FACTA UNIVERSITATIS
Series: **Working and Living Environmental Protection** Vol. 2, N° 2, 2002, pp. 135 - 142

NUSSELT NUMBER DEPENDENCE ON INCLINATION OF PARALELOGRAMIC ENCLOSURE UNDER NATURAL CONVECTION CONDITIONS

UDC 536.24

Miomir Raos, Ljiljana Živković

Faculty of Occupational Safety, University of Niš, Serbia, Yugoslavia

Abstract *A review of our investigations on the natural convection phenomena in parallelogram enclosures has been presented in this paper. In fact this is an investigation of the averaged and local Nusselt (Nu) numbers in mentioned conditions. We observed Nu number changes on the isothermal hot wall, which has different angle of inclination on horizontal plane. In order to find Nu number dependence, we had to make mathematical and numerical investigation of the phenomenon, and after that to solve it by an original computer code. Nu number dependence is based on the known temperature field in domain, especially on temperature values near hot and cold isothermal wall.*

Key words: *heat transfer, natural convection, enclosures*

1. INTRODUCTION

Nusselt (Nu) number represents one of the most important information in heat transfer phenomena, as well as in natural convection heat transfer phenomena in enclosed space. That information is significant across the enclosed space as well as on the boundary region or on the surrounding surfaces. Most of the Nu number analytical expressions of laminar and turbulent convection in enclosed space have been obtained by many numerical and real experiments. This paper represents a part of the wider study, which has to investigate natural convection phenomena especially in parallelogram enclosures. Therefore, realization of this subject has been done through many phases, in order to achieve better understanding of flow configuration, and better informations on velocity, temperature and pressure fields in enclosures.

The real physical model of the rectangular enclosure with different conditions on heated sides and adiabatic horizontal walls, has been defined. Physical model represents base for mathematical model, which defines basic variables for heat transfer and and flow processes.

Received November 07, 2002

Discretization of the defined mathematical model, with respect to the nature of the equations has been done with finite volume method (FVM). A computer code, which contains SIMPLE procedure in essence, and routines for solving field variables have solved the obtained system of algebraic equations.

Results of the numerical experiment have been compared with literature ones, and are in good agreement. Results of this study with some other analyses and researches present good base for definition of the phenomenon, which can be found in engineering practice.

2. Physical Model

In order to get good enough data for Nu number change, it's necessary to have information about temperature field in domain of calculation. That means we need to know temperatures in every point of the enclosure. This information about temperature field in enclosure is not available in advance because of impossibility to find out solution of the mathematical model directly. The same situation is with other variables. Therefore we could rely on discrete information about variables. Obviously we have to apply numerical procedure, which imply discretization of the enclosure domain. Therefore on the basis of numerical procedure we could obtained values of dependant variables in enclosure, like temperature, velocity, pressure, turbulent characteristics and so on.

Physical model is represented by paralelogramic enclosure, Fig.1. For better understanding heat transfer phenomena in enclosed rectangular space we will present 2D geometry results in our paper. That statement has reason in fact that the fields of variables in enclosure are similar for great depth of the enclosure.

Therefore, physical model has two isothermal walls and two adiabatic walls.

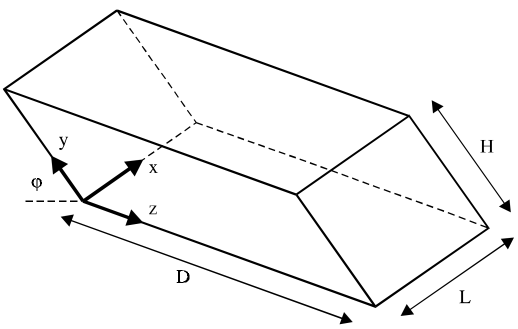

Fig. 1. Inclined rectangular enclosure

Dimension of the rectangular enclosure is L for length, H for height, and D for depth. The Cartesian coordinate system is fixed for one corner point and lies on the horizontal plane. We define a geometrical aspect A, as relation between H and L. Enclosure rotates around the z-axis and makes the angle of inclination to the horizontal plane. Therefore enclosure is tilted with respect to the gravitational acceleration vector g. As the consequence there are components of the gravitational accelerating vector represented by projection on the x and y axis. For $x = 0$ temperature of the hot wall is Th, and for $x = L$

temperature on the opposite colder wall is *Tc*. As we mentioned above the other walls are adiabatic.

