the dipole is fed by Dirac δ-generator. The integral equation is numerically solved by the point-matching method.*

Key words: *antennas, V-dipole, Hallen's integral equation, point matching method*

INTRODUCTION

It is well known that, when the arms of the conventional linear dipole are lengthened beyond wavelength, the directivity normal to the dipole begins to diminish due to an increasing sidelobe level. These usually undesirable caracteristics can be reduced if the arms of a dipole are tilted to make V-shape.

V-dipole is the first antena which was analised because of increasing directivity and effective length. The problem of input impedance based on the so-called mode theory, has been dealt before fifty years [4]. The entire analysis was identical to that used in the case of conventional straight dipole. Another approach, based on iterative solution of integral equation of Pocklington's type [1], was not different from those employed in analyzing the straight dipole [5], as far as the input impedance was concerned only the zeroth-order solution, which corresponds to infinitely thin arms, was presented. For the special case of rectangular V-dipole, the formula for input impedance was derived by variational method [9]. The essential shortage of this paper is the complete absence of numerical results. In addition, the same technique was already known from the theory of straight dipole [3, 5]. The first numerical results, [13], for input impedance and current distribution was obtained using point matching method [8], and with polynomial approximation for current

Received October 30, 2002

distribution along the V-dipole arms, originally introduced in [10]. Because of the lack of, until to then, published numerical results, only the particular results, corresponding to the straight dipole, were compared to known experimental and theoretical data. The variona-tional expression for input impedance of V-dipole was evaluated in [14]. Entire work on results obtained for straight dipole [12]. Finally, V-dipole was treated as opened two-wire line with tilted ends [16]. It is worthwhile to mention that the geometry of V-dipole antenna is the generalized geometry of the straight dipole.

The question why the V-dipole is so important has obvious answer. It was pointed out, [17], that some curvilinear structures, such as parabolic or circular bend wires, may yield a directivity considerably higher than that of straight dipoles. The geometry of V-dipole is the first approximation of such structures. The problem of current (or charge) distribution along curved (or straight) wires usually is solved by intersecting the wire and adopting some suitable function for an approximation of current (or charge) distribution on the each subsection. This standard technique has been applied for straight wires, circular arcs, short helical wires and no others, [7]. Only these tree structures lead to the convolution kernel of integral equation for current distribution that is necessary for replacement derivates between the current distribution function and the kernel. In the general case, the nature of the kernel of integral equation, which involve quasi-singularities of the first, second and third order, then produce non-physical current oscillations near source region [15], and consequently invalid input impedance, which is numerically generated. The reduced rank technique [22], is still based on modeling the curved wire by piece-wise-straight segments. Nevertheless, the rank of matrix and computing time have no importance for satisfactory accurate results, and this work is quite ambiguous and unnecessary at present.

Obviously, the wire antenna should be treated as the continual geometrical system and as the boundary value problem. This can be done only if an antenna problem is considered in conveniently chosen coordinate system, which enables entire-domain approximation for current distribution. Nevertheless, V-dipole is the special case of curved wires and, as far as the authors are informed, this antenna was not treated in that manner until now. The purpose of this paper is to present an integral equation for current distribution along the arms of V-dipole. This equation reduces to Hallen's one [2], in the case when the wires are co-linear, and thus has an advantage over all mentioned works. This advantage comes from the fact that the kernel of Hallen's integral equation involve has quasi-singularity of the first order which can be easily eliminated. The integral equation presented in this paper is only the special case of general integral formulation for curvilinear antenna designing [19, 20, 21].

