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Address: Univerzitetski trg 2, 18000 Niš, YU
Tel: +381 18 547-095, Fax: +381 18 547-950

GENERALIZED CANONICAL ANALYSIS OF THE FREE RAMOND STRING FIELD THEORY *

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M. Blagojević[1], B. Sazdović[1], M. Vasilić[2]

Institute of Physics, P.O. Box 57, 11001 Belgrade, Yugoslavia
Department of Theoretical Physics, Institute Vinča, P.O. Box 522, 11001 Belgrade Yugoslavia

Abstract. *Motivated by the canonical BRST quantization procedure, we analyze the classical structure of the free Ramond string field theory. The gauge symmetries of the theory are shown to be infinitely reducible, and the higher structure functions are explicitly found.*

Key words: *Ramond string, Batalin-Fradkin, reducibility*

1. INTRODUCTION

The presence of the inverse picture changing operator in the kinetic term for Witten's formulation of the Ramond string field theory [1] is the origin of extra gauge symmetries [2,3], additional to the ones generated by the BRST operator.The authors of Ref. 3 argue that these extra gauge symmetries are just what we need to be able to impose two gauge conditions on the Ramond string field, $\bar{c}_0\psi = 0$, $\bar{e}_0\psi = 0$ (the zero modes of the conformal antighost $\bar{c}_0$ and the superconformal antighost $\bar{e}_0$ anihilate the string field ψ).

For a proper quantization of this theory we apply the Batalin-Fradkin (BF) hamiltonian approach [4,5]. In sect. 2 we introduce the definitions of the action, BRST charge, inverse picture changing operator for the free Ramond string theories and some properties useful for later convenience. In sect. 3 we analyze the features of the classical hamiltonian dynamics which is a basis for the BRST quantization. We find that all the constraints are first class, and that the generators of the gauge transformations are associated with the already known gauge symmetries (sec. 4). In sect. 5 we show that all the symmetries are infinitely reducible, and that the symmetry generated by the BRST

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charge have growing reducibility. We also find explicitely the higher structure functions. Sect. 6 is devoted to conclusions.

2. LAGRANGIAN

The free field theory of the Ramond superstring is described by the action [1]

$$I = \overline{\psi} Y Q \psi \tag{1}$$

where ψ and $\overline{\psi}$ are Grassmann valued vectors belonging to the Fock representation of the commutation relations:

$$\begin{aligned}
\{c(\sigma), \overline{c}(\sigma')\} &= 2\pi\delta(\sigma-\sigma'), \\
[e(\sigma), \overline{e}(\sigma')] &= 2\pi\delta(\sigma-\sigma'), \\
\{d^\mu(\sigma), d^\nu(\sigma')\} &= 2\pi\eta^{\mu\nu}(\sigma-\sigma'), \\
[p^\mu(\sigma), p^\nu(\sigma')\} &= 2\pi i\eta^{\mu\nu}\partial_\sigma\delta(\sigma-\sigma').
\end{aligned} \tag{2}$$

The BRST charge of the first quantization Q and the inverse picture changing operator Y are given by

$$Q = \frac{1}{2\pi}\int_0^{2\pi} d\sigma [c(\frac{i}{2}p^2 + \frac{1}{2}d'd + c'\overline{c} - \frac{3}{2}e'\overline{e} - \frac{1}{2}e\overline{e}') + edp - ie^2\overline{c}], \tag{3a}$$

$$Y(\sigma) = -c(\sigma)\delta'(e(\sigma)), \quad Y \equiv Y(\pi/2), \tag{3b}$$

where primes denote first derivatives over the corresponding arguments.

