FACTA UNIVERSITATIS
Series: **Physics, Chemistry and Technology** Vol. 2, N° 3, 2001, pp. 149 - 157

NUMERICAL INVESTIGATION OF LAMINAR NATURAL CONVECTION IN INCLINED SQUARE ENCLOSURES

UDC 537.565/.568

Miomir Raos

Faculty of Occupational Safety, University of Nis, Serbia, Yugoslavia

Abstract. *The study represents investigation of the laminar natural convection phenomena in enclosed spaces. Realization of this subject has been done through number of phases, which have to make better understanding and configuration velocity, temperature and pressure fields in enclosures.*
The real physical model of the enclosure, which represents two - dimensional rectangular object with differentially heated sides and adiabatic horisontal walls, has been defined in order to predict good enough results. In fact, this two - dimensional object could be represent base for three - dimensional investigation, because all field are similar, for great object depth. Rotation of the enclosures is one of parameter of this study too.
Physical model represents base for mathematical model which defines valid parameters for temperature flow regime.
Solution of the defined mathematical model with respect the nature of the equations, has been done with numerical control volume method. Due to presence of the numerical procedure results there is made a computer code, which contains SIMPLE procedure in essence, and which contains routines for solving variables field.
Results of the numerical experiment has been compared with literature ones, and it gives a reliable agreements. Results of this study with some other analyses and research present good base for definition of the object parameters which can find in engineering practice, and which essentially contain natural convection phenomena.

1. NATURE OF PROBLEM

Natural convection heat transfer in enclosures involves different aspect of problems. This variety of problems comes from possibly geometry characteristic of enclosures, kind of fluid, nature of fluid flow, orientation of the enclosure etc. The most studies of natural convection in enclosures, based on 2D or 3D parallelogram enclosure investigation, annuli and cylinders with different aspect ratio or diameters, or caliber. It's very interesting because of sensibility of natural convection phenomena from geometry.

Important thing like orientation of enclosures according acceleration gravity vector, product variety of physical situation. Also there is type of fluid with influence on natural

Received June 07, 2002

convection phenomena. If we add new phenomena like radiation, change fluid phase, porous media, and chemical reaction and so on, we have very difficult physical models often unsolvable.

2. MATHEMATICAL FORMULATION

We defined 2D inclined square enclosure. Angle of inclination measured from horizontal plate to the hot wall of the enclosure. The square enclosure has opposite isothermal hot and cold walls and adiabatic horizontal walls. Physical model of the enclosure is represented on Fig 1.

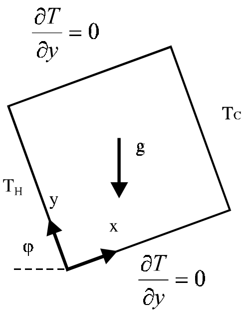


Fig. 1. 2D square enclosure

As we see, dimensions of the enclosure are L and H. For $x = 0$ temperature of the hot wall is Th and for $x = L$ temperature of the opposite cold wall is T_C.

Difference between these temperatures is $\Delta T = T_H - T_C$. Other sides of enclosure for $y = 0$, $y = H$ are adiabatic. We defined aspect ratio A, as relation between height over the length of the enclosure $A = H/L = 1$.

Enclosure rotates about some angle over the z axe, so the hot wall and horizontal plate make angle of rotation.

Pr number assumed as 0,73 (air filled enclosure), and there are laminar flow condition.

Cartesian coordinates rotate with enclosure. We consider constant some physical characteristic of the fluid like: dynamic viscosity, thermal conductivity, specific heat rate on the constant pressure for averaged temperature T_0. All temperatures are low intensity and therefore we can exclude radiation. If there are relatively small temperature difference in enclosure we can use Boussinesq approximation.

Density is also considered as constant value but for buoyant term it's linearised by relation:

$$\rho(T) = \rho(T_0) - \beta\rho(T_0)(T - T_0) \tag{1}$$

were β is thermal expansion coefficient for temperature T_0.

Described physical model represent base for mathematical model, otherwords, system of the partial differential equations, which describe physical phenomenon.

The mathematical model is defined by:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\beta g\left(T-T_0\right)\cos\varphi \tag{3}$$

$$u\frac{\partial \upsilon}{\partial x}+\upsilon\frac{\partial \upsilon}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 \upsilon}{\partial x^2}+\frac{\partial^2 \upsilon}{\partial y^2}\right)-\beta g\left(T-T_0\right)\sin\varphi \tag{4}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

In fact eq. 2-5 are system of partial differential equations. They are base for natural convection phenomenon for 2D enclosures, presented by mass, momentum and energy conservation equations. This equations system used to be named as p-v formulation, and we will discuss them.

