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NONLINEAR THEORY OF STIMULATED RAMAN BACKSCATTERING IN PLASMA

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Abstract: The equations for slowly–varying amplitudes of the three waves taking part in the stimulated Raman backscattering are derived with the nonlinear frequency–shift effect incorporated. Temporal evolution of such a system is studied. When linear damping of electron plasma wave is small or absent, the behaviour is nonstationary in the sense the reflectivity coefficient pulsates irregularly in time. Above some critical damping value this incoherency is removed and asymptotic system behaviour is stationary. The mean reflectivity level in the incoherent regime turns out very low (typically under 10%), just as detected in recent experiments.

1. Introduction

Stimulated Raman scattering (SRS) in plasmas has been studied theoretically [1–4], experimentally [5–7] and by computer simulations [2, 4, 8] for more than two decades. It corresponds to a parametric decay of an electromagnetic (transverse) wave into another (scattered) electromagnetic wave (EMW) and an electron plasma wave (EPW). SRS operates in the outer shell of corona (subquarter–critical density), and is responsible for anomalous reflection of the pump–wave energy in the fusion plasma and for a production of suprathermal electrons in the tail of the Maxwellian distribution function.

Characteristic Raman instability growth time is significantly shorter than a laser pulse duration. Thus, it is interesting to follow the temporal evolution of the waves taking part in the process. The special attention is payed here to the question whether SRS reaches the final stationary regime or it shows some kind of nonstationarity. The conditions which produce one or the other final state are discussed.

In the standard approach [1, 2], the waves taking part in the SRS process are assumed to obey a linear dispersion relation, although the interaction is intrinsically nonlinear. However, the powerful lasers, which are used nowadays in the controlled fusion experiments, necessarily cause the breaking of the linear wave structure. It is primarily typical for the EPW which expresses various forms of nonlinearity. We take into account the EPW nonlinear frequency–shift [4, 9], which is of great importance by significant pump powers. At the nonlinear stage the increased density fluctuation can cause an enhanced effective damping of the density fluctuations. Thus, we discuss here a relative influence of the

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EPW damping on the SRS evolution taking into account the convection and nonlinear frequency–shift of the EPW's.

2. Derivation of the model equations

Standard approach in theoretical investigations of SRS starts from combining of Maxwell and fluid equations [2]

$$\nabla \cdot \vec{E} = -\frac{e}{\epsilon_0}\delta n_e, \tag{1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\nabla \times \vec{B} = -e\mu_0 \left(n_0 + \delta n_e\right) + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}, \tag{3}$$

$$\frac{\partial \vec{v}_e}{\partial t} + \left(\vec{v}_e \cdot \nabla\right)\vec{v}_e = -\frac{e}{m_e}\left(\vec{E} + \vec{v}_e \times \vec{B}\right) - \frac{3\kappa T_e}{m_e\left(n_0 + \delta n_e\right)}\nabla \delta n_e - \nu_e \vec{v}_e, \tag{4}$$

$$\frac{\partial \delta n_e}{\partial t} + \nabla \cdot \left[\left(n_0 + \delta n_e\right)\vec{v}_e\right] = 0, \tag{5}$$

where n_0 is the equilibrium density of a homogeneous, isotropic plasma and δn_e represents EPW–driven density perturbation; $\vec{v}_e$ is electron fluid velocity, T_e – electron temperature, ν_e – linear EPW damping rate.

It is convenient to use plane geometry (Fig. 1.) and perfect matching conditions: $\omega_0 = \omega_1 + \omega_L$ and $\vec{k}_0 = \vec{k}_1 + \vec{k}_L$. These conditions provide that the backscattering effect is conserved in space and time. The scattering process is assumed one–dimensional, that is the three wave vectors $\vec{k}_0$, $\vec{k}_1$ and $\vec{k}_L$ lie on the same direction (x–axis here). The scattered EM wave vector $\vec{k}_1$ is pointed opposite to the incident one $\vec{k}_0$, so we consider the backscattering case here.

