

## DEFINING OF THE INTENSITY OF SOLAR RADIATION ON HORIZONTAL AND OBLIQUE SURFACES ON EARTH

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**Abstract.** *While making projects of passive and active solar systems for catching solar radiation it is necessary to know the intensity of solar radiation that comes to the receiving surface unit. On the basis of relevant factors, such as: local latitude, azimuth of the front of the object ( $\psi$ ), the angle of the slope of receiving surface ( $s$ ), coefficient of the reflection from the surroundings, etc., there has been made relations on the basis of which is given a mathematical model for calculating the intensity of solar radiation that comes to horizontal and vertical surfaces on Earth. By the help of mathematical model a programme has been made for defining the intensity of solar radiation on Earth surfaces.*

**Key words:** *The intensity of solar radiation, mathematical model, programme for defining the solar radiation intensity*

### 1. INTRODUCTION

The energy of Sun is created in its core in the course of fusion thermonuclear processes of oxygen into helium. The Earth which is  $150 \times 10^6$  kilometres far from the sun receives only  $0,5 \times 10^{-9}$  part of the Sun energy. The strength of solar radiation that comes to the Earth is  $175 \times 10^9$  MW and this surpasses for  $10^5$  times the strength of all power plants on Earth when they work with full strength.

The Sun is absolutely pure and free energy source for the whole mankind. The solar radiation can be used for obtaining electric power or for heating water, air or some other materials.

While making projects for passive and active systems for making use of solar radiation it is first necessary to define the intensity of solar radiation that comes to horizontal and oblique receiving surface of the particular solar system.

## 2. EXTRATERRESTRIAL SOLAR RADIATION

The solar radiation at the entrance into the Earth atmosphere is known as *extraterrestrial* radiation.

The intensity of extraterrestrial solar radiation is changeable because of the change in distance between the Earth and Sun and because of the Sun activity. The value of this radiation during the course of a year changes in the range from 1307 (W/m<sup>2</sup>) to 1393 (W/m<sup>2</sup>).

The intensity of extraterrestrial solar radiation that falls on the surface and that is at a right angle to the direction of the solar radiation can be calculated as follows:

$$I_{on} = I_{sc} \frac{r}{R^2} \quad [\text{W/m}^2] \quad (1)$$

where is:  $r$  – middle distance of the Earth from the Sun,  $R$  – instantaneous distance of the Earth from the Sun,  $I_{sc} = (1353 \pm 21)$  [W/m<sup>2</sup>] – solar constant [1].

The energy of extraterrestrial solar radiation that in a unit of time falls at a right on square meter of surface can be calculated as follows:

$$I_{on} = \left[ 1 + 0,0333 \cos\left(\frac{360n}{365}\right) \right] I_{sc} \quad [\text{W/m}^2] \quad (2)$$

where:  $n$  – a day in a year that counts from January 1<sup>st</sup>.

The energy of extraterrestrial radiation on horizontal surface can be calculated as follows:

$$I_{oH} = I_{on} \cos z \quad [\text{W/m}^2] \quad (3)$$

where:  $z$  – zenith angle ( the angle between direct solar beams and a line that is right angled on horizontal surface).

Zenith angle ( $z$ ) can be calculated as follows:

$$\cos z = \sin \alpha = \sin L \sin \delta + \cos L \cos \delta \cos h \quad (4)$$

By substitution of equation (4) in equation (3) the energy of extraterrestrial radiation on horizontal surface for particular day in a year can be calculated as follows:

$$I_{oH} = I_{sc} \left[ 1 + 0,033 \cos\left(\frac{360n}{365}\right) \right] (\sin L \sin \delta + \cos L \cos \delta \cos h) \quad [\text{W/m}^2] \quad (5)$$

where:  $I_{sc}$  – solar constant,  $n$  – a day in a year that counts from January 1<sup>st</sup>,  $L$  – local latitude,  $\delta$  – declination,  $h$  – hour angle.