For the purpose of the Hamiltonian analysis it is necessary to explicitly separate time derivatives in the kinetic operator YQ. To this end we make use of the Fourier decomposition

$$p^\mu(\sigma) = \sum_n e^{-in\sigma} p_n^\mu, \quad p_0^0 \equiv i\frac{\partial}{\partial t},$$

to see that Q can be written in the form

$$Q \equiv A\frac{\partial^2}{\partial t^2} + B\frac{\partial}{\partial t} + C. \tag{4}$$

Then the action $I = \int dtL$ defines the Lagrangian of the theory

$$L = -\dot{\overline{\psi}} YA\dot{\psi} + \frac{1}{2}(\overline{\psi}YA\dot{\psi} - \dot{\overline{\psi}}YB\psi) + \overline{\psi}YC\psi, \tag{5}$$

where the dot denotes time derivative.

To proceede we need some properties of the operators A, B, C and Y appearing in (5). We shall use the well known ones

$$Q^2 = 0, \quad \{Q, Y\} = 0, \tag{6a}$$

from which it follows

$$A^2 = C^2 = B^2 + \{A,C\} = 0\,, \quad \{A,B\} = \{B,C\} = 0\,,$$
$$\{A,Y\} = \{B,Y\} = \{C,Y\} = 0\,, \tag{6b}$$

and also

$$cY = Y\,c = 0\,, \quad e^2Y = Ye^2 = 0\,,$$
$$\{A,c\} = [A,e^2] = 0\,, \tag{7}$$

where we have adopted the notation $c \equiv c(\pi/2)$ and $e \equiv e(\pi/2)$ to simplify further exposition.

3. Hamiltonian and constraints

Let us begin with the definition of the canonical momenta:

$$\pi \equiv \partial_R L/\partial\dot{\overline{\psi}} = YA\dot{\psi} + \frac{1}{2}YB\psi\,,$$
$$\overline{\pi} \equiv \partial_R L/\partial\dot{\psi} = -\dot{\overline{\psi}}YA + \frac{1}{2}\overline{\psi}YB\,. \tag{8}$$

As a consequence of the singularity of the operator $Y\,A$ the following primary constraints appear:

$$\Phi_1 \equiv e^2\pi \approx 0\,, \quad \Phi_2 \equiv c\pi \approx 0\,,$$
$$\Phi_3 \equiv A(\pi - \frac{1}{2}YB\psi) \approx 0\,,$$
$$\overline{\Phi}_1 \equiv -\overline{\pi}e^2 \approx 0\,, \quad \overline{\Phi}_2 \equiv \pi c \approx 0\,,$$
$$\overline{\Phi}_3 \equiv (-\overline{\pi} + \frac{1}{2}\overline{\psi}YB)A \approx 0\,. \tag{9}$$

Up to these constraints the velocities in (8) can be expressed in terms of the momenta. To do that we need the "inverse" of YA:

$$ZYA = 1 + e^2\alpha_1 - \check{c}\,\alpha_2 + A\alpha_3\,,$$
$$YAZ = 1 + \overline{\alpha}_1 e^2 - \overline{\alpha}_2\,\check{c} + \overline{\alpha}_3 A\,, \tag{10}$$

where $\check{c} \equiv c - c_0$, c_0 being the zero component in the Fourie decomposition of $c(\sigma)$ (note that $A = c_0/2i$). In appendix A we explicitly found the form of the operator Z,

$$Z = -2\pi i\overline{A}\,\overline{c}_1[e\delta(i\overline{e}_0) + \delta(i\overline{e}_0)e]\,, \tag{11}$$

where $\overline{A} \equiv 2i\overline{c}_0$.

