UNIVERSITY OF NIŠ
The scientific journal "FACTA UNIVERSITATIS
Series: **Physics, Chemistry and Technology"** Vol.1, N° 1, 1994. pp 26-30
Editor of Series: Predrag Dimitrijević
Address: Trg bratstva i jedinstva 2 YU-18000 Niš
Tel: (018) 347-095 Fax: (018) 24-488

NONLINEAR CURRENTS IN A COLD MAGNETOACTIVE PLASMA UNDER THE INFLUENCE OF A HIGH-FREQUENCY ELECTROMAGNETIC FIELD

Yu. M. Aliev*
B. M. Jovanović°
A. A. Frolov*

*P.N.Lebedev Institute, Rusian Academy of Csieces
117924, Moscow, Russia

° Institute of Physics, Medical Faculty,
University of Niš,PO Box 174, 18000 Niš, Yugoslavia

Abstract: Starting from the tensor of nonlinear interaction [3], the expresions for nonlinear currents density as well as for generated magnetic field are obtained. These results are in agreement with those given in [1].

1. Introduction

The problem of plasma interaction with a high-frequency electromagnetic field has attracted many autors. Pustovalov and Silin [3] gave the nonlinear theory of plasma interaction with a high-frequency electromagnetic field. They introduced the tensor of nonlinear interactions in a plasma. Aliev et al. [2] solved this problem by kinetic theory and obtained expressions for the nonlinear current density as well as for the magnetic field generated in a cold isotropic plasma. The analogous expressions are given in [1] using the hydrodynamic description of magnetoactive plasma. Here, starting from the tensor of nonlinear interaction [3], we obtained the relations which agree with those obtained in [1].

2. Basic equations

To describe the evolution of the nonlinear interaction of self-oscilations of magnetoactive plasma, we can introduce the three-index tensor of nonlinear interaction $S_{ijs}(\omega, \vec{k}, \omega', \vec{k}')$

UDK: 533.9.74; Received April 15. 1992

determined in a cold plasma [3] by the expression:

$$\begin{aligned} S_{ijs}(\omega,\vec{k},\omega',\vec{k}') = &-\frac{ie}{m\omega_p^2}\left\{\frac{k_a}{\omega}\left[\omega'\delta\varepsilon_{is}(\omega')\delta\varepsilon_{aj}(\omega'')+\omega''\delta\varepsilon_{ij}(\omega'')\delta\varepsilon_{as}(\omega')\right]\right. \\ &+\frac{k''_b}{\omega'}\left[\omega'\delta\varepsilon_{js}(\omega)\delta\varepsilon_{ib}(\omega)-\omega\delta\varepsilon_{bs}(\omega')\delta\varepsilon_{ij}(\omega)\right] \\ &\left.+\frac{k'_c}{\omega'}\left[\omega''\delta\varepsilon_{sj}(\omega'')\delta\varepsilon_{ic}(\omega)-\omega\delta\varepsilon_{cj}(\omega'')\delta\varepsilon_{is}(\omega)\right]\right\} \end{aligned} \tag{1}$$

where $\omega = \omega' + \omega''$, $\vec{k} = \vec{k}' + \vec{k}''$ and ω_p is the electron plasma frequency. The two-index tensor $\delta\varepsilon_{ij}$ is given by the relation:

$$\varepsilon_{ij}(\omega,0) = \delta_{ij} + \sum \delta\varepsilon_{ij}(\omega). \tag{2}$$

Here $\varepsilon ij(\omega,0)$ is the dielectric tensor of a cold magnetoactive plasma.