Hour angle ( $h$ ) is expressed in degrees or radians but mostly in hour measure (an hour, minute, second). Hour angle changes by changing the place of observation and the apparent motion of the Sun by the circles parallel to the equator. The magnitude of hour angle ( $h$ ) can be calculated as follows:

$$h = \pm \frac{1}{4} (\text{number of minutes from local solar time}) \quad [^\circ] \quad (6)$$

where the + sign applies to afternoon hours and – sign to morning hours.

The maximum value of declination angle is  $\delta = 23^\circ 27'$ . It comes up with June 21<sup>st</sup> (summer longest daylight on the north hemisphere, i.e., winter shortest daylight on the south hemisphere). The minimal value of declination angle  $\delta = -23^\circ 27'$  comes up with

December 20<sup>th</sup> (summer longest daylight on the south hemisphere, i.e., winter shortest daylight on the north hemisphere). During the spring and autumn equinox, on March 21<sup>st</sup> and September 22<sup>nd</sup> the declination angle is 0°. The declination value ( $\delta$ ) can be calculated for every day in a year as follows:

$$\delta = 23,45 \sin \left[ \frac{360}{365} (284 + n) \right] \text{ [}^\circ\text{]} \quad (7)$$

where:  $n$  – a day in year ( $1 \leq n \leq 365$ ).

The whole daily extraterrestrial radiation from the sunrise to the sunset can be calculated as follows:

$$I_o = \frac{24}{\pi} I_{sc} \left[ 1 + 0,033 \cos \left( \frac{360n}{365} \right) \right] \left[ \cos L \cos \delta \cos h_s + \left( \frac{2\pi h_s}{360} \right) \sin L \sin \delta \right] \text{ [W/m}^2\text{]} \quad (8)$$

where:  $h_s$  – the time of sunrise (sunset) beyond the horizon.

In order to define the daily amount of energy of solar radiation on surfaces on the Earth, it is necessary to define the time of sunrise and sunset ( $h_s$ ) beyond the horizon. In the moment of sunrise (sunset) beyond the horizon the height angle of sun  $\alpha$  has 0 magnitude. By the substitution  $\alpha = 0$  in the following equation we get:

$$\sin \alpha = 0 = \sin L \sin \delta + \cos L \cos \delta \cos h_s \quad (9)$$

$$\cos h_s = -\text{tg } L \text{ tg } \delta \quad (10)$$

The equation (10) can be solved for  $h_s$  if  $-1 \leq -\text{tg } \delta \text{ tg } L \leq 1$ .

$$h_s = \text{arc cos } (-\text{tg } L \text{ tg } \delta) \quad (11)$$

The equation (11) has two solutions:

$$h_{S \text{ sunrise}} = -h_s \text{ and } h_{S \text{ sunset}} = h_s \quad (12)$$

where:  $h_{S \text{ sunrise}}$  and  $h_{S \text{ sunset}}$  – hour angles of sunrise and sunset. If:  $-\text{tg } \delta \text{ tg } L > 1$ , the Sun will not rise whole day (the Polar Night);  $-\text{tg } \delta \text{ tg } L < -1$ , the Sun will shine the whole day (the Polar Day).

Using the already obtained expressions for sunrise and sunset hour angles, the length of a day (time from sunrise to sunset beyond the horizon) can be calculated as follows:

$$t_{\text{day}} = \frac{2}{15} \cos^{-1} (-\text{tg } L \text{ tg } \delta) \quad (13)$$

[2, 3].

### 3. TERRESTRIAL SOLAR RADIATION

Passing through the earth atmosphere and because of dispersing and absorption on atoms and ions of present gases (oxygen, hydrogen, nitrogen, ozone, carbon dioxide, etc.) the intensity of solar radiation reduces for 25% – 30%. Solar radiation that comes to the earth is known as terrestrial radiation [1].

This reducing of energy because of its passing through the atmosphere can be presented by the help of *Bouguer – Lambert* law:

$$I_{Bn} = E_o e^{-\frac{B}{\sin \alpha}} = E_o e^{-Bm} \quad [\text{W/m}^2] \quad (14)$$

where:  $B$  – the attenuation coefficient of solar radiation in the earth atmosphere (table 1.),  $I_{Bn}$  – the energy of solar radiation that falls at the right angle on square meter of the earth surface in a unit of time,  $E_o$  – normal solar radiation on the Earth surface neglecting the existence of atmosphere (air mass  $m = 0$ ) (table 1.),  $m$  – optical air mass.

