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THE DIPOLE MODE DISPERSION RELATION OF ELECTRON SURFACE WAVES ON AN EQUIVALENT PLASMA/DIELECTRIC INTERFACE

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Abstract: The replacement of the dispersion relation $\Delta_1(\omega,\beta)=0$ of the electron surface dipole wave propagating on a plasma column placed in a glass tube of thickness δ and permittivity ε_g with the dispersion relation $\Delta_1^h(\omega,\beta)=0$ for the same surface wave on a plasma cylinder in a homogeneous dielectric is studied analytically. It is shown that this method of reducing the involved mathematics gives acceptable approximate results provided a conveniently chosen effective permittivity of the dielectric $\varepsilon(\delta,\varepsilon_g,\beta)$ instead of the constant ε_g is taken. The computations valid for a dipole mode surface wave are given and the necessary comparisons done.

1. Introduction

An immense experimental work on the propagation of surface waves along a plasma/dielectric interface has been done during the past two or three decades. The results and published papers are partly summarized and reviewed [1,2]. Many articles among them deal with the surface waves in gas-discharge plasmas [3]. As a rule, in such cases the plasma is created by ionizing a gas or vapour within a dielectric tube (a glass of permittivity ε_g) of known wall thickness δ. A similar situation exists in experiments with surfatron plasma sources [4,5,6,7,8]. In all these experiments, the involved geometries are cylindrical with the typical *plasma column/glass tube/air* structure. Theoretical paper often follow that fact in both linear and nonlinear treatments [9,10].

The dispersion relation $\Delta_1(\omega,\beta)=0$ of a dipole mode surface wave on a lossless cold plasma column surrounded by a glass cylinder of finite thickness and circular cross section implies a function of a fairly complex form. Even in the simplifying case of the quasistatic approximation, when the expression achieves the explicit structure of the type $\omega=f_1(\beta)$, the involved mathematics is still very complicated - the relevant parameters (glass permittivity, wall thickness) exist in clusters of modified Bessel functions I_p and K_p, and related derivatives I_p' and K_p' (where $p=0,1$), partly in their arguments. Denoting $x=\beta a$,

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$z = xd$, $y = \omega/\omega_p$ and $S_1(z) = K_1'(z)I_1(z) - \varepsilon_g K_1(z)I_1'(z)$, we obtain for the dipole mode (cf. ref. 11)

$$y = \left\{ 1 - \varepsilon_g \frac{I_1(x)}{I_1'(x)} \cdot \frac{(1-\varepsilon_g)\, I_1'(x)\, K_1(z)\, K_1'(z) - K_1'(x)\, S_1(z)}{(1-\varepsilon_g)\, I_1(x)\, K_1(z)\, K_1'(z) - K_1(x)\, S_1(z)} \right\}^{-1/2} . \tag{1}$$

Here, primes denote differentiations with respect to x, a and b are the radii of the tube and $d = b/a$.

On the other hand, much simpler is the dispersion relation $\Delta_1^h(\omega, \beta) = 0$ of the surface wave on a plasma column placed in a homogeneous, endless dielectric, i.e. in the limit $d = \delta/a + 1 \to \infty$. In the same approximation and for the same wave mode, we have (cf. ref. 11)

$$y = [1 - M_1(x)\, \varepsilon_g]^{-1/2} , \tag{2}$$

where $M_1 = I_1(x)\, K_1'(x)/[I_1'(x)\, K_1(x)]$. Using the above relations we can plot the dispersion curves for the dipole mode. Let us show and comment on some of them in order to explain our idea, which shell be elaborated in the next Section, in the most convenient way. The curve a), the Figure 1, is valid in the limit $\varepsilon_g \to 1$ or, which is the same, when $d \to 1$ (i.e. the tube is of negligible thickness). The curve b) represent the case of a very thick tube wall, $d \to \infty$. In the limit $x \to \infty$, the asymptotes are, respectively, $y_a(\text{air}) \simeq 0.707$ and $y_b(\text{glass}) \simeq 0.415$, both values being in accordance with the common expression $y_{asym} = 1/\sqrt{1+\varepsilon_g}$. The dashed line in fig. 1 gives the dispersion for $d = 1.2$, the points having been computed by means of (1). We see that the dashed line tends to join curve b) for large values of x.