Because of rotating the enclosure there is some angle of inclination of hot wall to the horizontal plane. The enclosure is filled by air ($Pr = 0{,}73$) and there are no chemical reaction in it. We may neglect radiate heat transfer phenomena because of average temperature *To* which is half of the temperature sum on the isothermal walls. If there is a small temperature difference in enclosure we have to use Boussinesq approximation, which assume fluid properties as μ (dynamic viscosity), λ (thermal conductivity), c_p (specific heat at constant pressure), to be constant, evaluated at the averaged temperature. Therefore, the fluid density may be assumed as constant value except for buoyancy terms in equations of motion.

3. Mathematical model

Equations for conservation of mass, momentum and energy are the basic equations of mathematical model. Equation for the conservation of mass or continuity equation, is expressed by:

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \rho \underline{u} = 0 \tag{1}$$

Fluid is assumed to behave as a Newtonian, and gravity is only external force. For example, conservation of the momentum and energy for the x_i can be expressed by:

$$\frac{\partial \rho u_i}{\partial t} + \underline{\nabla} \rho \underline{u} u_i = -\frac{\partial p}{\partial x_i} + \underline{\nabla} \mu \underline{\nabla} u_i + \rho g_i + \underline{\nabla} \mu \frac{\partial \underline{u}}{\partial x_i} - \frac{2}{3} \frac{\partial}{\partial x_i} (\mu - k) \underline{\nabla} \underline{u} \tag{2}$$

$$c_p \frac{\partial \rho T}{\partial t} + c_p \underline{\nabla} \rho \underline{u} T = \underline{\nabla} \lambda \underline{\nabla} T \tag{3}$$

where ρ is density, $\underline{u}$ - velocity vector, $\underline{\nabla}$ - gradient operator, κ - negligible bulk viscosity. For the laminar case mentioned equations for *x, y, z* axis are:

$$\underline{\nabla} \cdot \underline{u} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + \underline{\nabla} \cdot \underline{u} u = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \underline{\nabla} \cdot \nu \underline{\nabla} u + \underline{\nabla} \cdot \nu \frac{\partial \underline{u}}{\partial x} - \beta g \cos \varphi (T - T_0) \tag{5}$$

$$\frac{\partial v}{\partial t} + \underline{\nabla} \cdot \underline{u} v = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \underline{\nabla} \cdot \nu \underline{\nabla} v + \underline{\nabla} \cdot \nu \frac{\partial \underline{u}}{\partial y} - \beta g \sin \varphi (T - T_0) \tag{6}$$

$$\frac{\partial w}{\partial t} + \underline{\nabla} \cdot \underline{u} w = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \underline{\nabla} \cdot \nu \underline{\nabla} w + \underline{\nabla} \cdot \nu \frac{\partial \underline{u}}{\partial z} \tag{7}$$

$$\frac{\partial T}{\partial t} + \underline{\nabla} \cdot \underline{u} T = \underline{\nabla} \cdot \frac{\nu}{\mathrm{Pr}} \underline{\nabla} T \tag{8}$$

where $\nu = \mu / \rho$ is the kinematic viscosity, $\Pr = \mu\, c_p / \rho$ is Prandtl number. The density of the fluid is assumed to be constant except for buoyancy terms, for which the density is linearised as:

$$\rho(T) = \rho(T_0) - \beta\rho(T_0)(T - T_0) \tag{9}$$

In order to get complete mathematical model we are defining boundary conditions. The velocities on the walls are equal zero as well as temperature gradients on the adiabatic walls. Temperatures on the isothermal walls are *Th* for hot wall and *Tc* for cold wall. These combined boundary conditions are so called Neumann boundary conditions.

$$0 \le x \le L \quad (y = 0 \quad \vee \quad y = H) \quad u = v = 0 \qquad \frac{\partial T}{\partial y} = 0 \tag{10}$$

$$x = 0 \quad 0 \le y \le H \quad u = v = 0 \qquad T = T_H \tag{11}$$

$$x = L \quad 0 \le y \le H \quad u = v = 0 \qquad T = T_C \tag{12}$$

4. SOLVING PROCEDURE

Solving of the mentioned mathematical model is done by numerical procedure, using finite volume method (FVM) with pressure correction. Finite volume method gives a system of the discretizated equations, which means that the system of the partial elliptic equations is transformed to the system of the algebraic equations. Common Gauss - Seidel method helps in solving algebraic equations system.