Table 1. Normal solar radiation on the Earth surface ( $E_o$ ) and corresponding data for  $C$  (the factor of diffusion radiation) and  $B$  (attenuation coefficient of solar radiation in the Earth atmosphere) for the twenty first day each month [4]

Month	$E_o$ [W/m <sup>2</sup> ]	$B$	$C$
January	1209	0,142	0,058
February	1193	0,144	0,060
March	1164	0,156	0,071
April	1115	0,180	0,097
May	1084	0,196	0,121
June	1069	0,205	0,134
July	1066	0,207	0,136
August	1088	0,201	0,122
September	1131	0,177	0,092
October	1172	0,160	0,073
November	1199	0,149	0,063
December	1212	0,142	0,057

Spectral distribution of the intensity of extraterrestrial and terrestrial solar radiation are different. This differences are consequences of distance change between the Earth and Sun and raducing the solar radiation because of dispersing and absorbtion on atoms, molecules and ions of gases in the earth atmosphere (hydrogen, oxygen, ozone, water steam, carbon dioxide etc.).

On the earth falls about 97% of solar radiation of the wave lenght range from 0,3 to 2,5  $\mu\text{m}$  and about 3% of the wave length range longer than 2,5  $\mu\text{m}$ . About 3% of the radiation that comes to the Earth are in ultraviolet, 42% in visible and 55% in infrared region of the spectrum of electromagnetic radiation.

The solar radiation that comes to the Earth is 0,5 milliard part of emitted solar energy, and that corresponds to the power of  $175 \times 10^9$  MW. This power is  $10^5$  stronger than the power of all power plants on the Earth when they work in full power [1, 5, 6, 7].

#### 4. GLOBAL SOLAR RADIATION

Two components of solar radiation come to the Earth surface. One component comes directly from the Sun (*direct solar radiation*) and the other originates from dispersing of direct solar radiation in the atmosphere (*diffuse solar radiation*). Global solar radiation consists of direct and diffuse solar radiation.

The radiation that reflects from surroundings (so called albedo) is of importance for some surfaces that are inclined under some angle to the horizontal surface. This radiation

is mainly diffuse and comes to the receiving surface under different angles. The intensity of the reflected solar radiation depends on the surroundings that it is reflected from.

## 5. DIRECT SOLAR RADIATION

The direct solar radiation represents a component of global solar radiation that comes directly to the earth in a bright and clear day. The direction of the direct radiation can be defined on every spot on the earth surface by geometry method. The direct solar radiation is a dominant component of global solar radiation for clear days.

To define the intensity of the direct solar radiation that comes to the Earth surface under the right angle the equation (14) can be used. If the observed surface is not horizontal the following relations for defining the direct solar radiation are used:

$$\cos z = \frac{I_B}{I_{Bn}} \quad (15)$$

$$\cos i = \frac{I_{Bt}}{I_{Bn}} \quad (16)$$

where:  $I_{Bn}$  – direct solar radiation that comes normally on square meter of surface in a time unit,  $I_B$  – the direct solar radiation on horizontal surface,  $I_{Bt}$  – the direct solar radiation that comes on oblique surface,  $z$  – zenith angle,  $i$  – the angle that sunbeams come to the inclined surface.

For a surface, that is inclined under some angle to the horizontal flat, the direct solar radiation can be calculated according to the following equation:

$$I_{Bt} = I_{Bn} \cos i \quad [\text{W/m}^2] \quad (17)$$

## 6. DIFFUSE SKY RADIATION

Except the direct solar radiation every surface receives a part of solar radiation that comes to it indirectly. It is called the diffuse solar sky radiation. Even on a brightest day, with minimal amount of water steam, about 8% of the whole energy of solar radiation that comes to the earth originates from diffuse radiation. Near towns and cities, as the consequence of air pollution and ground configuration, the diffuse sky radiation comes even to 22% of the complete radiation. During the cloudy days almost complete radiation is diffuse [5, 7].