Now, we can write the induced current density in presence of a high-frequency electromagnetic field as a sum [2]

$$\vec{j}(\omega,\vec{k}) = \vec{j}_0(\omega,\vec{k}) + \vec{j}_1(\omega,\vec{k}) + \vec{j}_2(\omega,\vec{k}) + \dots \tag{3}$$

The current density $\vec{j}_0$ is the unperturbed current density in equilibrium and is taken zero; $\vec{j}_1$ is the high-frequency perturbed current density, while $\vec{j}_2$ is the low-frequency nonlinear current density, given by the tensor S_{ijs} in the following way:

$$\vec{j}_2(\omega,\vec{k}) = -\frac{i\omega}{8\pi}\int d\omega' d\vec{k}' S_{ijs}(\omega,\vec{k},\omega',\vec{k}')E_j(\omega'',\vec{k}'')E_s(\omega',\vec{k}') \tag{4}$$

This current density can be writen as the sum:

$$\vec{j}_2 = \vec{j}_s + \vec{j}_d \tag{5}$$

wher $\vec{j}_e$ and $\vec{j}_d$ are given by

$$\left(\vec{j}_e\right)_i = -\frac{e\omega}{4\pi m\omega_p^2}\int d\omega' d\vec{k}' E''_j E'_s \frac{k'_a}{\omega}\omega''\delta\varepsilon_{ij}(\omega'')\delta\varepsilon_{as}(\omega') \tag{6}$$

and

$$\begin{aligned} \left(\vec{j}_d\right)_i = &-\frac{e\omega}{4\pi m\omega_p^2}\int d\omega' d\vec{k}'\left[\frac{k''_a\omega''}{\omega}\delta\varepsilon_{ij}(\omega'')\delta\varepsilon_{as}(\omega')E''_j E'_s\right. \\ &\left.+\frac{k''_b\omega'}{\omega''}\delta\varepsilon_{js}(\omega')\delta\varepsilon_{ib}(\omega)E''_j E'_s - \frac{k''_b\omega}{\omega''}\delta\varepsilon_{bs}(\omega')\delta\varepsilon_{ij}(\omega)E''_j E'_s\right] \end{aligned} \tag{7}$$

Now, introduce new quantities $\vec{V}(\omega'',\vec{k}'')$, $\vec{R}(\omega',\vec{k}')$ and $\vec{v}(\omega'',\vec{k}'')$ by the following expresions:

$$\begin{aligned} V_i(\omega'',\vec{k}'') &= \omega''\delta\varepsilon_{ij}(\omega'')E_j(\omega'',\vec{k}'') \\ R_a(\omega',\vec{k}') &= \delta\varepsilon_{as}(\omega')E_s(\omega',\vec{k}') \\ v_j(\omega'',\vec{k}'') &= \frac{E_j(\omega'',\vec{k}'')}{\omega''} \end{aligned} \tag{8}$$

Having used (8) in equation (6) we obtain

$$\vec{j}_e = \frac{ie}{4\pi m\omega_p^2}\vec{V}\nabla\vec{R} \tag{9}$$

To represent the dependence of the nonlinear current density $\vec{j}_d$ on quantities $\vec{V}$ and $\vec{v}$ we use the relation [3]

$$\omega'\Gamma_{ia}(\omega)\Gamma_{aj}(\omega'') = \Gamma_{ij}(\omega'')\Gamma_{ij}(\omega), \tag{10}$$

where the tensor Γ_{ij} is determined by the following expresion:

$$\Gamma_{ij}(\omega) = -\frac{\omega}{\omega_p^2}\delta\varepsilon_{ij}(\omega). \tag{11}$$

After elementary transformations in equations (10) and (7) we get the slowly current density $\vec{j}_d$ in the form:

$$\vec{j}_d(\omega) = \frac{e\hat{\sigma}}{m\omega_p^4}\left\{\frac{1}{2}\nabla V^2 - \left[\vec{V}\times \mathrm{rot}\left(\vec{V}+\omega_p^2\vec{v}\right)\right]\right\}, \tag{12}$$

wher the conductivity tensor $\hat{\sigma}$ is given by

$$\hat{\sigma} = -\frac{i\omega}{4\pi}\delta\hat{\varepsilon}.$$

Now, we can introduce a complex amplitude $E_1(\vec{r},t)$ of the high-frequency electric field $\vec{E}(\vec{r},t)$

$$\vec{E}(\vec{r},t) = \frac{1}{2}e^{-i\omega t} + c.c. \tag{13}$$

Quantities $\vec{v}(\vec{r},t)$, $\vec{V}(\vec{r},t)$ and $\vec{R}(\vec{r},t)$ can be expressed in terms of the qmplitude $\vec{E}_1(\vec{r},t)$. So, starting from the relation:

$$\vec{v}(\vec{r},t) = \int d\omega e^{-i\omega t}\frac{\vec{E}(\omega,\vec{r})}{\omega} \tag{14}$$

and taking into consideration that $\omega \ll \omega_0'$, we obtain:

$$\vec{v}(\vec{r},t) = \frac{1}{2}\vec{v}_1 e^{-i\omega_0 t} - c.c \tag{15}$$

where $\vec{v}_1$ is given by:

$$\vec{v}_1 = \frac{1}{\omega_0}\left(1 - \frac{i}{\omega_0}\frac{\partial}{\partial t}\right)\vec{E}_1. \tag{16}$$

Using the relation

$$\vec{R}(\vec{r},t) = \int d\omega e^{-i\omega t}\delta\hat{\varepsilon}(\omega)\vec{E}(\omega,\vec{r}), \tag{17}$$

we have the following expression for the quantity $\vec{R}(\vec{r},t)$:

$$\vec{R}(\vec{r},t) = \frac{1}{2}\vec{R}_1 e^{-i\omega t} + c.c. \tag{18}$$

where $\vec{R}_1(\vec{r},t)$ is given by:

$$\vec{R}_1 = \left[\delta\hat{\varepsilon}(\omega_0) + i\frac{\partial}{\partial\omega_0}\delta\varepsilon(\omega_0)\frac{\partial}{\partial t}\right]\vec{E}_1. \tag{19}$$

In a similar way, $\vec{V}(\vec{r},t)$ can be written as follows:

$$\vec{V}(\vec{r},t) = \int d\omega e^{-i\omega t}\omega\delta\hat{\varepsilon}(\omega)\vec{E}(\omega,\vec{r}) = \frac{1}{2}\vec{V}_1 e^{-i\omega_0 t} + c.c. \tag{20}$$

where the amplitude $\vec{V}_1$ has the form:

$$\vec{V}_1 = \left[\omega_0\delta\hat{\varepsilon}(\omega_0) + i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}\left(\omega_0\delta\hat{\varepsilon}(\omega_0)\right)\right]\vec{E}_1 \tag{21}$$

Substituting the expresions (15), (18) and (20) into (9) and (12) we obtain relations for the nonlinear currents density:

$$\begin{aligned}\vec{j}_e =& \frac{ie}{16\pi m\omega_p^2}\left\{\left[\omega_0\delta\hat{\varepsilon}(\omega_0) + i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}(\omega_0\delta\hat{\varepsilon}(\omega_0))\right]\vec{E}_1\right.\\ &\left.\times \operatorname{div}\left[\delta\hat{\varepsilon}^*(\omega_0) - i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}\delta\hat{\varepsilon}^*(\omega_0)\right]\vec{E}_1^* - c.c.\right\}\end{aligned} \tag{22}$$

$$\begin{aligned}\vec{j}_d =& \frac{ie\omega\delta\hat{\varepsilon}(\omega)}{16\pi m\omega_p^4}\left\{\nabla\left|\left[\omega_0\delta\hat{\varepsilon}(\omega_0) + i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}(\omega_0\delta\hat{\varepsilon}(\omega_0))\right]\vec{E}_1\right|^2\right.\\ &- \left(\left[\omega_0\delta\hat{\varepsilon}^*(\omega_0) + i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}(\omega_0\delta\hat{\varepsilon}^*(\omega_0))\right]\vec{E}_1^*\right.\\ &\left.\left.\times\operatorname{rot}\left[\frac{\omega_p^2\delta\hat{\varepsilon}(\omega_0)}{\omega_0} + i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}\left(\frac{\omega_p^2\delta\hat{\varepsilon}(\omega_0)}{\omega_0}\right)\right]\vec{E}_1 + c.c.\right)\right\}.\end{aligned} \tag{23}$$