Domain is divided by orthogonal grid. Essence of the discretization is appropriate selection of discretization scheme. Otherwords, that is appropriate option of balance between convective and diffusive terms through the boundary of the each control volume. We used hybrid scheme for both laminar and turbulent regimes. General differential equation is:

$$\frac{\partial}{\partial t}(\rho\Phi) + div(\rho\vec{w}\Phi) = div(\Gamma_\Phi grad\Phi) + S_\Phi \tag{13}$$

Total fluxes, convective and diffusive, for one dimension, as example, can be express like:

$$J_x = \rho u\Phi - \Gamma_\Phi \frac{\partial \Phi}{\partial x} \tag{14}$$

Discretization of the east side flux by hybrid scheme gives:

$$\left(\rho u - \Gamma\frac{\partial\Phi}{\partial x}\right)_e \approx \begin{cases} (\rho u)_e & Pe_e \le -2 \\ (\rho u)_e \dfrac{\Phi_P + \Phi_E}{2} - \Gamma_e \dfrac{\Phi_E - \Phi_P}{\Delta x_e} & -2 < Pe_e < 2 \\ (\rho u)_e \Phi_P & Pe_e \ge 2 \end{cases} \tag{15}$$

where Φ_i is dependent variable, Γ_i diffusive term, *Pe* Peclet number, subscript letter P and E are nodal grid point. In order to obtain sufficient number of the grid points in the boundary regions we used non-linear point distribution along domain.

In order to obtain results of the numerical procedure, there is made original computer code, which basically uses SIMPLE solving procedure for calculating variables. The first calculation is started from zero fields, and the every next one takes previous as initial condition.

With additional solving routine from Mathlab software package we made visual presentation of the results. Also we made some additional calculation in order to obtain temperature gradients, maximum and minimum of the values for local and average Nu number and so on.

5. RESULTS

Two-dimensional steady calculation has been performed for laminar regime of fluid flow. The Rayleigh number is 10^6 with angle of inclination above 20°, and aspect ratio equal one. The results are presented in Fig. 2. Temperature distribution is plotted for temperature differences of ΔT=10°C between isothermal walls.

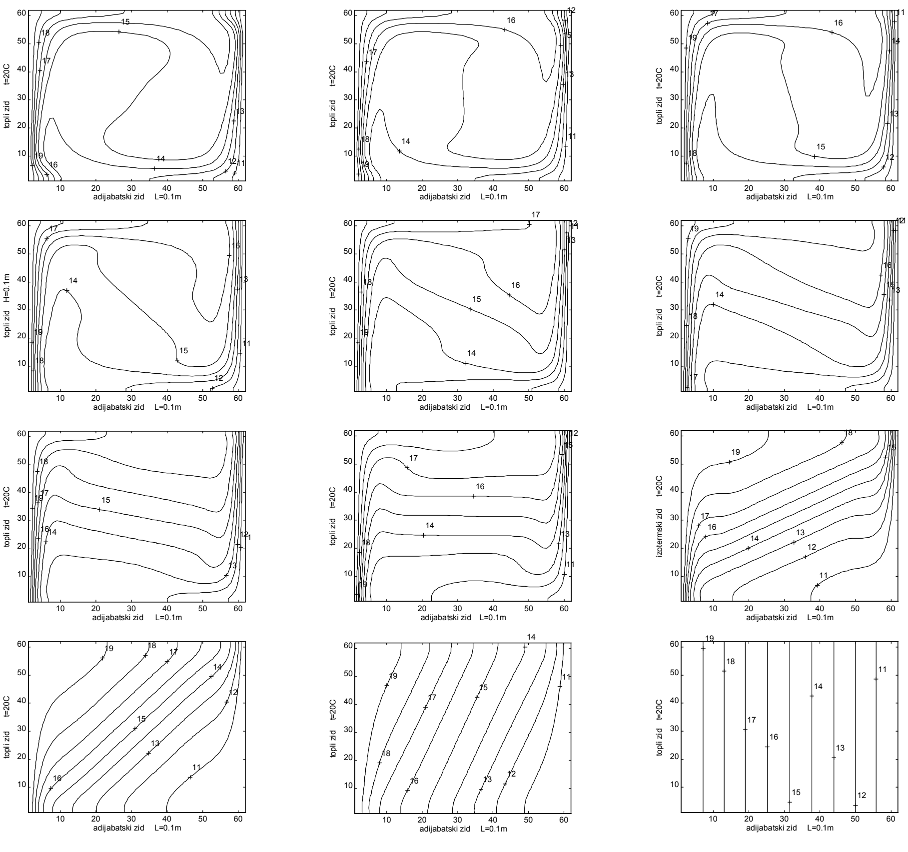

Fig. 2. Distribution of isotherms: angle of inclination: 20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°, 120°, 140°, 160°, 180°; Ra = $1{,}28\times10^6$, Air filed enclosure Pr = 0,73

5. ANALYSIS

Information about temperature field in domain is used to calculate values of local and average Nu number. In fact most important thing is to find out heat transfer rate in enclosures, especially on the isothemic walls. Local and average Nu number are expressed by eq. 17 and 18.

$$Nu = -\frac{H}{\Delta T}\left(\frac{\partial T}{\partial x}\right)_{x=0} \tag{16}$$

$$Nu_{sr} = \frac{1}{H}\int_{y=0}^{H} Nu dy \tag{17}$$

Results of named calculation are shown in Fig. 4. As we might see, Fig. 3., represents some area picture of the temperature gradient trough entire enclosed space. Calculation of the local and averaged Nu number are prepared for first line of temperature gradient near hot isothermal wall. Samples of temperature gradient are shown for inclination 60°and 140°.