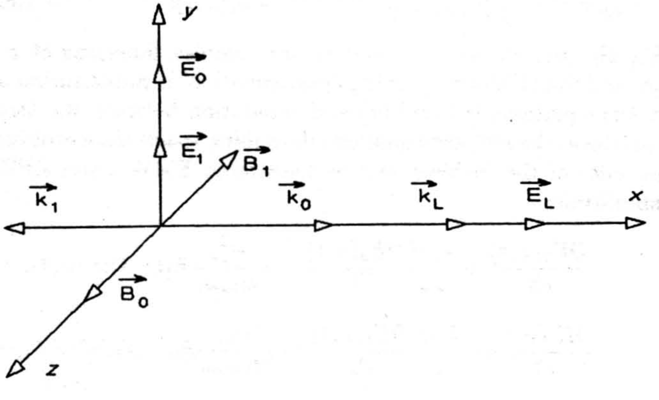

FIGURE 1. *Stimulated Raman backscattering: Directions of the characteristic vectors in the chosen geometry The process is one–dimensional as the three wave vectors $\vec{k}_0$, $\vec{k}_1$ and $\vec{k}_L$ lie all in the same direction.*

Thus, combining the above equations leads to the following differential equations in a one–dimensional case:

$$\frac{\partial^2 E_y}{\partial t^2} - c^2 \frac{\partial^2 E_y}{\partial x^2} + \omega_{pe}^2 E_y = -\frac{\omega_{pe}^2}{n_0} \delta n_e E_y + \frac{m_e}{e} \omega_{pe}^2 v_{ey} \frac{\delta n_e}{\partial t}, \tag{6}$$

$$\frac{\partial^2 \delta n_e}{\partial t^2} - 3v_{Te}^2 \frac{\partial^2 \delta n_e}{\partial x^2} + \nu_e \frac{\partial \delta n_e}{\partial t} + \omega_{pe}^2 \delta n_e = \frac{e n_0}{m_e} \frac{\partial}{\partial x} (v_{ey} B_z) + \frac{e}{m_e} \frac{\partial}{\partial x} (\delta n_e v_{ey} B_z) + \frac{n_0}{2} \frac{\partial^2 (v_{ex}^2)}{\partial x^2} + \frac{\partial}{\partial x} \left(\delta n_e v_{ex} \frac{\partial v_{ex}}{\partial x} \right) \frac{e}{m_e} \frac{\partial}{\partial x} (\delta n_e E_x) + \nu_e \frac{\partial}{\partial x} (\delta n_e v_{ex}). \tag{7}$$

Only the first term on the right–hand side of the equation (7) is retained, because the others are nonlinear terms of higher order.

Now we take into account a phase dependance of all entities related to the waves, assuming the scattered EM wave counterpropagating (backscattering):

$$E_x(x,t) = \mathrm{Re} \left\{ E_L(x,t) \mathrm{e}^{i(\omega_L t - k_L x)} \right\} = \frac{1}{2} \left[E_x(x,t) \mathrm{e}^{i(\omega_L t - k_L x)} + c.c. \right], \tag{8}$$

$$E_y(x,t) = \frac{1}{2} \left[E_0(x,t) \mathrm{e}^{i(\omega_0 t - k_0 x)} + E_1(x,t) \mathrm{e}^{i(\omega_1 t + k_1 x)} + c.c. \right], \tag{9}$$

$$B_z(x,t) = \frac{1}{2} \left[B_0(x,t) \mathrm{e}^{i(\omega_0 t - k_0 x)} - B_1(x,t) \mathrm{e}^{i(\omega_1 t + k_1 x)} + c.c. \right], \tag{10}$$

$$\delta n_e(x,t) = \frac{1}{2} \left[\delta n(x,t) \mathrm{e}^{i(\omega_L t - k_L x)} + c.c. \right], \tag{11}$$

$$v_{ex}(x,t) = \frac{1}{2} \left[v_L(x,t) \mathrm{e}^{i(\omega_L t - k_L x)} + c.c. \right], \tag{12}$$