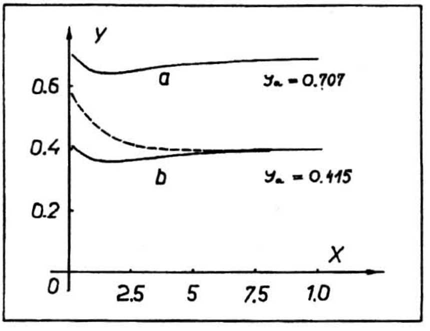

Fig. 1. The dispersion curves for the dipole mode wave; a) valid in the limit $\varepsilon_g \to 1$; b) valid in the limit $d \to \infty$ and $\varepsilon_g = 4.8$; the dashed line: $d = 1.2$ and $\varepsilon_g = 4.8$; the horizontal lines are the asymptotes for $x \to \infty$.

This fact has a simple physical explanation: for large wavenumbers we have $\lambda/\delta \ll 1$ and the wave "feels" mainly the glass as the medium surrounding the plasma column. For middle values of x the wavelength is comparable with the tube thickness. The dashed line is situated between the two limiting lines and exhibits a region with the negative derivative dy/dx. In fact, the curve exhibits a broad minimum, too, since it approaches the asymptote from the lower side. In any case, in the neighborhood of the point $x = 1$ the situation is more complicated: the wave "feels" the glass permittivity as well as the air permittivity, i.e. the physical picture can be understood by means of some ***equivalent dielectric*** representing the glass/air combination. Let us see what ***effective permittivity*** must have this equivalent dielectric. As far as we know, this problem has not been yet. The solution valid for the azimuthally symmetric electron plasma surface wave was recently found by the author [12]. In what follows we search for the solution applicable for the dipole mode.

2. Equivalent dielectric

In order to answer the above question, we shall analyse substitution sketched in the Figure 2. The problem is to find the effective permittivity $\varepsilon = \varepsilon(d, \varepsilon_g, x)$ of the homogeneous dielectric, which ensures the identity of the dispersion relations for the two configurations.

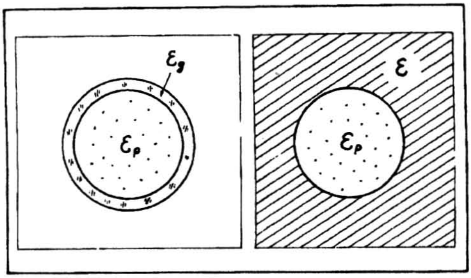

Fig. 2. The real experimental configuration (left) is to be replaced with the simpler model (right) without changing the dispersion relation.

The physical conditions that must be applied when formulating the permittivity ε are as follows: a) $\lambda \to \infty$ must result in $\varepsilon \to \tilde{\varepsilon}$; b) when $\lambda \to 0$ then $\varepsilon \to \varepsilon_g$. Here, an adequately chosen constant $\tilde{\varepsilon}$ should represent the equivalent permittivity in the limiting case of vanishingly small wavenumbers (what it exactly means will be discussed later). In addition we must take into account that the dependances on the thickness and the wavenumber should be in the form of the product: $\varepsilon = f(\delta\beta)$. The reason is that the crucial "measure" of the wall thickness is the wavelength of the surface wave. In other words, as the argument in the ε function appears the variable $\eta = (d-1)x$, where $d = b/a$ is the quotient of the tube radii, and $x = \beta a$. Therefore one could reasonable suggest the form

$$\varepsilon = \tilde{\varepsilon} + E_1(\varepsilon_g, \tilde{\varepsilon}) \cdot E_2(\eta) \,. \tag{3}$$

The functions E_1 and E_2 must satisfy the requirements of the mentioned physical conditions. The list of the requirements is:

1. $\varepsilon_g \to \tilde{\varepsilon} => E_1 \to 0$
2. $\eta \to 0 => E_2 \to 0$
3. $x \to \infty => E_2 \to 1$
4. $x \to \infty => E_1 \to (\varepsilon_g - \tilde{\varepsilon})$.