With the help of the operator Z we are able to construct the canonical Hamiltonian,

$$H_c = \overline{\Pi}Z\Pi - \overline{\psi}YC\psi\,, \tag{12}$$

where

$$\Pi \equiv \pi - \frac{1}{2} YB\psi \,, \quad \overline{\Pi} \equiv \overline{\pi} - \frac{1}{2} \overline{\psi} YB \,. \tag{13}$$

The total Hamiltonian is then obtained by adding to H_c all primary constraints with the corresponding multipliers:

$$H_T = H_c + \sum_{\alpha=1}^{3} (\overline{\lambda}_\alpha \Phi_\alpha + \overline{\Phi}_\alpha \lambda_\alpha) \,. \tag{14}$$

Now, to insure consistency of the theory, we demand that all the primary constraints be preserved in time [6]:

$$\dot{\Phi}_\alpha = \{\Phi_\alpha, H_T\} \approx 0 \,,$$
$$\dot{\overline{\Phi}}_\alpha = -\{H_T, \overline{\Phi}_\alpha\} \approx 0 \,.$$

Using the fact that the basic non-zero Poisson brackets are given by

$$\{\psi, \overline{\pi}\} = \{\pi, \overline{\psi}\} = 1$$

we find the following secondary constraints:

$$\begin{aligned} \Phi_4 &\equiv B\Pi + Y\, AC\psi \approx 0 \,, \\ \overline{\Phi}_4 &\equiv \overline{\Pi} B - \overline{\psi} C\, AY \approx 0 \,. \end{aligned} \tag{15}$$

These constraints should also be preserved in time, as a consequence of which we obtain tertiary constraints:

$$\begin{aligned} \Phi_5 &\equiv C(\Pi - BY\psi) \approx 0 \,, \\ \overline{\Phi}_5 &\equiv (-\overline{\Pi} + \overline{\psi} BY) C \approx 0 \,. \end{aligned} \tag{16}$$

In the next step we examine the consistency conditions of the tertiary constraints and find that these are identically satisfied. At that step the procedure is finished with the result that *no multiplier is determined and all the constraints are the first class ones*. Their Grassmann parities are given by $\varepsilon(\Phi_{\alpha_0}) = \varepsilon(\overline{\Phi}_{\alpha_0}) = \delta^1_{\alpha_0}$, $\alpha_0 = 1,2,...,5$.

The algebra of the constraints is easily checked to be Abelian:

$$\{\Phi, \overline{\Phi}\} = 0 \tag{17}$$

where, for the purpose of simplifying forthcoming exposition, we introduced the matrix notation :

$$\Phi \equiv \begin{pmatrix} \vdots \\ \Phi_{\alpha_0} \\ \vdots \end{pmatrix}, \quad \overline{\Phi} \equiv (\cdots \overline{\Phi}_{\alpha_0} \cdots) \,, \quad \alpha_0 = 1,2,...,5 \,. \tag{18}$$

On the other hand, the constraints commute with the Hamiltonian only weakly defining structure constants $\overline{V}_0$ and V_0:

$$\{\Phi, H_c\} = \overline{V}_0 \Phi \,, \qquad -\{H_c, \overline{\Phi}\} = \overline{\Phi} V_0 \,. \tag{19}$$

By a direct calculation one obtains

$$\overline{V_0} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ B\overline{\alpha}_1 & B\overline{\alpha}_2 & B\overline{\alpha}_3 & 1 & 0 \\ C\overline{\alpha}_1 & C\overline{\alpha}_2 & C\overline{\alpha}_3 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad V_0 \equiv \begin{pmatrix} 0 & 0 & -\alpha_1 B & \alpha_1 C & 0 \\ 0 & 0 & -\alpha_2 B & \alpha_2 C & 0 \\ 0 & 0 & -\alpha_3 B & \alpha_3 C & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \tag{20}$$