The diffuse solar sky radiation ( $I_{dn}$ ) is minimal when the sky is bright and clear, but even in this conditions it contains enough energy that it must be considered. In practical calculations the value of this diffuse radiation can be calculated in such a way that it is supposed that the sky emits diffuse radiation equally in all directions and that vertical surfaces receive equal amounts of energy from diffuse solar sky radiation. The simplified formula for calculating the intensity of diffuse sky radiation for any orientation of the surface in the conditions of a clear day is as follows:

$$I_{dn} = I_d \left( \frac{1 + \cos s}{2} \right) \quad [\text{W/m}^2] \quad (18)$$

$$I_d = C I_{Bn} F_{SS} \quad [\text{W/m}^2] \quad (19)$$

where:  $I_d$  – diffuse radiation on horizontal surface,  $C$  – factor of diffuse radiation (table 1.),

$$F_{SS} = \frac{1}{2}(1 + \cos s), \text{ where } s \text{ is the inclination angle of the observed surface.}$$

## 7. REFLECTED DIFFUSE RADIATION

The surfaces that make an angle with a horizontal flat receives radiation that reflects from surroundings. The intensity of reflected diffuse radiation depends on reflection ability of horizontal flat ( $\rho_o$ ) and can be calculated as follows:

$$I_{dr} = (I_B + I_d)\rho_o \frac{1 - \cos s}{2} \quad [\text{W/m}^2] \quad (20)$$

where:  $I_d$  – diffuse solar radiation on horizontal flat,  $I_B$  – direct solar radiation on horizontal flat,  $s$  – inclination angle of optionally oriented surface to the horizontal flat,  $\rho_o$  – reflection coefficient known as ground albedo in meteorology.

Table 2. Albedo of horizontal surfaces [4]

Reflecting surface	$\rho_o$ [%]
Fresh snow	85
Quartz sand	35
River sand	29
Humus	26
Green grass	26
City as the whole	10
Water surface	2-80
Concrete	40
Red brick	44

If a city surroundings is considered, the value 0,2 can be taken for  $\rho_o$ . The albedo for water surface depends on the height of the sun and waves of water. It is important to take it into consideration for vertical surfaces near the coast [4].

## 8. ENTIRE DIFFUSE RADIATION

The entire diffuse solar radiation that comes to an optionally placed surface on the Earth is:

$$I_{dif} = I_d \left( \frac{1 + \cos s}{2} \right) + (I_d + I_B) \rho_o \left( \frac{1 - \cos s}{2} \right) \quad [\text{W/m}^2] \quad (21)$$

where:  $I_{dn}$  – diffuse sky solar radiation on obliquely placed surface,  $I_{dr}$  – diffuse reflecting solar radiation on obliquely placed surface,  $I_d$  – diffuse sky solar radiation on horizontal surface,  $s$  – inclination angle of optionally oriented surface towards horizontal flat,  $I_B$  – direct solar radiation on horizontal surface [3].

## 9. THE ENTIRE SOLAR RADIATION ON HORIZONTAL SURFACE

The entire daily solar radiation that to a horizontal surface is:

$$I_{uh} = I_B + I_d \quad [\text{W/m}^2] \quad (22)$$

where:  $I_B$  – direct solar radiation on horizontal surface,  $I_d$  – diffuse solar radiation on horizontal surface.

## THE ENTIRE SOLAR RADIATION ON OBLIQUE SURFACE

The intensity of the entire solar radiation that comes to obliquely placed surface can be calculated as follows:

$$I_u = I_B R_B + I_d \left( \frac{1 + \cos s}{2} \right) + (I_B + I_d) \rho_o \left( \frac{1 - \cos s}{2} \right) \quad [\text{W/m}^2] \quad (23)$$

or

$$I_u = R I_{uh}$$

where:  $I_{dn}$  – diffuse sky solar radiation that comes to obliquely placed surface,  $I_{dr}$  – diffuse reflecting solar radiation that comes to obliquely placed surface,  $I_B$  – direct solar radiation on horizontal surface,  $I_d$  – diffuse solar radiation on horizontal surface,  $\rho_o$  – reflection coefficient,  $s$  – inclination angle of optionally oriented surface towards horizontal flat,  $R_B$  – relation between direct solar radiation that comes to horizontal surface,  $R$  – relation between entire solar radiation to horizontal surface.

