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DEPENDENCE OF SECOND HARMONIC GENERATION ON THE PARAMETERS OF THE MAGNETIZED PLASMA

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Abstract: Second harmonic generation of extraordinary wave in homogeneous magnetized plasma occupying the half-space by extraordinary or ordinary electromagnetic waves, as the "pump" wave, is considered. The efficiency of harmonic generation is calculated for various values of plasma parameters: external magnetic field, plasma density and propagation angle. The phase-synchronism conditions for the pump wave and its second harmonic are discussed.

1. Introduction

The phenomenon of the second harmonic generation of electromagnetic waves has attracted attention in connection with the use of powerful lasers for plasma heating [1]. The spectral properties of the emitted second harmonic depend on the laser pulse duration, intensity and polarization and the geometry of both the focusing optics and the target [2]. The other direction of investigation was the application of obtained results in plasma diagnostics [3-5]. The observation of two bands of emissions of type III radio bursts separated in frequency by a ratio of about 1:2 confirms the general idea of harmonic emission as one of important effects in cosmos plasmas [6].

Many authors [7-10] have analyzed the phenomenon of the second harmonic generation by the extraordinary modes in a magnetized plasma (**eee**-phase synchronism). Some authors [6,10,11] demonstrate that fundamental ordinary waves can generate the extraordinary wave of double frequency as well (**ooe**-synchronism). The only condition required for such a process is that the generated wave can propagate through the plasma, i.e. that the relevant dispersion equation is simultaneously satisfied for both waves. The effect shows a resonance enhancement when the wave-number-matching condition, i.e. $k(2\omega) = 2k(\omega)$, is satisfied. It will be confirmed in this paper that in the vicinity of plasma resonance $(\omega = \omega_p)$ the efficiency of harmonic generation is additional enhanced [9].

The aim of this work is to investigate how the efficiency of the second harmonic, at some given location x in plasma, depends on the three parameters mentioned in abstract, which define the phase-synchronism condition. The other parameters, relevant for the efficiency, have fixed values.

We suppose that the source of electromagnetic radiation of frequency ω is located in vacuum $(x < 0)$ and the wave vector $\vec{k}$ is along x axis. The polarization of the incident

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wave is defined by the angle Φ between the wave electric field $\vec{E}$ at the boundary ($x = 0$) and the normal to the plane specified by a constant external magnetic field vector $\vec{B}_0$ ($\vec{B}_0 = B_{0x}\vec{e}_x + B_{0z}\vec{e}_z$) and the wave vector $\vec{k}$.

2. Basic equations

Starting from Maxwell equations one can obtain wave equation for electric field:

$$\nabla \times (\nabla \times \vec{E}) + \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{1}{c^2 \epsilon_0}\frac{\partial \vec{j}}{\partial t}, \tag{1}$$

where $\vec{j}$ is the current density, which can be separated into its linear and nonlinear components ($\vec{j} = \vec{j}_l + \vec{j}_{nl}$). Now, we introduce several assumptions:

1) thermal effects end dynamics of ions are neglected,
2) the electric field amplitudes vary slowly with x,
3) considerations are restricted to waves of frequencies ω and 2ω,
4) the electric field amplitudes are stationary and depend only on the coordinate x.

In accordance with these assumtions (1) yields the following system of coupled equations for electric fields:

$$\nabla \times (\nabla \times \vec{E}^{(s)}) - \frac{\omega^2}{c^2}\hat{\epsilon}^{(s)}\vec{E}^{(s)} = \frac{i\omega}{c^2\epsilon_0}\vec{j}^{(s)}_{nl}, \quad s = 1, 2 \tag{2}$$

which describes the interaction between fundamental wave and its second harmonic in a weak turbulence approximation [7]. The dielectric tensors $\hat{\epsilon}^{(s)}$ are given by Ginzburg [12]. The nonlinear current densities contain perturbation quantities which follow from the standard hydrodynamic equations [7] for electrons.