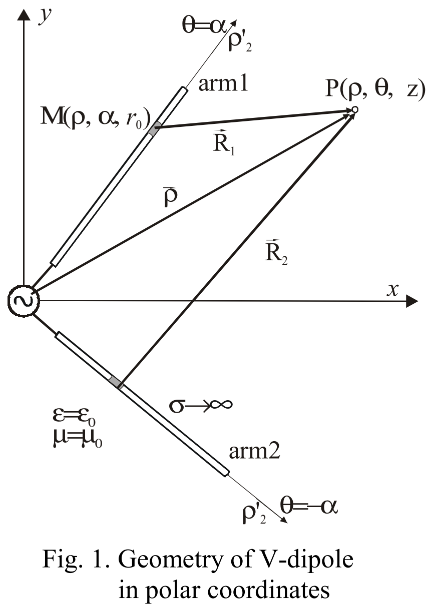

Fig. 1. Geometry of V-dipole in polar coordinates

THE INTEGRAL EQUATION

Consider a thin symmetric V-dipole antenna, the arms of which are inclined at arbitrary angle 2α with respect to each other, Fig.1. The antenna conductor are assumed to be perfect, $\sigma\rightarrow\infty$. Both arms have equal lengths h and the same radius $r_0(r_0<<h)$ and then, for historical reasons, $\Omega = 2\ln(2h/r_0)$ let be Hallens's parameter. In order to simplify the analysis, an idealized model of the excitation zone is adopted. This means that the real generator is replaced with impressed electric field, i.e. the antenna is driven by a Dirac δ-generator with scalar difference U across the base of V-dipole and with the angular frequency ω. It is assumed that currents I_1 and I_2 is localized on the antenna axis ρ'_1 and ρ'_2, which coincide with $\theta = \alpha$ and $\theta = -\alpha$ of polar-cylindrical coordinate system. The currents vanished at the antenna ends, and due to the symmetry,

$$I_1 = I_1(\rho'_1)\,,\; I_2 = I_2(\rho'_2)\,,\; I_1(\rho) = -I_2(\rho)\,,\; I_1(h) = I_2(h) = 0 \tag{1}$$

Using boundary condition that the tangential component of the total electric field along the antenna surface vanish the following differential equation can be establish

$$\frac{\partial}{\partial\rho}(\operatorname{div}\vec{A}) + k^2 A_\rho = 0 \quad \text{for } \mathrm{M}(\rho,\alpha,r_0), \quad 0<\rho\le h\,, \tag{2}$$

any point on the surface of arm 1 of antenna, where $\vec{A}$ is magnetic vector potential and $k = 2\pi/\lambda = \omega/c = \omega\sqrt{\varepsilon_0\mu_0}$ is free space propagation constant and λ is wavelength. The magnetic vector potential have components

$$A_\rho = A_1\cos(\alpha-\theta) + A_2\cos(\alpha+\theta) \;\text{ and }\; A_\theta = A_1\cos(\rho-\theta) + A_2\cos(\rho+\theta) \tag{3}$$

where is

$$A_n = \int_0^h I_n(\rho'_n)\frac{e^{-jkR_n}}{R_n}\mathrm{d}\rho'_n \quad n = 1,2 \tag{4}$$

$$R_n = |\rho-\rho'_n| = \sqrt{\rho^2 + \rho'^2_n - 2\rho\rho'_n\cos(\alpha-(-1^n)\theta) + z^2}\,, \quad n = 1,2 \tag{5}$$

For the divergence of magnetic vector potential at the point $\mathrm{M}(\rho,\theta,r_0)$ one can obtain

$$\operatorname{div}\vec{A}\Big|_{\substack{\theta=\alpha\\ z=r_0}} = \left(\frac{\partial A_1}{\partial\rho} + \cos 2\alpha\frac{\partial A_2}{\partial\rho} - \frac{1}{\rho}\sin 2\alpha\frac{\partial A_2}{\partial\theta}\right)\Bigg|_{\substack{\theta=\alpha\\ z=r_0}} \tag{6}$$

Adding to the both sides of differential equation (3) the term $\dfrac{1}{h_s^2}\dfrac{\partial^2 A_\rho}{\partial\rho^2}, h_s \in R$ equation (3) becomes

$$\frac{\partial^2 A_\rho}{\partial\rho^2} = (kh_s)^2 A_\rho = \frac{\partial}{\partial\rho}\left(\frac{\partial A_\rho}{\partial\rho} - h_s^2\operatorname{div}\vec{A}\right)\!, \text{ for } \theta=\alpha,\, z=r_0\,. \tag{7}$$

