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SOME CHARACTERISTICS OF THE ION-ION HYBRID INSTABILITY IN WEAKLY IONIZED TWO-ION PLASMAS

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Abstract: The ion-ion hybrid (IIH) modes are studied theoretically, on the ground of kinetic equations with BGK model collision integrals, in weakly ionized, magnetized, uniform and infinite two-ion plasmas. Attention is focused on the longwave domain (modal wavelengths larger than electron mean-free-path, $\nu_e \gg k_{\|} v_{Te}$), and general expressions for the spectra and the instability criteria are derived. Apart from the Buchsbaum mode ($\omega \approx \omega_B$) known long ago, modes pertaining to higher-order Buchsbaum harmonics ($\omega \approx N\omega_B$, $N = 1, 2, 3, ...$) could also be studied, owing to the kinetic (rather than hydrodynamic) approach. Particular consideration is given to the situations characterized by $\omega\nu_e \ll k_{\|}^2 v_{Te}^2$ which can only be studied kinetically.

1. Introduction

The ion-ion hybrid (IIH) modes are weakly damped modes existing in magnetized two-ion plasmas, provided that the requirement $\omega_{B1} \gg \omega_{B2}$ is met (ω_{B1} and ω_{B2} are the ionic gyrofrequencies). These modes are characterized by perpendicular [1,2] or nearly perpendicular [3,4,5] propagation relative to the magnetic lines of force, and are subject to spontaneous onset of the pertaining instabilities (in the form of waves of growing amplitude) if an electron drift directed along the lines of force is present and if its magnitude exceeds some threshold value. Although some attempts to include the effects of thermal motion by way of a kinetic treatment are to be noted at the early stage of theoretical investigations of IIH modes [6], the majority of studies carried out so far was based on hydrodynamic models, so that only the mode with modal frequencies satisfying the requirement $\omega_{B2} \ll \omega \ll \omega_{B1}$ could be investigated. Those studies were usually combined with further simplifications based on approximations of cold and collisionless plasma.

In the present paper an attempt is made to approach the problem of the IIH waves

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from the viewpoint of kinetic theory. The calculations are based on kinetic equations with model collision integrals BGK; it is assumed that the ionization is weak, so that only the collisions between the charged particles and the neutrals are to be accounted for. It is hoped that the kinetic approach will not only shed some additional light on the role of collisions and thermal motion of the plasma constituents in conjuction with the IIH mode studied so far, but that it will also enable the investigation of the possible existence of new IIH modes, presumably appearing in the frequency ranges $N\omega_{B2} \ll \omega \ll N\omega_{B1}$, where N is any integer other than 1 (this was not possible within the frames of hydrodynamic models). Attention will be focused on slow electrostatic waves with wavelengths pertaining to the longwave domain (this amounts to the requirements $\nu_e \gg k_\parallel v_{Te} \gg \omega$), and with quasi-perpendicular propagation (i.e. for ω, $|\omega - R\omega_{Bs}| \gg k_\parallel v_{Ts}$, ν_s, the index $s = 1,2$ labelling the two ion species present).

2. Outlines of the theory

Under the conditions specified above, the weakly damped IIH waves may be expected to appear if one has either (a) $\omega\nu_e \gg k_\parallel^2 v_{Te}^2$, or (b) $\omega\nu_e \ll k_\parallel^2 v_{Te}^2$. It should be pointed out that only the situation corresponding to former of these requirements (and only for $N = 1$) has been investigated in some detail so far [4,5]. Indeed, it was shown long ago [7] that this is the only situation analysable with the aid of hydrodynamic models. The latter of the above requirements inevitably calls for kinetic treatment, but it has not received sufficient attention in conjuction with the IIH waves so far. The dispersion equation for the electrostatic waves generally reads

$$\delta\varepsilon_e + \delta\varepsilon_1 + \delta\varepsilon_2 = 0, \tag{1}$$

and the electronic contributions to it, pertaining to the two situations considered, (a) and (b), are given by [7]:

$$\delta\varepsilon_e^{(a)} = i\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\nu_e(\omega - k_\parallel u)}, \qquad (\omega\nu_e \gg k_\parallel^2 v_{Te}^2), \tag{2a}$$

$$\delta\varepsilon_e^{(b)} = \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left(1 + i\frac{\nu_e}{k_\parallel v_{Te}}\frac{\omega - k_\parallel u}{k_\parallel v_{Te}}\right), \quad (\omega\nu_e \ll k_\parallel^2 v_{Te}^2). \tag{2b}$$