The values  $R_B$  and  $R$  can be calculated by the help of the following formulas:

$$R_B = \frac{\cos i}{\cos z} = \frac{\sin L \sin \delta \cos s - \cos L \sin \delta \sin s \cos \psi + \cos L \cos \delta \cos h \cos s + \sin L \cos \delta \cos h \sin s \cos \psi + \cos \delta \sin h \sin s \sin \psi}{\cos L \cos \delta \cos h + \sin L \sin \delta} \quad (24)$$

$$R = \frac{I_B}{I_{uh}} R_B + \frac{I_d}{I_{uh}} \left( \frac{1 + \cos s}{2} \right) + \rho_o \left( \frac{1 - \cos s}{2} \right) \quad (25)$$

where:  $i$  – fall in angle,  $z$  – zenith angle,  $\delta$  – declination,  $L$  – local latitude,  $s$  – inclination angle of optionally oriented surface to horizontal flat,  $I_B$  – direct solar radiation on horizontal surface,  $\psi$  – solar wall azimuth (the angle that defines the place of wall in relation to the sunbeams) [8, 9, 10].

## 10. SIMULATION RESULTS

On the base on already stated mathematical model, it has been made a computer programme **InSunZra**, for calculating daily hour intensity of solar radiation that comes to horizontal and obliques surfaces on the Earth.

By using the programme **InSunZra**, it has been calculated the intensity of solar radiation that comes to vertical wall surface ( $s = 90^\circ$ ) that has orientation  $\psi = 0^\circ$  (the wall on which sunbeams come is orientated to south, i.e., the wall is orientated to the east –

west direction by its longer side);  $\psi = +80^\circ$  (the wall is by its longer side oriented in the South West – North East direction, i.e., it is oriented to the east);  $\psi = -80^\circ$  (the wall is by its longer side oriented in the Sout East – North West direction, i.e., it is oriented to the west).

In the fig.1., fig. 2. and fig. 3. it is shown the intensity of solar radiation that comes to vertical wall surface in January, Febryary and March for:

$\psi = 0^\circ$ ;  $s = 90^\circ$ ;  $L = 43^\circ$ (Nis);

$\psi = +80^\circ$ ;  $s = 90^\circ$ ;  $L = 43^\circ$ (Nis);

$\psi = -80^\circ$ ;  $s = 90^\circ$ ;  $L = 43^\circ$ (Nis).

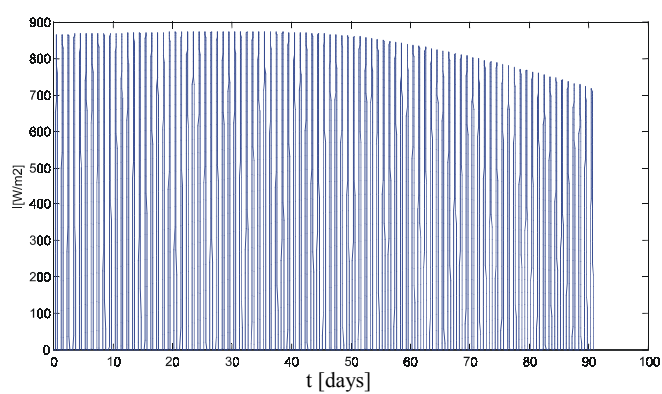


Fig. 1. The intensity of solar radiation that comes to vertical wall surface ( $s = 90^\circ$ ) for orientation  $\psi = 0^\circ$

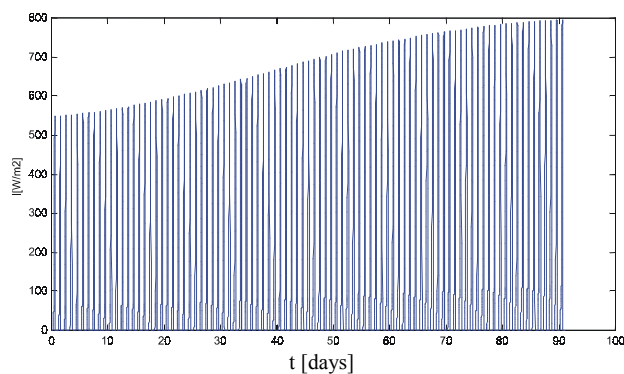


Fig. 2. The intensity of solar radiation that comes to vertical wall surface ( $s = 90^\circ$ ) for orientation  $\psi = +80^\circ$