The solution of this differential equation, via Lagrange's method of constants variation, leads to the integral equation for current distribution on the arm 1.

$$C_1 \cos(kh_s\rho) + C_2 \sin(kh_s\rho) = \\ = kh_s \int_0^{\rho} A_\rho \Big|_{\substack{\theta=\alpha \\ z=r_0}} \sin kh_s(\rho - s)\,\mathrm{d}s + h_s^2 \int_0^{\rho} \operatorname{div}\vec{A}\Big|_{\substack{\theta=\alpha \\ z=r_0}} \cos kh_s(\rho - s)\,\mathrm{d}s \tag{8}$$

Obviously the constant $C_1 = 0$, and the value of constant C_2 can be determined from the condition for electric scalar potential in the arm1 driven point, $\varphi(\rho \to 0) = U/2$. After differentiation the integral equation (8) in the limiting case, $\rho \to 0$, and applying Lorentz's gauge immediately follows $C_2 = -jU\,h_s/2c$. In the case of V-dipole antenna, which is the special case of curvilinear structures, the general method for substituting all derivates of $\operatorname{div}\vec{A}$ with derivate of current density along the thin opened antenna wires [19, 21], can be simplified because this is the case when the kernel of an integral equation is of closed-cycle type [7].

$$\operatorname{div}\vec{A}\Big|_{\substack{\theta=\alpha \\ z=r_0}} = \frac{\mu}{4\pi}\int_0^h \frac{\mathrm{d}I(\rho')}{d\rho'}[K_1(\rho,\rho') - K_2(\rho,\rho',\alpha)]\,\mathrm{d}\rho' \tag{9}$$

and (4), becomes

$$A_\rho\Big|_{\substack{\theta=\alpha \\ z=r_0}} = \frac{\mu}{4\pi}\int_0^h I(\rho')[K_1(\rho,\rho') - \cos(2\alpha)K_2(\rho,\rho',\alpha)]\,\mathrm{d}\rho' \tag{10}$$

where is

$$K_1(\rho,\rho') = \frac{e^{-jkR_1}}{R_1}\Big|_{\substack{\theta=\alpha \\ z=r_0}},\quad K_2(\rho,\rho',\alpha) = \frac{e^{-jkR_2}}{R_2}\Big|_{\substack{\theta=\alpha \\ z=r_0}},\quad K_1(\rho,\rho,\pi/2) = K_2(\rho,\rho') \tag{11}$$

NUMERICAL SOLUTION OF THE INTEGRAL EQUATION

Numerical testing shows, Tab.1, whenever h_s can be quite arbitrary, the best value is $h_s = \pm 1$ when the numerical difficulties are same as those appeared in numerical solving Hallen's integral equation for current distribution along the arms of the straight dipole. The other values for is h_s can produced unstable solu tions for input impedance. In the case $h_s = 0$ integral equation (11) has the solution too, as the limiting value. The proposed integral equation is numericaly soved by the point matching method and the polynomial aproximation for current distribution.

Table 1. $\Omega = 10, h = 0.25\lambda$

h_s	G [mA/V]	B [mA/V]
2.0	18.1	−5.4
1.5	19.4	−5.2
1.0	20.3	−5.1
0.5	20.8	−5.0
0.0	21.0	−5.0

Table 2. $\Omega = 10, h = 0.25\lambda, \mathrm{Mp} = 2, h_s = 1$

2α	G [mA/V]	B [mA/V]	R [Ω]	X [Ω]
12	0.27076	16.0.3955	1.02	-62.33
30	1.78338	18.67779	5.06	-53.06
45	8.50278	24.93930	12.25	-35.92
60	27.14523	22.10132	22.15	-18.04
75	29.43511	15.91281	33.87	-18.31
90	20.32133	-5.12197	46.27	11.60
105	14.99606	-5.12197	46.27	11.60
120	12.21303	-5.30861	68.87	29.93
135	10.67809	-4.86366	77.56	35.32
150	9.81561	-4.53777	83.94	38.80
165	9.37071	-4.34673	87.82	40.74
180	9.23246	-4.28416	89.12	41.36