The ionic contributions, appropriately modified for the quasi-perpendicular propagation (see, for example, [8]), are of the form:

$$\delta\varepsilon_s = \frac{\omega_{ps}^2}{k^2 v_{Ts}^2}\left[1 - \sum_{m=-\infty}^{+\infty}\frac{\omega A_m(\mu_s)}{\omega - m\omega_{Bs}} + i\sum_{m=-\infty}^{+\infty}\frac{\omega\nu_s A_m(\mu_s)}{(\omega - m\omega_{Bs})^2}\right], \quad (s = 1,2). \tag{3}$$

In the above expressions, i.e. (2a,b) and (3), $\omega_{p\alpha}$, $v_{T\alpha}$ and ν_α ($\alpha = e,1,2$) are, respectively, plasma frequency, thermal velocity and collision frequency of the constituent α, $k_\perp$ and $k_\parallel$ are the projections of the wave vector with respect to the magnetic lines of force, $\mu_s = k_\perp^2 v_{Ts}^2/\omega_{Bs}^2$ are the ionic finite Larmor radius parameters ($s = 1,2$), and $A_m(x) = I_m(x)\exp(-x)$.

Contrary to the hydrodynamic approaches, the expression (3) permits the study of the IIH waves with $N\omega_{B2} \ll \omega \ll N\omega_{B1}$, even if $N \neq 1$. Indeed, for any chosen N, one has to single out to the terms $m = 0, \pm N$ in (3), and to re-write consequently the above infinite sums as:

$$\delta\varepsilon_s = \frac{\omega_{ps}^2}{k^2 v_{Ts}^2}\left[1 - A_0(\mu_s) - \frac{2\omega^2 A_N(\mu_s)}{\omega^2 - N^2\omega_{Bs}^2} - \mathcal{S}^{(N)}(\mu_s) + i\frac{\nu_s}{\omega}R_N(\mu_s)\right], \quad (s = 1, 2), \qquad (4)$$

where:

$$R_N(\mu_s) = A_0(\mu_s) + \frac{2\omega^2(\omega^2 + N^2\omega_{Bs}^2)A_N(\mu_s)}{(\omega^2 - N^2\omega_{Bs}^2)^2} + \mathcal{D}^{(N)}(\mu_s). \qquad (5)$$

In the above relations, (4) and (5), $\mathcal{S}^{(N)}(\mu_s)$ and $\mathcal{D}^{(N)}(\mu_s)$ are infinite sums of (individually small) terms in (3) not taken into account explicitely in writing out (4) and (5). For the sake of brevity, they will be called "off-mode" terms. Hence:

$$\mathcal{S}^{(N)}(\mu_s) = \sum_{m\neq 0,\pm N} \frac{\omega A_m(\mu_s)}{\omega - m\omega_{Bs}}, \qquad \mathcal{D}^{(N)}(\mu_s) = \sum_{m\neq 0,\pm N} \frac{\omega^2 A_m(\mu_s)}{(\omega - m\omega_{Bs})^2}. \qquad (6a,b)$$

These "off-mode" terms were usually entirely omitted in the papers on IIH waves published so far. It may be mentioned, in particular, that for $N = 1$ and $\mu_s \ll 1$ (hydrodynamic limit, cold plasma), and with the "off-mode" terms and collisions completely neglected, (4) reduces to $\delta\varepsilon_s = -\omega_{ps}^2/(\omega^2 - \omega_{Bs}^2)$; if this is subsequently combined with (2a), the early Buchsbaum result $\omega = \omega_B$ is obtained [1,2]. Here, ω_B is the Buchsbaum frequency given by:

$$\omega_B = \left(\frac{\omega_{B1}^2\omega_{p2}^2 + \omega_{B2}^2\omega_{p1}^2}{\omega_{p1}^2 + \omega_{p2}^2}\right)^{1/2} = \omega_{B2}\left[\frac{1 + \dfrac{m_2}{m_1}\dfrac{\bar{n}_2}{\bar{n}_1}}{1 + \left(\dfrac{z_2}{z_1}\right)^2 \dfrac{m_1}{m_2}\dfrac{\bar{n}_2}{\bar{n}_1}}\right]^{1/2}; \qquad (7)$$

it obviously depends not only on the characteristics of the two ion species present (i.e. their mases m_s and charge numbers z_s, $s = 1, 2$), but also on the composition of the plasma in question (by way of the ratio of the fractional ion number densities $\bar{n}_s = \dfrac{n_s}{n_1 + n_2}$, $s = 1, 2$; one has $\bar{n}_1 + \bar{n}_2 = 1$).