In accordance with the previous assumptions, the electric field in plasma, after sufficiently long time, can be expressed in the form:

$$\vec{E}(x,t) = \vec{A}^{(1)}(x)e^{-i(\omega t - k^{(1)}x)} + \vec{A}^{(2)}(x)e^{-i(2\omega t - k^{(2)}x)} + c.c. \tag{3}$$

Here $\vec{A}^{(1)}(x)$ and $\vec{A}^{(2)}(x)$ are the complex electric field amplitudes of the fundamental mode and its second harmonic, respectively, and $k^{(1)}$ and $k^{(2)}$ are the wave numbers of these waves.

If one separates the real and imaginary parts in equations (2), the system for the real electric field amplitudes $a^{(1)}$ and $a^{(2)}$ can be readily obtained:

$$\begin{aligned} \frac{\partial a^{(1)}}{\partial x} &= -C_{12}a^{(1)}a^{(2)}sin\Psi \\ \frac{\partial a^{(2)}}{\partial x} &= C_{11}{a^{(1)}}^2 sin\Psi \\ \frac{\partial \Psi}{\partial x} &= C_{13}cos\Psi - \frac{2\omega\Delta N}{c}, \end{aligned} \tag{4}$$

where Ψ is the generalized phase depending on phases of complex amplitudes $A^{(1)}$ and $A^{(2)}$. In (4) ΔN is the difference of refraction indices for the fundamental wave and its second harmonic, i.e. $\Delta N = N^{(1)} - N^{(2)}$ and

$$N_{o,e}^{(s)^2} = 1 - \frac{2v^{(s)}(1-v^{(s)})}{2(1-v^{(s)}) - u^{(s)}sin^2\theta \pm \sqrt{u^{(s)^2}sin^4\theta + 4u^{(s)}cos^2\theta(1-v^{(s)})^2}} \tag{5}$$

$$v^{(s)} = \frac{\omega_p^2}{s^2\omega^2}, \quad u^{(s)} = \frac{\omega_c^2}{s^2\omega^2}.$$

Here ω_p and ω_c are the electron plasma and cyclotron frequencies, respectively and θ is the propagation angle (between wave vector $\vec{k}$ and external magnetic field $\vec{B}_0$). The subscripts "+" and "-" in (5) correspond to the ordinary ("o") and extraordinary ("e") mode, respectively.

The coupling constants C_{11}, C_{12} and C_{13} in (4) are functions of the parameters $u^{(s)}$, $v^{(s)}$ and θ.

If we assume that the second harmonic is absent on the boundary ($a_2(0) = 0$), the solutions of the system (4) are given by:

$$a^{(1)}(x) = a^{(1)}(0)\sqrt{1 - \kappa sn^2\left(a^{(1)}(0)x\sqrt{\frac{C_{11}C_{12}}{\kappa}};\kappa\right)},$$

$$a^{(2)}(x) = a^{(1)}(0)\sqrt{\frac{\kappa C_{11}}{C_{12}}}sn\left(a^{(1)}(0)x\sqrt{\frac{C_{11}C_{12}}{\kappa}};\kappa\right),$$

$$cos\Psi(x) = \frac{\Delta_1 V^{(2)}(x)}{1 - V^{(2)^2}(x)}, \tag{6}$$

where $sn(p;\kappa)$ is the Jacobian elliptic sine function; $p = \left[a^{(1)}(0)\sqrt{\frac{C_{11}C_{12}}{\kappa}}\right]x$. The quantity Δ_1 is given by $\Delta_1 = \frac{1-\kappa}{\sqrt{\kappa}}$, where κ is a function of ω, ω_p, ω_c, θ and ΔN. In the case of the phase synchronism, $\Delta N = 0$ or $\kappa = 1$ and $sn(p;1) = thp$, but when ΔN is large enough, $\kappa \to 0$ and $sn(p,0) = sinp$. The quantity $V^{(2)}(x)$ is the second harmonic dimensionless amplitude.

