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VISUALIZATION AS A CRITERION OF INVARIANT FINITE ELEMENT APPROXIMATION NATURALNESS

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Abstract: The fundamental criterion of the naturalness of a physical law is its invariance (covariance). Such a principle is adopted (not proved), and we shall attempt to support (in a finite element area) by a visualization procedure, which is certainly the most convincing method.

1 Introduction

In order to be law of nature, a physical law must be invariant under the coordinate transformations. This invariance (covariance) is certainly achieved if the physical law is expressed in a full tensorial form.

However, even though what we call "natural laws" are only *approximative* forms of the true laws of nature, their invariance is still requested. Hence, there is no reason to abandon this request, for example, in the case of finite element (FE) approximations of natural laws described by tensor equations, i.e. from the coordinate invariance of FE approximations of tensor fields appearing in these equations (s.[3]).

An attempt to use the *visualization process* in order to facilitate *geometric reasoning* in comparing the naturalness of proposed, ***physically based FE modeling*** (i.e. the invariant (even *fundamental?) algorithm* for FT approximations) and the usual, scalar one is the aim of this note.

2 Invariant versus scalar finite element approximation

Let us, for the sake of comparison of the scalar and invariant approach in finite element approximations, consider the interpolation of a vector field $\vec{v}$. In the usual,scalar approximation, one starts from the representation[1] (cf. with (7.43) in [2])

$$v^i = \Psi^N v^i_N, \qquad (1)$$

where Ψ^N are the interpolation functions, and v^i are the contravariant components of the field $\vec{v}$ in arbitrary curvilinear coordinates x^i (in three-dimensional Euclidian space); v^i_N are the nodal values of this field.

[1] Einstein's summation convention for diagonally repeated indices will be used. Lowercase Latin indices have the range {1,2,3}; index N relates to the nodes in the space where the values of the vector field were done.

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In the invariant approach, the representation to be used reads[2] (s. (2) in [3])

$$v^i = \Psi^N g^i_{g(N)j} v^j_N, \tag{2}$$

where $g^i_{g(N)j}$ are the Euclidian shifters given by

$$g^i_{g(N)j} = \left.\frac{\partial x^i}{\partial z^k}\right|_{x^m} \left.\frac{\partial z^k}{\partial x^j}\right|_{x^m_N} \tag{3}$$

(s. p. 807 in [1]), and z^k are rectangular Certain coordinates. Representation (2) is essentially different from (1); only in the case of rectangular Cartesian coordinates, when the shifters (3) are the Kronecker delta, formula (2) reduces to the form (1).

Generally, the geometrical and physical correctness, i.e. the naturalness, of the invariant approximation (2) is verified by the fact that all nodal quantities are shifted to the *same* point before the summation process is performed. However, the fact that the invariance covariance) is, as a rule, expressed in a *coordinate* form, while, on the other hand, an observer (no matter how unobjective) perceives an object (vector) as a *whole* (and not its components!), is the reason for a heuristic attempt to confirm the naturalness of the above proposed invariant approach in FE approximations.

3 Visualization as a criterion of invariant finite element approximation naturalness

Hence, we shall consider a vector field $\vec{v}$ defined along a circular arc. Let us prescribe the values of this field at the points A and B (s. Fig. 1). Using in approaches (1) and (2) the plane polar coordinates and, for example, the following interpolation functions

$$\Psi^A = 1 - \xi, \qquad \Psi^B = \xi \quad (\xi \in [0,1]) \tag{4}$$

(corresponding to the points (nodes) A and B, respectively), one obtains the usual scalar and invariant field distribution, respectively (s. Fig. 1). An ad hoc conclusion as to which approach is more natural is obviously doubtful.

However, in the situation depicted in Fig. 2, it seems that the scalar "radial" distribution (when the values of the components in the polar system are certainly preserved) is more uniform (more natural) than the invariant one?

But, whatever criterion of naturalness of an (FE) approximation we adopt, it should pass any *simple* test. Such a test is, for example, the one in Fig. 3, where the field $\vec{v}$ with the *same* values at A and B is considered. In the absence of any other data is most natural to suppose the constancy of the whole field $\vec{v}$, independently of the coordinate system in question. In this example the invariant approach (2) generates just such a distribution[3], while the scalar one (1) (with the polar system again) does not!

[2] The placement of index N in the parentheses in (2) means that the summation convention is not applied to the corresponding member – when summing over N the member is simply associated to the other members with this index.

[3] The significance of such a homogeneous vector field distribution is evidenced in the fact that the FE model capability to the produce a homogeneous stress state is a necessary condition for the problem convergence (s. [4])

Such an example of the break-down of the usual, scalar approach (1) is quite sufficient to discredit its general validity. On the other hand, it is clear that one example like this, however geometrically correct, is not sufficient to justify the invariant approach (2) as universally correct. but, as long as the above formulated criterion of physical law naturalness is generally accepted, there is no reason to desist from the proposed approach (2).

It should be noted that, if the above consideration concerning the FE approximation naturalness is adopted for a vector field, *visible* due to the "bristled" representation on Fig. 1–3, a tensor field can also be treated similarly[4], even though such a generalization remains *invisible* for a tensor of order two, three, ... Nevertheless, this would not be the first introduction of something not being visible, but producing only "traces". The "traces" of invisible tensorial objects will be the subject of another note.

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VIZUELIZACIJA KAO KRITERIJUM PRIRODNOSTI INVARIJANTNOG APROKSIMIRANJA KONAČNIM ELEMENATIMA

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Sadržaj: Osnovni kriterijum prirodnosti nekog fizičkog zakona jeste njegova invarijantnost (kovarijantnost). Takav princip se usvaja (ne dokazuje), a ovde se nastoji da se on potkrepi (u oblasti konačnih elemenata) postupkom vizualizacije koji je svakako najuverljiviji.

[4]In the paper [5] it has been explained how it is possible to prescribe boundary conditions forces in invariant two – field finite element approximations.