Local Nu numbers dependent on angle of inclination are shown in Fig. 5. At 180° there is no flow in enclosure, although we have some negligible incorrect results for variables as result of imperfect numerical procedure and allowed numerical errors. As we see on the Fig. 5, local Nu number for named case is close to one, which means that the main heat transfer mechanism is conduction.

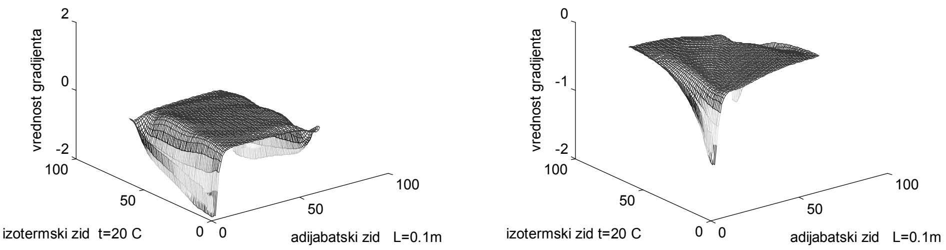

Fig. 3. Temperature gradient, angle of inclination 60°, 140°

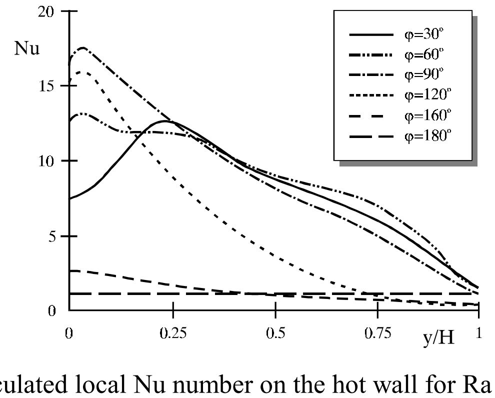

Fig. 4. Calculated local Nu number on the hot wall for Ra = $1{,}28\times10^6$

There is some flow activity occurs in enclosures with decreasing of rotation angle. Some negligible quantity of the fluid rises up on hot isothermal wall as well as on the cold wall (opposite situation). There is a nucleus of velocity field in enclosures as two slow independently rotated cells. With further decreasing of rotation the motion of the fluid become stronger. Therefore convection heat transfer mechanism become more basic than the diffusion one. Characteristic angle of inclination is about 90° with vertical boundary layer structure along the hot and cold walls. In the core region there is stable flow stratification and therefore stable isothermal stratification as can be seen on Fig. 2. The maximum values of the local Nu number on the hot wall are function of the angle of inclination. Therefore as we can see in Fig. 4., for angle of inclination 30° the maximum local Nu number is about 0,25y/H values. For angle 90° we have maximum local Nu number near lower corner point of hot isothermal wall and the opposite near upper corner of cold wall. On the other side, local Nu number on the cold isothermal wall shows the same but opposite picture. Therefore we have to know only values on the one of the isothermal walls. Those symmetrical presentations are valid for angle of inclination above 20°. Below this angle some inconsistent results are occurring as time dependent or some kind of three-dimensional results. As we've got some unstable flow structures we didn't investigate field variables for angles below 20°.

Averaged Nu number dependence from angle of inclination is shown in Fig. 5. There are some investigation results of many authors as we can see.

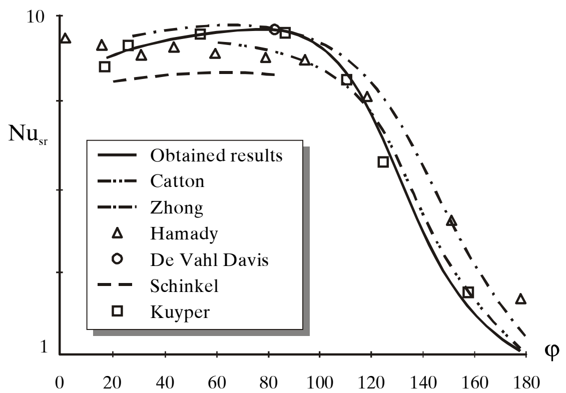

Fig. 5. Calculated averaged Nu number on the hot wall for $Ra = 1{,}28\times10^6$

Results express local maximum and minimum values. The maximum Nu number value is obtained for angle about 80°. The local minimum is obtained for the lowest angle of inclination. In fact, the obtained results have to satisfy two basic requests in order to correlate with literature ones. Those requests are: The local Nu number for angle 180° has to be equal to averaged Nu number and equal one, as well as change of the Nu number in function of angle is equal zero. As we can see in Fig. 5., there is some negligible disagreement in the results. It is from using very sensible measurement equipment and different of numerical procedure.