The second harmonic generation efficiency $W(x)$ at some location x in plasma is defined it in the form [11]:

$$W(x) = \frac{< S^{(2)}(x) >}{< S_x^i >}. \tag{7}$$

Here $< S^{(2)}(x) >$ and $< S_x^i >$ are the time averages, taken over one period, of wave energy fluxes for the second harmonic and the incident wave, respectively. Starting from the relation:

$$< S_x^{(2)}(x) >= \frac{c^2\epsilon_0}{2}R_e\left[E_y^{(2)}(x)B_z^{(2)}(x)^* - E_z^{(2)}(x)B_y^{(2)^*}(x)\right], \tag{8}$$

we obtain:

$$< S_x^{(2)}(x) >= \frac{N_e^{(2)}c\epsilon_0(1+B_3^2)}{2(1+B_3^2+B_4^2)}a^{(2)}(x)^2, \tag{9}$$

where B_3 and B_4 couple x,y and z components of the extraordinary electric field $E_e^{(2)}$, i.e. $a_{ez}^{(2)}(x) \equiv B_3 a_{ey}^{(2)}(x)$ and $a_{ex}^{(2)}(x) \equiv B_4 a_{ey}^{(2)}(x)$.

Taking into account the relations (6) and (9) as well as the boundary conditions for the system vacuum - plasma, for the **eee**-phase synchronism it follows:

$$W(x) = \frac{4N_e^{(2)} C_{11}(B_1^2 cos^2\Phi + sin^2\Phi)(B_2^2 + B_5^2 + 1)(B_3^2 + 1)\kappa}{C_{12}(N_e^{(1)} + 1)^2(B_1 - B_2)^2(B_3^2 + B_4^2 + 1)} sn^2(p;\kappa), \qquad (10)$$

where

$$p = \frac{\pi Y^2 x}{\lambda_0 \sqrt{Y^2 + 2\Delta N(\Delta N - \sqrt{\Delta N^2 + Y^2})}},$$

$$Y = \sqrt{2fC_{11}' C_{12}' \Sigma},$$

$$\Sigma = \frac{\lambda_0^2 S^i}{K^2},$$

$$K = \frac{2\pi mc^2\sqrt{c\epsilon_0}}{e} = 1.65597 \times 10^5 \quad \text{SI units},$$

$$\kappa = 1 + \frac{2\Delta N}{Y^2}(\Delta N - \sqrt{\Delta N^2 + Y^2}).$$

In these relations λ_0 is the wavelength in vacuum, f is the coefficient defined by expression:

$$a^{(1)}(0)^2 = \frac{4(B_1^2 cos^2\Phi + sin^2\Phi)(B_2^2 + B_5^2 + 1)}{(N_e^{(1)} + 1)^2(B_1 - B_2)^2} a^2 \equiv fa^2,$$

and dimensionless functions C_{11}' i C_{12}' are connected with C_{11} i C_{12} by way of:

$$C_{11}' = -i\frac{2mc^2}{e}C_{11} \quad C_{12}' = -i\frac{2mc^2}{e}C_{12}.$$

For the **ooe**-phase synchronism (when $\theta = 90^0$) we obtain:

$$W = \frac{4N_e^{(2)}\kappa sin^2\Phi}{(N_o^{(1)} + 1)^2} sn^2(p;\kappa), \qquad (10)$$

where

$$p = \frac{2\pi sin\Phi Y^2 x}{\lambda_0 (N_o^{(1)} + 1)\sqrt{Y^2 + 2\Delta N(\Delta N - \sqrt{Y^2 + \Delta N^2})}},$$

$$Y = \frac{2\sqrt{2}N_e^{(2)}\epsilon_2^{(2)}\sqrt{\Sigma}}{N_o^{(1)}\epsilon_1^{(2)}},$$

$$\kappa = 1 + \frac{(N_o^{(1)} + 1)\Delta N}{2Y^2 sin^2\Phi}\left[(N_o^{(1)} + 1)\Delta N - \sqrt{(N_o^{(1)} + 1)^2\Delta N^2 + 4Y^2 sin^2\Phi}\right]$$

$$\epsilon_1^{(2)} = 1 - \frac{\omega_p^2}{4\omega^2 - \omega_c^2}, \quad \epsilon_2^{(2)} = -\frac{\omega_p^2\omega_c}{\omega(4\omega^2 - \omega_c^2)}.$$

The coefficients B_1, B_2 and B_5 in (10) couple x, y and z components of electric field $\vec{E}^{(1)}_{o,e}$, i.e. $A^{(1)}_{oz}(0) = iB_1A^{(1)}_{oy}(0)$, $A^{(1)}_{ez}(0) = iB_2A^{(1)}_{ey}(0)$ and $a^{(1)}_{ex}(x) = B_5a^{(1)}_{ey}$, where the subscripts "o" and "e" pertain to ordinary and extraordinary wave, respectively. The coefficients B_1, B_2, B_3, B_4 and B_5 are the functions of parameters ω, ω_c, ω_p and θ. But we can notice in relations (10) and (11) that the second harmonic generation efficiency $W(x)$ is the function of seven parameters: ω, ω_p, ω_c, θ, Φ, S^i and x. The first four parameters define the phase synchronism condition ($\Delta N = 0$), because the refractive indices are the functions of them, too.