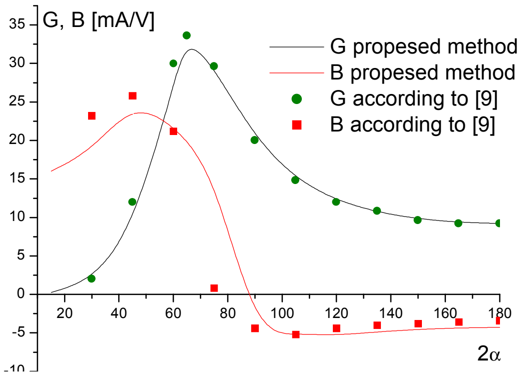

Fig. 2. $\Omega = 10, h = 0.25\lambda, \mathrm{M} = 2, h_s = 1$

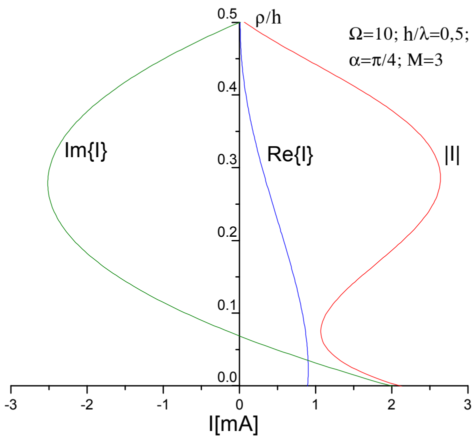

Fig. 3. Current distribution

Table 2. $\Omega = 10, h_s = 1, 2\alpha = \pi / 2$, M is the order of polymomial

M	$h = 0.25\lambda$				$h = 0.5\lambda$			
	propesed method		according to [9]		propesed method		according to [9]	
	G [mA/V]	B [mA/V]	G [mA/V]	B [mA/V]	G [mA/V]	B [mA/V]	G [mA/V]	B [mA/V]
2	20.32	–5.12	19.64	–4.35	0.94	1.94	0.96	2.00
3	20.26	–5.32	19.55	–4.80	0.90	1.98	0.92	2.04
4	19.64	–5.48	18.57	–4.80	0.90	2.10	0.93	2.17

CONCLUSION

This paper presents the new integral equation for current distribution along the arms of V-dipole antenna. Equation is derived from the boundary condition for the tangential condition of the total electric field on the antenna surface and Lorenz's gauge. The general method for transforming differential equation, the original idea which enables this transformation and quite new formulae for the divergence of magnetic vector potential was developed earlier, by the first titled author, for the curvilinear antennas with opened ends. In this paper V-dipole is treated as the special case of curved structures. This approach leads to the integral equation the kernel which involves the quasi-singularity of the first order only. Further the proposed approach enables, in the case when angle between the arms are very small V-dipole can be treated as inhomogeneous two-wire line, or when the arms co-linear, as the straight dipole, because of that in this case the integral equation becomes the Hallen's one. The polynomial approximation for current distribution along the arms is adopted, and the excitation zone is approximated by δ-generator. Then integral equation is solved by the point-matching method. The numerical results for input admittance was compared to some known theoretical (and experimental) data and excellent agreement was found. Further more, the antenna composed of two arms and section of the circular arc which connects the end of the arms, can be treated on the same way. Obviously, when the arms make 2π angle this antenna becomes the circular loop driven by two-wire line.