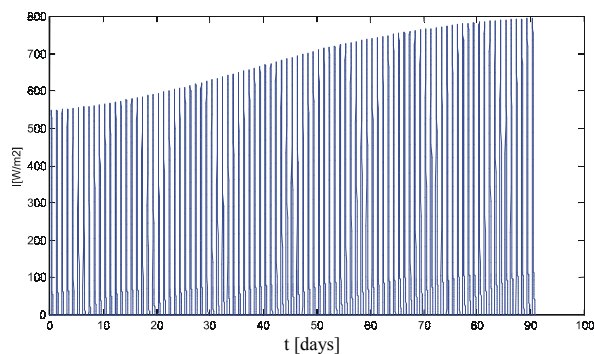


Fig. 3. The intensity of solar radiation that comes to vertical wall surface ( $s = 90^\circ$ ) for orientation  $\psi = -80^\circ$

In fig. 4. it is shown the intensity of solar radiation that comes to vertical wall surface for orientation  $\psi = 0^\circ$ ;  $\psi = \pm 80^\circ$  for  $L = 43^\circ$ (Nis) during January 15 and 16.

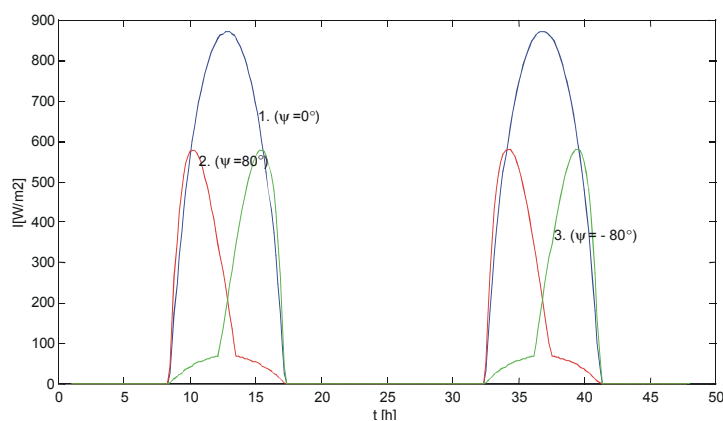


Fig. 4. The intensity of solar radiation that comes to vertical wall for:  
1.  $\psi = 0^\circ$ ; 2.  $\psi = +80^\circ$ ; 3.  $\psi = -80^\circ$

## 11. CONCLUSION

By using the **InSunZra** programme it can be calculated the intensity of solar radiation that comes in receiving surface unit, for passive and active solar systems, for optional location. The receiving surface can be optionally oriented ( $\psi = 0^\circ \div \pm \pi/2$ ) and optionally to horizontal surface ( $s = 0^\circ \div 90^\circ$ ).

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## ODREĐIVANJE INTENZITETA SUNČEVOG ZRAČENJA NA HORIZONTALNE I KOSE POVRŠINE NA ZEMLJI

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*Prilikom projektovanja pasivnih i aktivnih solarnih sistema za zahvatanje sunčevog zračenja neophodno je poznavati intenzitet sunčevog zračenja koji dospeva na jedinicu prijemne površine. Na osnovu relevantnih faktora, kao što su: geografska širina mesta, azimut fasade objekta ( $\psi$ ), ugao nagiba prijemne površine ( $s$ ), koeficijent refleksije od okoline, itd., urađene su relacije na osnovu kojih je dat matematički model za izračunavanje intenziteta sunčevog zračenja koji dospeva na horizontalne i vertikalne površine na Zemlji. Pomoću matematičkog modela urađen je program za određivanje intenziteta sunčevog zračenja na površine na Zemlji.*

**Ključne reči:** *Intenzitet sunčevog zračenja, matematički model, program za određivanje intenziteta sunčevog zračenja*