The current density $\vec{j}_e$ results from the influence of the high-frequency density and the velocity oscillations, while $\vec{j}_d$ comes from the slowly varying velocity [1]. Besides the nonlinear currents density $\vec{j}_e$ and $\vec{j}_d$, the linear current density $\vec{j}_0$ is induced in plasma too. In accordance with the Maxwell equation:

$$\operatorname{rot}\vec{B} = \frac{4\pi}{c}\left(\vec{j}_0 + \vec{j}_e + \vec{j}_d\right) \tag{24}$$

we can write the equation for the quasistationary magnetic field $\vec{B}$ generated in plasma:

$$\frac{\partial \vec{B}}{\partial t} + \mathrm{rot}\frac{c^2}{\omega_p^2}\left[\frac{\partial}{\partial t}\mathrm{rot}\vec{B} + Q_0(\vec{h}\times \mathrm{rot}\vec{B}\right] = \frac{4\pi c}{\omega_p^2}\mathrm{rot}\left[\frac{\partial}{\partial t}\vec{j}_e + Q_0(\vec{h}\times\vec{j}_e)\right]$$
$$+ \frac{ec}{4m\omega_p^4}\mathrm{rot}\left\{\left[\omega_0\delta\hat{\varepsilon}^*(\omega_0) - i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega}(\omega_0\delta\hat{\varepsilon}^*(\omega_0))\right]\vec{E}_1^*\right.$$
$$\left.\times\mathrm{rot}\left[\frac{\hat{I}\omega_p^2 + \omega_0^2\delta\hat{\varepsilon}(\omega_0)}{\omega_0} + i\frac{\partial}{\partial t}\frac{\partial}{\partial\omega_0}\left(\frac{\hat{I}\omega_p^2 + \omega_0^2\delta\hat{\varepsilon}(\omega_0)}{\omega_0}\right)\right]\vec{E}_1 + c.c.\right\} \quad (25)$$

where $Q_0 = \dfrac{eB_0}{mc}$ is the electron cyclotron frequency and $\vec{h} = \dfrac{\vec{B}_0}{B_0}$ is the unit vector along the direction of the external magnetic field.

3 Discussion

The expressions (22), (23) and (25), in the case of a very slowly varying amplitude $\vec{E}_1$, yield the expressions for the nonlinear currents density and the generated magnetic field obtained in [3] by the use of the hydrodynamic approach. This agreement of results account for description of the nonlinear current density $\vec{j}_2$ through the tensor S_{ijs}.

In an isotropic plasma ($Q_0 = 0$), the current density $\vec{j}_e$ is the only source of magnetic field generation and then the equation (25) turns into the same as obtained in [2]:

$$\left(c^2\Delta - \frac{\partial^2}{\partial t^2} - \omega_p^2\right)\left\langle\vec{B}\right\rangle = \frac{iec\omega_p^2}{4m\omega_0^3}\mathrm{rot}\left(\vec{E}_1^*\mathrm{div}\vec{E}_1 - c.c.\right) \quad (26)$$

This equation was obtained by many other autors too.

References

[1] Yu.M. Aliev, M.B. Jovanović and A.A. Frolov: Generation of quasistationary magnetic fields by external electromagnetic radiation in a cold magnetoactive plasma. To be published in *J. Plasma Phys.*

[2] Yu.M. Aliev, V.Yu. Bychenkov, M.S. Jovanović and A.A.Frolov: Kinetic theory of nonlinear low-frequency response of collisionless plasma to influence of high-frequency electromagnetic radiation. To be published in *J. Plasma Phys.*

[3] V.V. Pustovalov and V.P. Silin: Nelineynaya teoriya vzaimodeystviya voln v plazme. Trudy FIAN 61 (1972), 42-280.

NELINEARNE STRUJE U HLADNOJ MAGNETOAKTIVNOJ PLAZMI NA KOJU DELUJE VISOKOFREKFENTNO ELEKTROMAGNETNO ZRAČENJE

Yu. M. Aliev
B. M. Jovanović
A. A. Frolov

Polazeći od tenzora nelinearne interakcije [3] dobijeni su izrazi za nelinearne gustine struje i generisano magnetno polje. Ovi rezultati se slažu sa onima koji su dati u radu [1].