In this paper the influence of parameters $u^{(1)}$, $v^{(1)}$ and θ on W is considered, when the **eee** or **ooe** phase synchronism condition is satisfied, or partially satisfied. In view of a complex analytical expression for $W(x)$, the problem is investigated numerically and the results are plotted in the figures given below.

3. Numerical results

There are two separate **eee**-phase synchronism regions where the considered second harmonic emission occurs namely the region of lower magnetic field ($u^{(1)} \leq 1$), larger electron density ($1 \leq v^{(1)} \leq 2$) embiacing all propagation angles θ and the domain of strong magnetic field ($u^{(1)} \geq 9$), low electron density ($v^{(1)} \leq 1$) and propagation angles θ up to 45^0, as well as one **ooe**-phase synchronism region 3) the domain with "moderately strong" magnetic field ($2.25 \leq u^{(1)} \leq 3$), low electron density ($v^{(1)} \leq 1$) and propagation angles in the interval $55^0 \leq \theta \leq 90^0$.

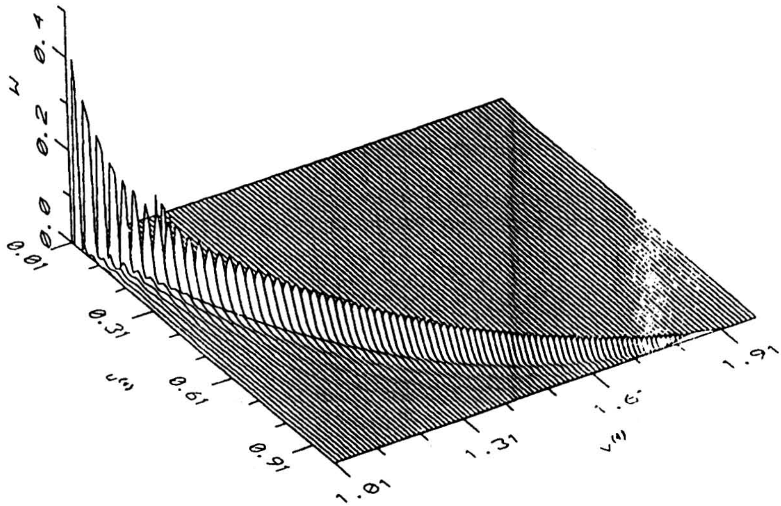

Figure 1: Second harmonic efficiency W as function of magnetic field and plasma density at location of $x = 20\lambda_0$ in plasma for parameter values: $\theta = 90^0, \Phi = 45^0, \Sigma = 10^{-4}$.

Figure 1 shows the second harmonic efficiency W as a function of external magnetic field ($u^{(1)} = \frac{\omega_c^2}{\omega^2}$) and electron density ($v^{(1)} = \frac{\omega_p^2}{\omega^2}$), when $\theta = 90^0$ (region 1). The other parameters are fixed and their values are given in the figure caption. We can see that $W \neq 0$

holds only in the vicinity of resonant values of parameters $u^{(1)}$ and $v^{(1)}$ (which satisfy the phase synchronism condition $N_e^{(1)} = N_e^{(2)}$). The plot indicates that the most efficient generation of the second harmonic occurs when $\omega_p \approx \omega$ ($v^{(1)} \approx 1$) and $\omega_c \approx 0$ ($u^{(1)} \approx 0$). These values satisfy the phase synchronism condition, but the plasma resonance ($\omega = \omega_p$) as well [9]. It means that the enhanced efficiency of harmonic generation for $v^{(1)} = 1$, $u^{(1)} = 0$ results from two resonant conditions. For the given parameters the maximum value decreases from $W \approx 0.4$ as $u^{(1)} \to 1$ and $v^{(1)} \to 2$. Away from the point of the phase synchronism condition efficiency is practically zero.