$$v_{ey}(x,t) = \frac{1}{2} \left[v_{osc}(x,t) \mathrm{e}^{i(\omega_0 t - k_0 x)} + v_1(x,t) \mathrm{e}^{i(\omega_1 t + k_1 x)} + c.c. \right], \tag{13}$$

where E_0, E_1, B_0, B_1, δn, v_L, v_{osc} and v_1 are complex functions of x and t that vary slowly in space and time ("slowly–varying approximation"). Substitution of the expressions (8)–(13) into the equations (6) and (7) and separation between the terms by the phase dependance produces the coupled equations describing space–time evolution of the slowly–varying amplitudes of the incident and backscattered EMW's and EPW–driven density fluctuation amplitude:

$$\frac{\partial E_0(x,t)}{\partial t} + \frac{k_0 c^2}{\omega_0} \frac{\partial E_0(x,t)}{\partial x} = i \frac{\omega_{pe}^2}{4 n_0 \omega_1} E_1(x,t) \delta n(x,t), \tag{14}$$

$$\frac{\partial E_1(x,t)}{\partial t} - \frac{k_1 c^2}{\omega_1} \frac{\partial E_1(x,t)}{\partial x} = i \frac{\omega_{pe}^2}{4 n_0 \omega_0} E_0(x,t) \delta n^*(x,t), \tag{15}$$

$$\frac{\partial \delta n(x,t)}{\partial t} + \nu_e \delta n(x,t) + \frac{3 k_L v_{Te}^2}{\omega_L} \frac{\partial \delta n(x,t)}{\partial x} - i \sigma \omega_L \left| \frac{\delta n}{n_0} \right|^2 \delta n(x,t) = i \frac{\epsilon_0 \omega_{pe}^2 k_L^2}{4 m_e \omega_0 \omega_1 \omega_L} E_0(x,t) E_1^*(x,t). \tag{16}$$

The last term on the left–hand side of (16) is new in comparison to the former papers on this subject. It is added into the EPW equation to account for EPW nonlinearity, that could originate from a ponderomotive or relativistic corrections effects. This term expresses nonlinear wave detuning, and has a form of nonlinear frequency shift of the EPW. It will be demonstrated that the inclusion of the nonlinear frequency–shift effect changes basically the behaviour of the waves taking part in the SRS process.

It is convenient to perform a normalization of all quantities involved in (14)–(16) in order to obtain more usable operative equations:

$$\xi = \frac{x}{L}, \qquad \tau = \gamma_0 t,$$

$$a_0(\xi,\tau) = \frac{E_0(x,t)}{\hat{E}},\ a_1(\xi,\tau) = \sqrt{\frac{\omega_0}{\omega_0-\omega_{pe}}}\frac{E_1(x,t)}{\hat{E}},\ b(\xi,\tau) = -i\frac{\omega_{pe}}{k_L v_{osc}}\sqrt{\frac{\omega_{pe}}{\omega_0}}\frac{\delta n(x,t)}{n_0}, \tag{17}$$

where γ_0 is the Raman instability linear growth rate in a homogeneous lossless plasma, following from the equations (15) and (16):

$$\gamma_0 = \frac{ek_L\hat{E}}{4m_e\omega_0}\sqrt{\frac{\omega_{pe}}{\omega_0-\omega_{pe}}},$$

and $\hat{E} \equiv E_0(x=0)$, $v_{osc} = e\hat{E}/m_e\omega_0$.

In this way we come to the following model equations:

$$\frac{\partial a_0}{\partial \tau} + V_0\frac{\partial a_0}{\partial \xi} = -a_1 b, \tag{18}$$

$$\frac{\partial a_1}{\partial \tau} - V_1\frac{\partial a_1}{\partial \xi} = a_0 b^*, \tag{19}$$

$$\frac{\partial b}{\partial \tau} + V_L\frac{\partial b}{\partial \xi} + \Gamma b + i\Omega|b|^2 b = a_0 a_1^*, \tag{20}$$

with the constants:

$$V_0 = \frac{4k_0c^2}{k_L L v_{osc}\omega_0}\sqrt{\frac{\omega_0-\omega_{pe}}{\omega_{pe}}},\quad V_1 = \frac{4\,(k_L-k_0)\,c^2}{k_L L v_{osc}\sqrt{\omega_{pe}\,(\omega_0-\omega_{pe})}},$$

$$V_L = \frac{12v_{Te}^2}{\omega_{pe}Lv_{osc}}\sqrt{\frac{\omega_0-\omega_{pe}}{\omega_{pe}}},\quad \Gamma = \frac{\nu_e}{2\gamma_0},\quad \Omega = \frac{4k_L v_{osc}\omega_0\sigma}{\omega_{pe}^2}\sqrt{\frac{\omega_0-\omega_{pe}}{\omega_{pe}}}.$$

The NLFS coefficient σ is given the value 0.1 here, although it can take different values, depending on the NLFS generics.

We assume the homogeneous plasma slab spreading in the region $0 \le x \le L$ (i.e. $0 \le \xi \le 1$). The following boundary conditions for $a_0(\xi,\tau)$, $a_1(\xi,\tau)$ and $b(\xi,\tau)$ are employed in a numerical treatment of the equations (18)–(20): $a_0(\xi,0) = 0$, except $a_0(0,0) = 1$ (the pump wave falls onto plasma "from left"); $a_1(\xi,0) = \epsilon$, ϵ designating the thermal noise level; $b(\xi,0) = 0$, $a_0(0,\tau) = 1$, $a_1(1,\tau) = \epsilon$, $b(0,\tau) = 0$.

Now, we are able to study the behaviour of the system for different sets of plasma parameters.

3. Results and Discussion

We are most interested in calculations of SRS reflectivity – relative fraction of incident laser energy reflected via SRS process:

$$r(t) \equiv \frac{|E_1(x=0,t)|^2}{|E_0(x=0,t)|^2} = \left(1 - \frac{\omega_{pe}}{\omega_0}\right) |a_1(\xi=0,\tau)|^2 .$$

It is convenient to introduce a relative reflectivity R through

$$R(\tau) = \frac{r(\tau)}{r_{max}} = \frac{V_1}{V_0} |a_1(0,\tau)|^2 ,$$

r_{max} designating the maximum theoretical value of the reflectivity r in the stationary regime $(\frac{\partial}{7}\partial t = 0)$:

$$r_{max} = \frac{V_0}{V_1} \left(1 - \frac{\omega_{pe}}{\omega_0}\right) .$$

First we study the nonlinear temporal evolution of the reflectivity in a lossless case, it is the coefficients $\Gamma = \mu = 0$ in the equation (21). The other parameters used in calculations are typical for controlled fusion experiments [10]. Fig. 2.a represents a temporal evolution of the SRS reflectivity with the EPW nonlinear frequency–shift neglected [11], while Fig. 2.b accounts for this nonlinear effect. It is shown in Fig. 2.b that SRS grows linearly in the beginning with a growth rate $\gamma_0 \approx 0.2$ (SRS growth rate of linear parametric theory). However, after attaining a maximum reflectivity the SRS process becomes incoherent. Namely, the reflectivity pulsates irregularly in time, which is a physical consequence of the inclusion of the nonlinear frequency–shift in the EPW equation. Thus, in this case ($\Gamma = 0$) an asymptotically stable state does not establish.