REFERENCES

1. H.E. Pocklington, "Electric oscillation in wires", *Camb. Phil. Soc. Proc.*, vol. 9., pp. 334-332, 1897.
2. E. Hallen, "Theoretical investigation into the transmitting and receiving qualities of antennas", *Nova acta, Soc. Sci.*, Upsala, pp. 1-44,1938.
3. J. E. Storer "Solution to thinwire anntena problems by variational methods" Doctoral dissertation, Harvard University, 1951.
4. S.L. Schelkunoff, *Advanced antenna theory*, John Wiley & Sons, 1952.
5. R.W.P. King *The theory of linear antennas,* Harvard University Press, 1956.
6. F.G. Trikomi, *Differentsial'nye Uravneniya*, Izdalstvo inostraani literaturi, Moskva, 1960.
7. C.H. Tang, "Input impedance of arc antennas and short helical radiators", *IEEE Trans. AP-12*, pp. 2-9, 1964.
8. R.F. Harington, *Field Computation by Moment Method,* Macmillan comp, N.York, 1968.
9. J.V. Surutka, D.M. Veličković, "Input impedance of rectangular V-dipole annena derived by varionational method", *Publ. El. Fak.*, Beograd, ser. math. & phys. No. 247-273, pp. 87-95,1969.
10. B.D. Popović, "Polynomial approximation of current along thin symmetrical cylindrical dipoles", *Proc. IEE*, vol 117. pp. 873-878, 1970.

11. B.D. Popović, Z.D. Popović "Thin cilindrical antenna: variational solution with polynomial current approximation, *The Radio And Electronic Engineer*, vol. 41., No. 11, p. 493, 1971.
12. B.D. Popović, J.V. Surutka, "Variational solution to the problem of asymmetrical cylindrical dipole", *IEEE Trans. AP-19*, p.17, 1971.
13. J.V. Surutka, D.M. Veličković, "Current and admittance of symmetric center-driven V-antenna", *Publ. El. Fak.* Beograd ser. elect., No. 79-96, pp. 21-31, 1973.
14. J.V. Surutka, D.M. Veličković, "Variational approach to V-dipole antenna analysys", *The Radio And Electronic Engineer*, vol. 44., No. 7, pp. 367-372, 1974.
15. R. Mittra, *Numerical and Asymptotic Techniques in Electromagnetics*, Spring-Verlag, New York, 1975.
16. J.V. Surutka, D.M. Veličković, "Theory of V-dipol antenna driven by a two-wire line", *Publ. El. Fak.*, Beograd ser. elect., No. pp. 102-106, 1976.
17. F.M. Landstorfer, R.R. Sacher: *Optimization of Wire Antennas*, John Wiley & Sons Inc, Chichester, 1986.
18. D.M. Petković, "Numerical solution of a new integral equation for curvilinear antennas" (on Serbian), *ETAN-'89*, Vol 5, pp. 3-10., Novi Sad, 1989.
19. D.M. Petković, "A new method for designing curvilinear antennas", Doctoral dissertation (on Serbian), University of Nish, Faculty of electronics, 1992.
20. D.M. Veličković, D.M.Petković, "A new integral equation method for thin-wire curvilinear antennas designig", *TELSIKS '93*, pp. 2.8-2.12, University of Nish, Faculty of electronics, 1993.
21. D.M.Petković, D.D. Krstić, "On the divergence of magnetic vector potential in the thin-wire antenna analysis", *TELSIKS '97*, University of Nish, Faculty of electronics, 1997.
22. S.D.Rogers, C.M. Butler: "An Efficient Curved-Wire Integral Equation Solution Technique", *IEEE Trans. AP 49*, No. 1, pp. 70-79, 2000.

NOVI PRISTUP PROBLEMU V-DIPOLA

Dejan M. Petković, Dejan D. Krstić

U radu je prikazana nova integralna jednačina za raspodelu struje duž antenskih provodnika simetrično napajanog V-dipola. Kad su antenski kraci opruženi ova integralna jednačina se redukuje na dobro poznatu Hallenovu integralnu jednačinu, što je znatna prednost u odnosu na do sada poznate pristupe rešavanju ovog problema. Za aproksimaciju raspodele struje upotrebljeni su algebarski polinomi, a za aproksimaciju zone napajanja Diracov delta generator. Integralna jednačina je numerički rešena metodom podešavanja u tačkama, i dobijeni rezultati za ulaznu admitansu se veoma dobro slažu sa ranije poznatima.

Ključne reči: *antene, V-dipol, Hallenova integralna jednačina, metod podešavanja u tačkama*