Complete omission of the "off-mode" terms is not satisfactory enough; indeed, these terms are shown not only to improve the results obtained without them, but also to lead, in some instances, to qualitatively new efects, as demonstrated for some other quasi-perpendicular electrostatic modes in magnetized plasmas [8]. For a better accuracy, therefore, it is a necessary to take the "off-mode" terms into account. In principle, this may be effectuated by an iteration procedure. This means to evaluate the spectra with the terms (6a,b) completely omitted first, introduce the resulting ω into (6a,b), and calculate these sums, insert the results back into (4) and (5), and then repeat the whole procedure. In the present paper, however, a simpler way of accounting for the "off-mode" terms is adopted. Namely, one simply lets $\omega = N\omega_B$ in all these terms, and thus arrives at the following form of the sums (6a,b):

$$\bar{\mathcal{S}}^{(N)}(\mu_s) = \sum_{m\neq 0,\pm N} \frac{N\omega_B A_m(\mu_s)}{N\omega_B - m\omega_{Bs}}, \qquad \bar{\mathcal{D}}^{(N)}(\mu_s) = \sum_{m\neq 0,\pm N} \frac{N^2\omega_B^2 A_m(\mu_s)}{(N\omega_B - m\omega_{Bs})^2}. \qquad (8a,b)$$

3. IIH Spectra

Inserting (8a,b) into (4) and (5), and then combining the obtained expressions with (2a) or (2b), the dispersion equation (1) is easily formed. From Re $(\delta\varepsilon_e + \delta\varepsilon_1 + \delta\varepsilon_2) = 0$ one obtains the folowing relation for the spectrum:

$$\gamma + \frac{\omega_{p1}^2}{\omega_{pe}^2}\frac{v_{Te}^2}{v_{T1}^2}\left[1 - A_0(\mu_1) - \frac{2\omega^2 A_N(\mu_1)}{\omega^2 - N^2\omega_{B1}^2} - \bar{S}^{(N)}(\mu_1)\right] + \\ + \frac{\omega_{p2}^2}{\omega_{pe}^2}\frac{v_{Te}^2}{v_{T2}^2}\left[1 - A_0(\mu_2) - \frac{2\omega^2 A_N(\mu_2)}{\omega^2 - N^2\omega_{B2}^2} - \bar{S}^{(N)}(\mu_2)\right] = 0. \tag{9}$$

Here, γ is a coefficient, having numerical values 0 and 1, respectively for the situations (a) and (b). It may be useful to mention that, in view of the quasi-neutrality condition $n_e = z_1 n_1 + z_2 n_2$, one has $\dfrac{\omega_{ps}^2}{\omega_{pe}^2} = \dfrac{m_e}{m_s}\dfrac{z_s^2 \bar{n}_s}{z_1\bar{n}_1 + z_2\bar{n}_2}$, so that only the fractional ion number densities, introduced in the comment of (7), will appear throughout in the results that follow. After some re-arrangements, (9) becomes a quadratic equation in ω^2. For the given type and composition of the plasma (i.e. for fixed values of m_s, z_s, T_s and $\bar{n}_s$, $s = 1, 2$), both solutions of (9) can be regarded as functions of the variable μ_1 only; indeed, it can be easily verified that μ_2 is reducible to μ_1, and $\mu_2 = \dfrac{T_2}{T_1}\dfrac{m_2}{m_1}\left(\dfrac{z_1}{z_2}\right)^2 \mu_1$. However, only one of the solutions yields modal frequencies lying in the interval $(N\omega_{B1},\ N\omega_{B2})$, and this one corresponds to the problem investigated. Its explicit form is obtained in a straightforward manner, but it will not be given here in view of its unwieldiness.

4. Evaluation of the threshold drifts

The condition of marginal instability is Im $(\delta\varepsilon_e + \delta\varepsilon_1 + \delta\varepsilon_2) = 0$. If solved for u/v_{T1} (index 1 labels the ion species with larger gyrofrequency), this condition yields:

$$\left(\frac{u}{v_{T1}}\right)_N^{(a)} = \frac{\omega}{k_\parallel v_{T1}} + \frac{k_\parallel v_{T1}}{\omega}\frac{\omega^2}{\nu_e\left[\dfrac{\omega_{p1}^2}{\omega_{pe}^2}\nu_1 R_N(\mu_1) + \dfrac{\omega_{p2}^2}{\omega_{pe}^2}\nu_2\left(\dfrac{v_{T1}}{v_{T2}}\right)^2 R_N(\mu_2)\right]}, \tag{10a}$$