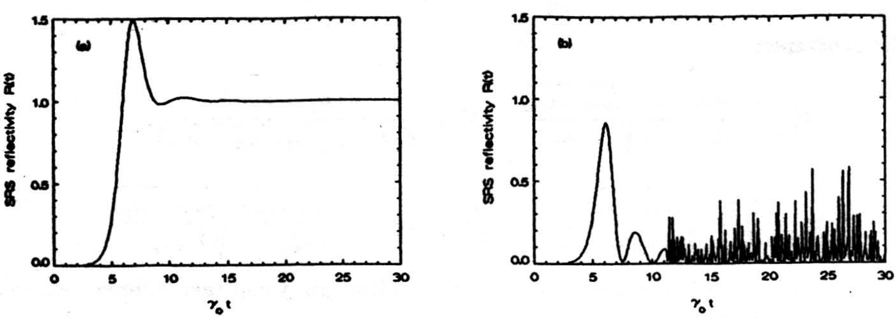

Figure 2. *Temporal evolution of the SRS reflectivity in a lossless ($\nu_e = 0$) homogeneous plasma slab for the following plasma parameters: $L = 100c/\omega_0$, $n_0 = 0.1\,n_{crit.}$, $T_e = 1\,keV$, $v_{osc} = 0.1\,c$, $\epsilon = 0.01$ with the EPW nonlinear frequency–shift neglected (a) and taken into account (b).*

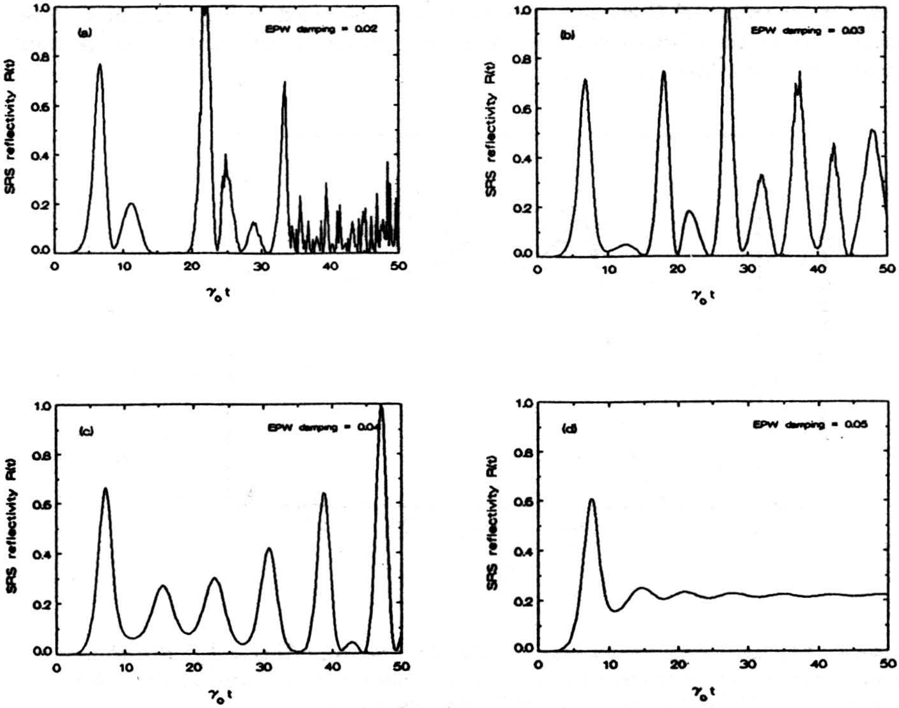

FIGURE 3. *Temporal evolution of the SRS reflectivity for different values of the linear damping of the EPW (nonlinear frequency–shift taken into account) for the following plasma parameters:* $L = 100c/\omega_0$, $n_0 = 0.1\, n_{crit.}$, $T_e = 1\, keV$, $v_{osc} = 0.1\, c$, $\epsilon = 0.01$.

It is important to find out how the damping affects the described incoherent character of the scattering. In order to do this we follow the SRS evolution in the cases when linear damping coefficient ν_e takes different nonzero values. The results are presented in Fig. 3.

It is clear from Fig. 3. that the introduction of the linear damping of the EPW suppresses the incoherent wave pulsations. It is reasonable to suppose that there exists a critical damping value ν_{cr} that entirely removes a chaotic temporal behaviour of the SRS reflectivity presented in Fig. 2.b. This means that there establishes a final stationary state when $\nu_e \geq \nu_{cr}$.