Of course, $\eta \to 0$ can be realized in the two ways: $x \to 0$ and $d \to 1$. Also, $\eta \to \infty$ can be realized in the two ways: $x \to \infty$ and $d \to \infty$.

3. Effective permittivity

We have found that the satisfactory choice is $E_1 = \varepsilon_g - \widetilde{\varepsilon}$ and $E_2 = \tanh(\eta)$, when the effective permittivity takes the form

$$\varepsilon(d, \varepsilon_g, x) = \widetilde{\varepsilon} + (\varepsilon_g - \widetilde{\varepsilon}) \tanh[(d-1)x] \ . \tag{4}$$

Here, the new parameter $\widetilde{\varepsilon}$ must be treated as a function of ε_g and d. Our computations lead to the conclusion that this function is of the form

$$\widetilde{\varepsilon} = \varepsilon_g - \frac{\varepsilon_g - 1}{d} \ . \tag{5}$$

Figure 3 shows the dispersion curves obtained a) from (1) and b) from (2) with the substitution $\varepsilon_g \to \varepsilon$ in accordance with the above model.

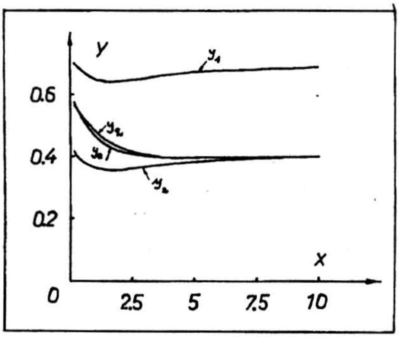

Fig. 3. The quasistatic dispersion curves compared with the approximate ones; y_1 valid for $\varepsilon_g \to 1$; y_2 valid for $d \to \infty$ and $\varepsilon_g = 4.8$; y_q as the dashed line in the Fig. 1; y_e is the approximate curve.

The curve designated as y_1 represents the dispersion in the case $d \to 1$ (a negligible thin glass envelope). Here, the original quasistatic results are practically the same as the results stemming from the equivalent dielectric concept, and no differences are detectable on the drawing. Also, the agreement is excellent in the other limiting case $d \to \infty$ (a very thick glass envelope) when the two curves join giving the curve designated as y_∞. For intermediary values of d the equivalent dispersion points (y_e) are slightly below the points which represent the original quasistatic procedure (y_q). Both curves are valid for $d = 1.2$. The maximal discrepancies (of the order of 2%) lie in the region of wavenumbers $x \sim 1$. Again, the differences are practically undetectable for $x > 2.5$ and are minor for $x < 0.5$. For a given wavenumber the y_e–values as a function od $d = b/a$ show a narrow region of minimal reliability (see the Figure 4 which holds for $x = 1$). The curve exhibits a sharp maximum in the vicinity of the point $d = 1.1$ but the relative deviation $(y_q - y_e)/y_q$ do not surpasses 3 %. This can be check plotting Δy versus x. Figure 5 shows such a graph for $d = 1.1$. After having maximum in the region $x \sim 1$ the deviation curve smoothly approaches $x-$ axes in the limit $x \to \infty$.

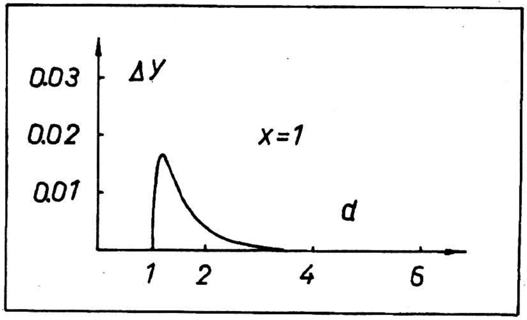

Fig. 4. The deviation parameter as a function of d, for $x = 1$; see the Section 3.