Figure 2 demonstrates the same dependence as the figure 1, but in region 2, when $\theta = 10^0$, and for values of other parameters given below this figure. We can notice that the resonant maximum is obtained for $u^{(1)} \approx 12$ and $v^{(1)} \approx 0.75$. This pair of values $(u^{(1)}, v^{(1)})$ satisfies the **eee**-phase synchronism and agrees with those obtained earlier [7] for the emission source located on the boundary of plasma. It means that the boundary does not influence the extraordinary transmission coefficient in domain of strong magnetic fields (region 2). In this case the plasma resonance ($\omega = \omega_p, v^{(1)} = 1$) does not play an important role related to the phase synchronism.

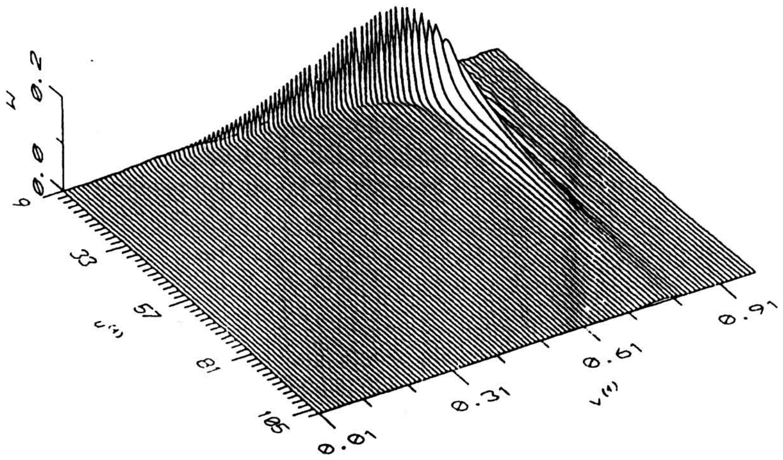

Figure 2: Second harmonic efficiency W as function of magnetic field and plasma density at location of $x = 100\lambda_0$ in plasma for parameter values: $\theta = 10^0, \Phi = 45^0, \Sigma = 10^{-4}$.

Figure 3 gives the dependence of W on external magnetic field ($u^{(1)}$) and electron density ($v^{(1)}$) when $\theta = 90^0$ in region 3, where the **ooe**-phase synchronism is satisfied. Decreasing the magnetic field and increasing of electron density (in accordance with the **ooe**-phase synchronism condition) it is marked enhancement of W. The finite width of the emission peak, especially for larger values of magnetic fields ($u^{(1)} \to 2.85$), indicates that the second harmonic emission is still important even though the system is no longer in strict phase synchronism condition.

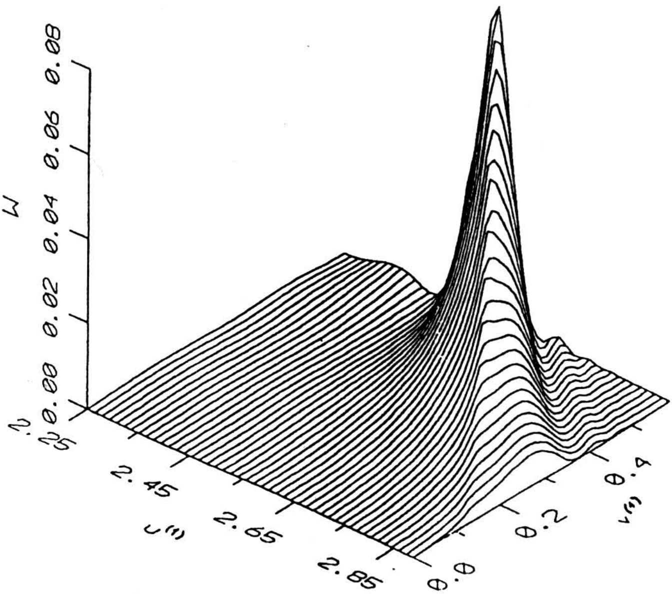

Figure 3: Second harmonic efficiency W as function of magnetic field and plasma density at location of $x = 10\lambda_0$ in plasma for parameter values: $\theta = 90^0, \Phi = 45^0, \Sigma = 10^{-4}$.