4. Gauge symmetries

The presence of first class constraints in any theory indicates that the theory posseses a gauge symmetry with as many gauge parameters as is the number of primary first class constraints. Applying the well known algorithm for constructing gauge generators [7] to our case we find

$$\begin{gathered} G_1 = -\overline{\eta}\Phi_1\,, \quad \overline{G}_1 = -\overline{\Phi}_1\eta\,, \\ G_2 = \overline{\omega}\Phi_2\,, \quad \overline{G}_2 = \overline{\Phi}_2\omega\,, \\ G_3 = -\ddot{\overline{\varepsilon}}\Phi_3 + \dot{\overline{\varepsilon}}\Phi_4 - \overline{\varepsilon}\Phi_5\,, \quad \overline{G}_3 = -\overline{\Phi}_3\ddot{\varepsilon} + \overline{\Phi}_4\dot{\varepsilon} - \overline{\Phi}_5\varepsilon\,, \end{gathered} \tag{21}$$

where η, ω, ε and $\overline{\eta}, \overline{\omega}, \overline{\varepsilon}$ are gauge parameters. Their action on fields ψ will then be

$$\begin{gathered} \delta_1\psi = e^2\eta\,, \quad \delta_2\psi = c\omega\,, \\ \delta_3\psi = A\ddot{\varepsilon} + B\dot{\varepsilon} + C\varepsilon \equiv Q\varepsilon\,, \end{gathered} \tag{22}$$

which is just the form of gauge symmetries of the action (1) we have expected.

5. Reducibility and higher structure functions

It is easy to see that not all the gauge parameters in Eq. (22) are independent. From the nilpotency of the BRST charge of the first quantization Q, for example, it follows that changing the parameter ε according to $\delta\varepsilon = Q\varepsilon_1$ will not change the gauge transformation $\delta_3\psi$. Similarly, the change of the parameters η and ω by $\delta\eta = Y\eta_1$ and $\delta\omega = Y\omega_1$ will be ineffective as follows from Eq.(7). We say that the theory is reducible. In the Hamiltonian approach this fact is manifested in the existence of "constraints on constraints". For example, one easily finds that

$$\delta'(e)\Phi_1 = 0\,, \quad c\Phi_2 = 0\,, \quad A\Phi_3 = 0\,, \tag{23a}$$

and also

$$\begin{gathered} B\Phi_3 + A\Phi_4 = 0\,, \quad C\Phi_3 + B\Phi_4 + A\Phi_5 = 0\,, \\ C\Phi_4 + B\Phi_5 = 0\,, \quad C\Phi_5 = 0\,. \end{gathered} \tag{23b}$$

Using the matrix notation we can rewrite Eq.(23) as

$$T_1\Phi = 0\,, \tag{24a}$$

where T_1 is 5×7 matrix of the form

$$T_1 \equiv \begin{pmatrix} \delta'(e) & 0 & & & \\ & & & 0 & \\ 0 & c & & & \\ & & A & 0 & 0 \\ & & B & A & 0 \\ & 0 & C & B & A \\ & & 0 & C & B \\ & & 0 & 0 & C \end{pmatrix} . \tag{24b}$$

The relations (24) define the so-called zero modes of the constraints Φ, and are the consequence of the fact that not all Φ's are independent. But neither are the relations among them. There exists another matrix T_2 whose null–vectors define zero modes of the relations (24) themselves:

$$T_2 T_1 = 0 . \tag{25}$$

The matrix T_2 is 7×9 matrix and it has zero modes of its own. The procedure is never ending and the theory is, consequently, infinitely reducible. Moreover, we have here the so-called growing reducibility, since the number of zero modes at any stage is bigger than the one of the preceding stage. The complete set of zero modes is then

$$T_{k+1} T_k = 0 , \quad k \geq 1 \tag{26}$$

with

$$T_k \equiv \begin{pmatrix} N_k & 0 \\ 0 & M_k \end{pmatrix} , \quad k \geq 1 , \tag{27a}$$

where N_k are 2×2 matrices of the form

$$N_{2k-1} \equiv \begin{pmatrix} \delta'(e) & 0 \\ 0 & c \end{pmatrix} , \quad N_{2k} \equiv \begin{pmatrix} e^2 & 0 \\ 0 & c \end{pmatrix} , \tag{27b}$$

and M_k are $(2k+1) \times (2k+3)$ matrices given by

$$M_k \equiv \begin{pmatrix} A & & & \\ B & A & & 0 \\ C & B & & \\ & C & \ddots & \\ & & & A \\ & 0 & & B \\ & & & C \end{pmatrix} . \tag{27c}$$