Another fact is interesting as well. If we ignore the nonlinear frequency–shift term (20), the SRS saturates at a pretty high level (Fig. 2.a). However, taking into account this effect leads to a significant reduce of the SRS reflectivity. This is checked by an averaging method and the result is presented in Fig. 4. for the EPW damping value $\nu_e/\omega_{pe} = 0$. The mean value of reflectivity is calculated as

$$\overline{R}(t) = \frac{1}{t}\int_0^t R(t)\, dt,$$

so, the maximum values of $\overline{R}$ are less than those demonstrated in Figs. 2. and 3. Nevertheless, the asymptotic SRS reflectivity levels are obtained low, just as the relevant experimental evidence shows [7, 10, 12].

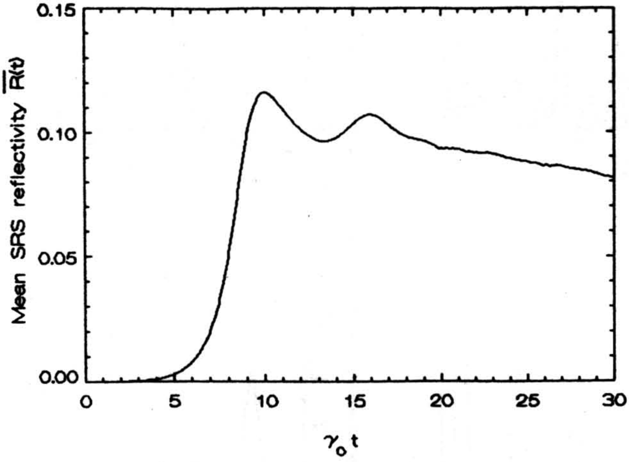

FIGURE 4. *The mean value (time average) of the SRS reflectivity ($\overline{R}(t) = \frac{1}{t}\int_0^t R(t)\,dt$) in a lossless case for the following plasma parameters: $L = 100c/\omega_0$, $n_0 = 0.1\, n_{crit.}$, $T_e = 1\, keV$, $v_{osc} = 0.1\, c$, $\epsilon = 0.01$ (nonlinear frequency–shift taken into account).*

Thus, the nonlinear frequency–shift is an important effect, that considerably affects the SRS temporal behaviour. Namely, SRS necessarily transits to a nonstationary (incoherent) regime if linear EPW damping is not too strong. On the other hand the mean SRS reflectivity turns out to be very low as compared to the case without nonlinear frequency–shift.

4. Conclusion

The set of partial differential equations describing three waves behaviour in the SRS process was derived. The derivation is based on the Maxwell and fluid (hydrodynamic) equations and an assumption that the EPW–driven electron density fluctuations are comparable in magnitude with the equilibrium electron density in plasma. A weak dependance of the wave amplitude on the space coordinates and time is assumed as well. The nonlinear frequency–shift in the EPW equation raises, which gives entirely new temporal behaviour as compared to the earlier known results. The effects of the frequency–shift and the EPW damping compete in determination of the asymptotic behaviour of the system. For any set of chosen parameters there exists a critical value of the EPW damping, depending on the nonlinear frequency–shift, so that the wave amplitudes vary irregularly (incoherently) in time for EPW damping under the critical one, and the system attains a stationary asymptotic state for the damping above critical. The averaged value of the SRS reflectivity in the incoherent regime is found to be at very low level (under 10%), which is in accordance with the recent experimental evidence.

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NELINEARNA TEORIJA STIMULISANOG RAMANOVOG RASEJANJA UNAZAD U PLAZMI

M.S. Jovanović and M.M. Škorić

Sadržaj: Izvedene su jednačine za sporo promenljive amplitude talasa u procesu stimulisanom Ramanovom rasejanju, pri čemu je uzet u obzir nelinearni frekventni pomeraj. Proučava se vremenska evolucija takvog sistema. Kada je linearno slabljenje elektronskog plazmenog talasa vrlo malo ili ga nema, ponašanje sistema je nestacionarno u smislu da koeficijent refleksije pulsira nepravilno u vremenu. Iznad neke kritične vrednosti slabljenja ova nekoherentnost nestaje i asimptotsko stanje sistema je stacionarno. Srednja reflektivnost u nekoherentnom režimu je vrlo niska (oko 10%), kao što je izmereno u nedavnim eksperimentima.