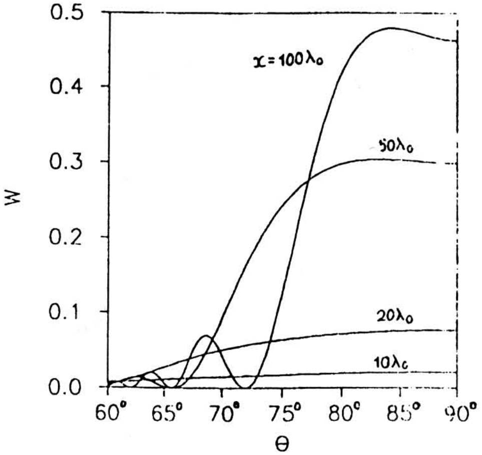

Figure 4: Second harmonic efficiency W in the vicinity of resonant propagation angle ($\theta = 90^0$) at various locations x, shown again the curves for the following fixed parameters: $\Phi = 45^0$, $\Sigma = 10^{-4}$, $u^{(1)} = 0.55$, $v^{(1)} = 1.25$.

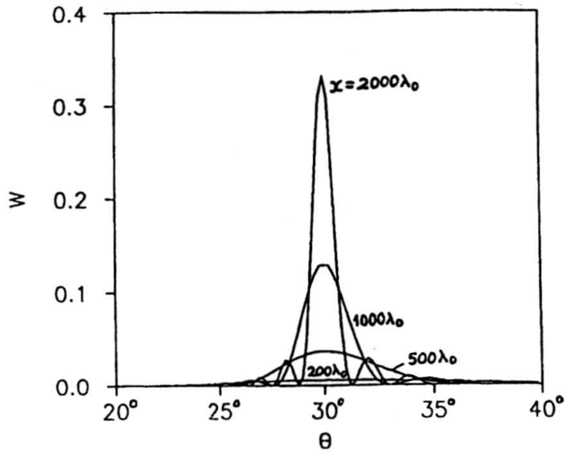

Figure 5: Second harmonic efficiency W in the vicinity of resonant propagation angle ($\theta = 30^0$) at various locations x, in plasma, shown again the curves for the following values of parameters: $\Phi = 45^0$, $\Sigma = 10^{-4}$, $u^{(1)} = 20$, $v^{(1)} = 0.142$.

Finally, figures 4 and 5 show the efficiency W for the two resonant domains plotted against propagation angle θ for various locations x in plasma and for other parameters. Specified in the captions we can notice that the width of resonant peak is larger in the case of lower magnetic field (figure 4) as compared to the one pertaining to strong magnetic field (figure 5). It means that the effect of second harmonic generation is important in the vicinity of resonant angle θ as well as for resonant one, when the external magnetic field is low (region 1).

4. Conclusions

The phase synchronism condition plays a predominant role in the second harmonic generation in a magnetized plasma. In the regions of weak ($\omega_c \to 0$) and moderately strong ($\omega_c \to \sqrt{2.25}\omega$) magnetic fields, the vicinity of plasma resonance also contributes to enhancement of efficiency of the process considered here. In the region of strong magnetic fields there are particular values of magnetic field, plasma density and propagation angle which result in the efficiency reaching the maximum, far from plasma resonance. We notice that the conversion efficiency is very significant and additionally increases when other parameters (fixed in present consideration) have optimum values.

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ZAVISNOST GENERACIJE DRUGOG HARMONIKA OD PARAMETARA MAGNETIZOVANE PLAZME

Bratislav M. Jovanović

Sadržaj: Razmatra se generacija drugog harmonika neredovnog talasa u homogenoj magnetizovanoj plazmi smeštenoj u poluprostoru, kada kao pumpa za ovaj proces služi neredovan ili redovan elektromagnetni talas. Efikasnost generacije harmonika se računa za razne vrednosti parametara plazme: spoljašnjeg magnetnog polja, gustine plazme i ugla prostiranja. Proučavaju se uslovi faznog sinhronizma za talas osnovne frekvence i njegov drugi harmonik.