Similarly, we find the zero modes of the constraints $\overline{\Phi}$:

$$\overline{\Phi}\overline{T}_1 = 0 \tag{28}$$

and also

$$\overline{T}_k \overline{T}_{k+1} = 0\,, \quad k \geq 1\,, \tag{29}$$

where $\overline{T}_k \equiv dT_k^+ d(d/\sqrt{2} \equiv d_0^0 =$ the zero component in the Fourier decomposition of $d^\mu(\sigma)$). Explicitly:

$$\overline{T}_k \equiv \begin{pmatrix} \overline{N}_k & 0 \\ 0 & \overline{M}_k \end{pmatrix}, \tag{30a}$$

$$\overline{N}_{2k-1} \equiv \begin{pmatrix} \delta'(e) & 0 \\ 0 & -c \end{pmatrix}, \quad \overline{N}_{2k} \equiv \begin{pmatrix} e^2 & 0 \\ 0 & -c \end{pmatrix}, \tag{30b}$$

and

$$\overline{M}_k \equiv \begin{pmatrix} A & -B & C & & & & \\ & A & -B & C & & & 0 \\ & & & & & & \\ & 0 & & & & & \\ & & & & A & -B & C \end{pmatrix} \tag{30c}$$

are $(2k + 3) \times (2k + 1)$ matrices.

The coefficients T_k and $\overline{T}_k$ completely define reducibility of the theory and, owing to its Abbelian nature, exhaust the set of structure functions necessary for the construction of the BRST charge.

To construct the BRST invariant Hamiltonian, on the other hand, we need another set of structure functions. The way of defining those is close to what we did in the Lagrangian treatments of the superparticle and the superstring [8]. We start with the relation that defines structure constants V_0 and multiply it by $\overline{T}_1$ to obtain

$$0 = -\{H_c, \overline{\Phi}\overline{T}_1\} = \overline{\Phi} V_0 \overline{T}_1\,. \tag{31}$$

Since Φ's are not all independent, the general solution for the coefficient multiplying $\overline{\Phi}$ will have the form:

$$V_0 \overline{T}_1 = \overline{T}_1 V_1\,, \tag{32}$$

which defines new structure constants V_1. Similarly, multiplying Eq.(32) by $\overline{T}_2$ from right, and using the relation $\overline{T}_1\overline{T}_2 = 0$ we find

$$\overline{T}_1 V_1 \overline{T}_2 = 0\,, \tag{33}$$

whose general solution

$$V_1 \overline{T}_2 = \overline{T}_2 V_2 \tag{34}$$

serves as a definition of new structure constants V_2. Continuing this procedure we obtain an infinite set of structure constants V_k:

$$V_{k-1}\overline{T}_k = \overline{T}_k V_k\,, \quad k \geq 1\,. \tag{35}$$

In a similar way, starting with the relation that defines $\overline{V}_0$, we can define structure constants $\overline{V}_k$:

$$T_k \overline{V}_{k-1} = \overline{V}_k T_k\,, \quad k \geq 1\,. \tag{36}$$

As will be seen in the next section the coefficients V_k, $\overline{V}_k$ $(k \geq 1)$ exhaust the number of structure functions that are necessary for the construction of the BRST invariant Hamiltonian.