$$\left(\frac{u}{v_{T1}}\right)_N^{(b)} = \frac{\omega}{k_\parallel v_{T1}} + \frac{k_\parallel v_{T1}}{\omega}\left(\frac{v_{Te}}{v_{T1}}\right)^2\left[\frac{\omega_{p1}^2}{\omega_{pe}^2}\left(\frac{v_{Te}}{v_{T1}}\right)^2\frac{\nu_1}{\nu_e}R_N(\mu_1) + \\ + \frac{\omega_{p2}^2}{\omega_{pe}^2}\left(\frac{v_{Te}}{v_{T2}}\right)^2\frac{\nu_2}{\nu_e}R_N(\mu_2)\right], \tag{10b}$$

respectively for the situations (a) and (b). The above equations define the critical drifts for the onset of the IIH instability in the given direction of propagation (specified by $k_\parallel$ and $k_\perp$; the latter argument comes in by way of μ_1). To determine the threshold drifts, one has to evaluate the minimum of the right-hand sides of (10a) and (10b). To this effect, it might be useful to introduce the dimensionless quantities:

$$X_N = \frac{N\omega_B}{k_\parallel v_{T1}}, \qquad Y_N = \frac{\omega}{N\omega_B}; \tag{11}$$

they conveniently substitute the variables $k_\parallel$ and $k_\perp$, since $\omega = \omega(\mu_1)$. Relations (10a) and (10b) are then re-written in a unified manner:

$$\left(\frac{u}{v_{T1}}\right)_N^{(a,b)} = X_N Y_N(\mu_1) + \frac{Q_N^{(a,b)}(\mu_1)}{X_N Y_N(\mu_1)}, \tag{12}$$

with

$$Q_N^{(a)}(\mu) = \frac{(N\omega_b)^2 Y_N(\mu_1)^2}{\nu_e \left[\frac{\omega_{p1}^2}{\omega_{pe}^2}\nu_1 R_N(\mu_1) + \frac{\omega_{p2}^2}{\omega_{pe}^2}\nu_2 \left(\frac{v_{T1}}{v_{T2}}\right)^2 R_N(\mu_2)\right]}, \tag{13a}$$

$$Q_N^{(b)}(\mu) = \left(\frac{v_{Te}}{v_{T1}}\right)^2 \left[\frac{\omega_{p1}^2}{\omega_{pe}^2}\left(\frac{v_{Te}}{v_{T1}}\right)^2 \frac{\nu_1}{\nu_e} R_N(\mu_1) + \frac{\omega_{p2}^2}{\omega_{pe}^2}\left(\frac{v_{Te}}{v_{T2}}\right)^2 \frac{\nu_2}{\nu_e} R_N(\mu_2)\right]. \tag{13b}$$

If the derivatives of (12) with respect to X_N and μ_1 are equated to zero, and the ensuing equations solved (cf. [8]), one eventually arrives at the following exppression for the threshold drifts:

$$\left(\frac{u}{v_{T1}}\right)_N^{(a,b)} = 2\sqrt{\left(Q_N^{(a,b)}\right)_{\min}}. \tag{14}$$

Thus, the evaluation of the threshold drifts amounts to the numerical determination of the minima of (13a) and (13b), and this is an elementary task.

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NEKE KARAKTERISTIKE JON-JONSKIH HIBRIDNIH NESTABILNOSTI U SLABO JONIZOVANIM PLAZMAMA SA DVE VRSTE JONA

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Sadržaj: Teorijski se proučavaju jon-jonski hibridni modovi u slabo jonizovanoj, zamagnetisanoj, homogenoj i beskonačnoj plazmi, na osnovu kinetičkih jednačina sa modelnim kolizionim integralom BGK. Pažnja je koncentrisana na dugotalasno područje (talasne dužine veće od srednjeg slobodnog puta elektrona, $\nu_e \gg k_{\parallel} v_{Te}$), za koje su izvedeni opšti izrazi za spektre i kriterijume nestabilnosti. Osim Buchsbaumovog moda ($\omega \approx \omega_B$) koji je poznat i od ranije, bilo je moguće, zahvaljujući kinetičkom (a ne hidrodinamičkom) prilazu, izučavanje i modova koji odgovaraju višim Buchsbaumovim harmonicima ($\omega \approx N\omega_B$, $N = 1, 2, 3, ...$). Posebno je obraćena pažnja na situacije u kojima je dodatno zadovoljen uslov $\omega\nu_e \ll k_{\parallel}^2 v_{Te}^2$ i koje se mogu analizirati samo kinetički.