In fact, dependence of averaged Nu number on angle has two regimes. Nu number strongly decreases from 90° to 180° where is near by one. If the angle is less than 90°, the heat transfer rate is not changing drastically. We may conclude that when the hot wall is bellow the cold one the heat transfer rate is stronger. On the other side when the hot wall is above the cold one we have strong decreasing of heat transfer rate due to the diffusion. We might conclude that the enclosures behave like some thermal diode.

It has to remember that results of the investigation like this one, with some additional analyses, can be the base for defining the parameters of process in practice. There are different phenomenon of heating, cooling electronic equipment, cooling nuclear reactors, global natural phenomena and so on.

REFERENCES

1. Catton I., "Natural Convection in Enclosures", Proceeding of 6th International Heat Transfer Conference, vol. 6, pp 13-31, 1978.
2. Graham de Vahl Davis, "Finite Difference Methods for Natural and Mixed Convection in Enclosures", Proceeding of the Eighth International Heat Transfer Conference, Vol. 4, pp 101-109, San Francisco, 1986.
3. Hoogendoorn C. J., "Natural Convection in Enclosures", Proceeding of the Eighth International Heat Transfer Conference, Vol. 1, pp 111-121, San Francisco, 1986.
4. Kuyper R. A., "Transport Phenomena in Underground Coal Gasification Channels", ", Phd Thesis, Technishe Universitet Delft, The Netherlands, 1994.
5. Maekawa T., Tanasawa I., "Natural Convection Heat Transfer in Parallelogrammic Enclosures", Proceeding of the Seventh International Heat Transfer Conference, Vol. 2, pp 227-233, Munchen, 1982.
6. Mossel M. D., "Thermal Stratification of Turbulent air Flows in a Rectangular Channel", Phd Thesis, Technishe Universitet Delft, The Netherlands, 1995.
7. Ostrach S., "Natural Convection Heat Transfer in Cavities and Cells", Proceeding of the 7th International Heat Transfer Conference, vol. 1, pp 365-379, Munchen, 1982.
8. Ostrach S., "Advances in Heat Transfer", Academic Press, New York 1972.
9. Patankar S., "Numerical Heat Transfer and Fluid Flow", Hemisphere Publishing Co., New York, 1980.
10. Patankar S. V., "Numerical Methods in Heat Transfer", Proceeding of the Seventh International Heat Transfer Conference, Vol. 1, pp 83-90, Munchen, 1982.
11. Raos M.., 1999, Laminar Natural Convection in Enclosures. M.Sc. thesis, University of Nis, Faculty of Mechanical Engineering, Nis, Yugoslavia
12. Reddy C. S., "Numerical Simulation of Laminar Natural Convection in Shallow Inclined Enclosures", Proceeding of the International Heat Transfer Conference, Vol. 2, pp 263-269, Munchen, 1982.

ZAVISNOST NUSSELTOVOG BROJA OD UGLA NAGIBA PROSTORA OBLIKA PARALELOGRAMA U USLOVIMA PRIRODNE KONVEKCIJE

Miomir Raos, Ljiljana Živković

U radu je dat pregled istraživanja prenosa toplote u prostorima oblika paralelograma u uslovima prirodne konvekcije. U suštini rad je deo veće studije i predstavlja istraživanje lokalnog i srednjeg Nusselt-ovog (Nu) broja u pomenutim uslovima. Analizirane su promene lokalnog i srednjeg Nu broja na izotermskom toplom zidu u funkciji ugla nagiba prostora prema horizontalnoj ravni. Vektori polja promenjivih dobijeni numerickom procedurom i originalnim softverom dodatno su obradjivani rutinama paketa Mathlab i na osnovu toga date odredjene grafičke prezentacije u radu. Zaključci se mogu primeniti i nasuprotan hladni izotermski zid jer su slike strujanja i polja promenjivih simetrična i okrenuta, kao i vrednosti Nu brojeva.

Ključne reči: *prenos toplote, prirodna konvekcija, zatvoreni prostor*