Solving Eqs. (35) and (36) in Appendix B we have found the explicit form of $(2k+5) \times (2k+5)$ matrices V_k and $\overline{V}_k$, $(k \geq 1)$:

$$V_k = \begin{pmatrix} 0 & 0 & & \cdots & & & 0 \\ 0 & 0 & & \cdots & & & 0 \\ 0 & 0 & \overline{A}B & -\overline{A}C & 0 & \cdots & 0 \\ & & 1 & & & & \\ \vdots & \vdots & & 1 & & 0 & \vdots \\ & & 0 & & \ddots & & \\ 0 & 0 & & & & 1 & 0 \end{pmatrix} \tag{37a}$$

$$\overline{V}_k = \begin{pmatrix} 0 & 0 & 0 & & \cdots & & 0 \\ 0 & 0 & 0 & & \cdots & & 0 \\ & & -B\overline{A} & 1 & & & \\ & & -C\overline{A} & & 1 & 0 & \\ \vdots & \vdots & 0 & & & \ddots & \vdots \\ & & \vdots & & 0 & & 1 \\ 0 & 0 & 0 & & \cdots & & 0 \end{pmatrix}. \tag{37b}$$

6. CONCLUSIONS

The BRST quantization of the free Ramond string field theory has been studied using the systematic BF method of canonical quantization.

By analyzing the classical hamiltonian structure we have found the generators of the gauge transformations. They are associated with the already known gauge symmetry (generated by the first quantization BRST charge also present in the bosonic case) and the new gauge symmetry specific for the Ramond theory [2,3]. We have also shown that this is an infinite stage reducible theory. The extra gauge symmetry has the same number of constraints at any level of reducibility while for the old gauge symmetry the number of the constraints grows with the stage of reducibility.

APPENDIX A

In this appendix the operator Z, which is defined in (11) as the "inverse" to YA, is constructed and the explicit form of the operators α_k, $\overline{\alpha}_k$ is found.

We begin with the quantum mechanical problem of finding "inverse" to $\delta'(\hat{x})$, where $\hat{x} = \hat{x}^+$ is the well known position operator. Since $\delta'(x)x^2 = 0$ the operator $\delta'(\hat{x})$ can only have the "inverse up to terms proportional to $\hat{x}^2$":

$$\hat{U}\delta'(\hat{x}) = 1 + \hat{x}^2\hat{\xi} . \tag{A1}$$

In the x-representation this equation is written as

$$\int dy U(x,y)\delta'(y)\psi(y) = \psi(x) + O(x^2) , \quad \forall\psi(x) , \tag{A2}$$

where U(x,y) $\equiv \langle x \,|\, \hat{U} \,|\, y \rangle$ is the kernel of the operator $\hat{U}$. Solving (A2) one easily finds that

$$U(x,y) = -x - y . \tag{A3}$$

In the momentum representation $(\hat{p} = \hat{p}^+ ,\ [\hat{x}, \hat{p}] = -i)$ this kernel takes the form

$$\left\langle k \,|\, \hat{U} \,|\, p \right\rangle = 2\pi i[\delta(p)\delta'(k) - \delta'(p)\delta(k)] . \tag{A4}$$

It is not difficult to show then that the operator

$$\hat{U} = -2\pi[\hat{x}\delta(\hat{p}) + \delta(\hat{p})\hat{x}] \tag{A5}$$

is just the operator whose kernel in the momentum representation is given by (A4).

Now, having solved the equation (A1), we can easily pass to the problem of finding "inverse" of $\delta'(e)$. Defining

$$\hat{x} \equiv e , \quad \hat{p} \equiv -i\overline{e}_0 \tag{A6}$$

and using $[e, \overline{e}_0]$ we convince ourselves that $[\hat{x}, \hat{p}] = -i$ and the problem boils down to (A1) with the same answer (A5). Consequently,

$$U = -2\pi[e\delta(i\overline{e}_0) + \delta(i\overline{e}_0)e] \tag{A7}$$

is the solution of

$$U\delta'(e) = 1 + e^2\xi . \tag{A8}$$

The next step is the construction of the "inverse" to $Y = -c\delta'(e)$. Since $\{c, \overline{c}_1\} = i$ we look for the solution of

$$XY = 1 + O(e^2) + O(c) \tag{A9}$$

in the form $X \sim \overline{c}_1 U$. Direct calculation leads us to

$$X = -2\pi i\overline{c}_1[e\delta(i\overline{e}_0) + \delta(i\overline{e}_0)e] . \tag{A10}$$

Finally, having in mind the relation $\{A, \bar{A}\} = 1$, one can easily verify that the equation

$$ZYA = 1 + O(e^2) + O(c) + O(A) \tag{A11}$$

has the solution displayed in Eq. (11).