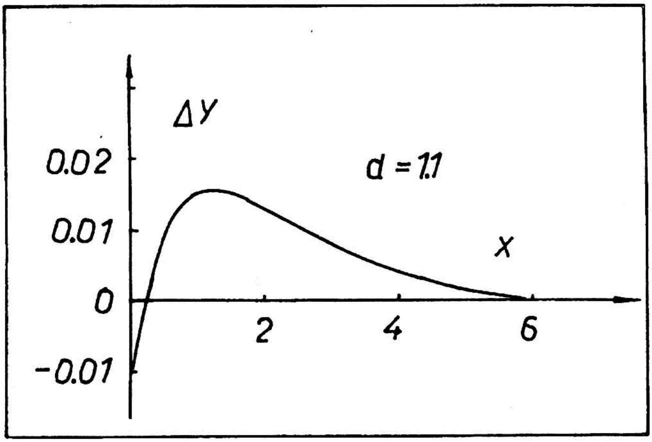

Fig. 5. The deviation parameter as a function of x, for $d = 1.1$; see the Section 3.

The negative values near the origin are of no importance - the quasistatic approximation itself is not applicable for $x \to 0$. We see that the concept of an equivalent dielectric with its effective permittivity defined by Eqs. (4) and (5) is applicable in the region where the most experiments take places ($\varepsilon_g \sim 4, d \sim 1.2, x \sim 1$). Of course, in the case when an another dielectric spreads around the glass tube (instead of air), a generalization of Eqs. (4), (5) can be easily done via an obvious modification.

2. Conclusion

In conclusion it should be noted that the dipole mode surface wave propagating on a plasma column placed in a glass tube of finite thickness may be conceived as a wave propagating on a plasma column placed in an equivalent homogeneous dielectric of given effective permittivity. This statement is true in the sense that the dispersion relations of the two waves should be the same. In the case of the quasistatic approximation the agreement is almost complete. Using the full dispersion relations, the method gives correct semi-quantitative results. The concept of the equivalent dielectric enables a considerable reduction of involved mathematical expressions, whenever the direct use of dispersion relations is necessary [13,14]. Also, in that way we may quickly obtain some qualitative answers concerning the behaviour of the bulky full dispersion relation. Further, our model enables an affective exploitation of Eq. (2), connected with Eqs. (4) and (5), in various limiting cases. First of all, one could try to find more elegant approximations in the region $x < 1$, where several attempts of experimental verifications of the dipole mode dispersion relation already exist. We shall add that there are specialists inclined to a deep reservations whenever a talk about the easy and doubtful observation of the dipole mode is going [15]. Also, in the literature on the topic there are items denying the autonomous position of the dipole branch on the frequency-wavelength curve [16].

The proposed model includes several assumptions which require more detailed investigation. In particular, an unification of the procedures for axially symmetric and dipole mode could be of some interest. The work is in progress and results will be presented elsewhere.

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DISPERZIONA RELACIJA ZA DIPOLNI TIP ELEKTRONSKOG POVRŠINSKOG TALASA NA EKVIVALENTNOJ CILINDRIČNOJ VODJICI

V. M. Babović

U radu se analizira mogućnost zamene složene disperzione relacije za dipolni tip elektronskog površinskog talasa na stubu hladne plazme u staklenoj cevi jednostavnijom disperzionom relacijom za taj isti talas na plazmi u homogenom dielektriku. Pokazuje se da se na taj način dobija vrlo dobar aproksimant koji znatno uprošćava matematički tretman prostiranja talasa, pod uslovom da se pogodno izabere efektivna permitivnost ekvivalentnog dielektrika. Priložena su odgovarajuća izračunavanja i procenjena greška aproksimiranja. Data su uporedjivanja sa kvazistatičkom disperzionom relacijom dipolnog tipa talasa.