To find the operators α_1, α_2 and α_3 of the equation (10) we multiply YA from the left by Z and use (A8) to obtain

$$ZYA = -i\bar{c}_1\breve{c}\bar{A}A - ie^2\xi(e,\bar{e}_0)\bar{c}_1\breve{c}\bar{A}A\,. \tag{A12}$$

wherefrom we read the operator α_1:

$$\alpha_1 = \beta_1 A\,,\ \ \beta_1 \equiv -i\xi(e,\bar{e}_0)c_1\breve{c}\bar{A} \tag{A13}$$

Then, performing commutations of $\bar{c}_1$ with $\breve{c}$ and $\bar{A}$ with A leads to

$$ZYA = 1 - \bar{A}A + i\breve{c}\bar{c}_1\bar{A}A + e^2\alpha_1\,. \tag{A14}$$

Comparing (A14) with (10) gives

$$\alpha_2 = \beta_2 A\,,\ \ \beta_2 \equiv -i\bar{c}_1\bar{A}\,,\ \ \alpha_3 = -\bar{A}\,. \tag{A15}$$

The operators $\bar{\alpha}_k$ ($k = 1,2,3$) are easily obtained after using $Z^+ = dZd$ in (10). The result is

$$\bar{\alpha}_k = d\alpha_k^+ d \tag{A16}$$

APPENDIX B

In this appendix the structure functions V_k and $\bar{V}_k$ are evaluated.

To find the higher structure functions V_k and $\bar{V}_k$ ($k \geq 1$) we need not know the operators α_k, $\bar{\alpha}_k$ in all details. It will be sufficient to use the fact that they can be written in the form

$$\begin{aligned} &\alpha_1 = \beta_1 A\,,\ \alpha_2 = \beta_2 A\,,\ \alpha_3 = -\bar{A} \\ &\bar{\alpha}_1 = A\bar{\beta}_1\,,\ \bar{\alpha}_2 = A\bar{\beta}_2\,,\ \bar{\alpha}_3 = -\bar{A}\,. \end{aligned} \tag{B1}$$

Then, a direct calculation leads to

$$V_0\bar{T}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\bar{A}AB & \bar{A}AC & 0 & 0 & 0 \\ 0 & 0 & A & -B & C & 0 & 0 \\ 0 & 0 & 0 & A & -B & C & 0 \end{pmatrix} \tag{B2}$$

which is easily checked to equal $\bar{T}_1V_1$, provided the matrix V_1 is given by

$$V_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{A}B & -\overline{A}C & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \tag{B3}$$

Knowing V_1 one can now evaluate $V_1\overline{T}_2$ and show that it is proportional to $\overline{T}_2$ from the left. The corresponding coefficient will be V_2. These two steps turn out to provide enough motivation for the assumption that V_k has the form (37a). The verification of the assumption is straightforward.

The structure constants $\overline{V}_k$ are obtained from Eq. (35) by using $\overline{T}_k = dT_k^+ d$, wherefrom it follows

$$\overline{V}_k = dV_k^+ d\ . \tag{B4}$$

A direct evaluation gives the result (37b).

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UOPŠTENA KANONSKA ANALIZA SLOBODNE RAMONDOVE TEORIJE POLJA STRUNA

M. Blagojević, B. Sazdović, M. Vasilić

Motivisani BRST procedurom kanonske kvantizacije, analizirali smo klasičnu strukturu slobodne Ramondove teorije polja struna. Pokazano je da su kalibracione simetrije teorije beskonačno reducibilne i eksplicitno su nađene više strukturne funckije.