There is inertia, pressure, viscous and buoyancy terms in momentum equations and convective and diffusion term in energy equation.

In order to obtain results of the conservation equations we define boundary conditions.

All velocities on the walls and temperature gradients on the adiabatic walls are equal zero. There is defined temperature values as Th and Tc on the hot and cold walls.
Mathematic interpretation is:

$$0\le x\le L \quad y=0 \quad u=v=0 \qquad \frac{\partial T}{\partial y}=0 \tag{6}$$

$$0\le x\le L \quad y=H \quad u=v=0 \qquad \frac{\partial T}{\partial y}=0 \tag{7}$$

$$x=0 \quad 0\le y\le H \quad u=v=0 \qquad T=T_H \tag{8}$$

$$x=L \quad 0\le y\le H \quad u=v=0 \qquad T=T_C \tag{9}$$

3. Numerical procedure

The essence of the natural convection phenomenon is connection between equations in mathematical model. In forced convection we have not such problem but in our situation we have temperature in buoyancy term in momentum equations for both direction x and y. Therefore we have some difficulties in solving this system elliptic partial differential equations and we have to solve them by numeric procedure.

Numeric procedure is based on finite volume method (FVM) and discretization procedure.

Therefore system of the partial differential equations approximated with algebraic equations system by discretization procedure. The new algebra equations system can easy be solved using some of known techniques, like Gauss method.

Enclosure divided with orthogonal grid on the subvolumes. Each of control volume surround one nodal point in its center called discretization point.

The aim is that we have to balance influence of convection and diffusion terms on the control volume sides. Therefore we use some of the discretization chemes. In our paper we used hybrid cheme of discretization which is mixture of central-difference cheme and upwind cheme.

General differential equation is:

$$\frac{\partial}{\partial t}(\rho\Phi) + div(\rho\vec{w}\Phi) = div(\Gamma_\Phi grad\Phi) + S_\Phi \tag{10}$$

Integration of differential equation for control volume and using of Gauss divergence theorem we find balance of the source terms and fluxes for each side of control volume.

$$\int_{V_P} \frac{\partial}{\partial t}(\rho\Phi)dV + \int_{S_P}(\rho\vec{w}\Phi - \Gamma_\Phi grad\Phi)\vec{n}dS = \int_{V_P} S_\Phi dV \tag{11}$$

Total fluxes, convective and diffusive, for one dimension can be express like:

$$J_x = \rho u\Phi - \Gamma_\Phi \frac{\partial\Phi}{\partial x} \tag{12}$$

and if total flux are constant value through surface between two neighbourgh control volume:

$$J_e = \int J_x dy = \left(\rho u\Phi - \Gamma_\Phi \frac{\partial\Phi}{\partial x}\right)_e \Delta y \tag{13}$$

as we generalized problem for other sides of control volume:

$$(J_e - J_w) + (J_n - J_s) = \int S_\Phi dV \tag{14}$$

Source term on the right side of the of the equation can be linearised by:

$$\int S_\Phi dV = (S_c + S_p\Phi_P)\Delta x\Delta y \tag{15}$$

and *Sc*, *Sp* are constant values and independent from Φ*p*. Sp term must be positive to achieve numeric stability and convergation .

Hybrid cheme gives:

$$\left(\rho u - \Gamma\frac{\partial\Phi}{\partial x}\right)_e \approx \begin{cases} (\rho u)_e & Pe_e \le -2 \\ (\rho u)_e\dfrac{\Phi_P+\Phi_E}{2} - \Gamma_e\dfrac{\Phi_E-\Phi_P}{\Delta x_e} & -2 < Pe_e < 2 \\ (\rho u)_e\Phi_P & Pe_e \ge 2 \end{cases} \tag{16}$$

otherwords:

$$\begin{aligned} &Pe_e \le -2\, J_e = F_e\Phi_E \\ &-2 < Pe_e < 2 J_e = 0{,}5F_e(\Phi_P + \Phi_E) - D_e\frac{\Phi_E - \Phi_P}{\Delta x_e} \\ &Pe_e \ge 2\, J_e = F_e\Phi_P \end{aligned} \tag{17}$$

were *F* is flux, and Γ*e* diffusion coefficient. The result of all efforts is discretization equation:

$$a_P\Phi_P = a_E\Phi_E + a_W\Phi_W + a_N\Phi_N + a_S\Phi_S + b \tag{18}$$

with a_p, a_e, a_w, a_n, a_s as coefficients for respect discretization point.

Connection between pressure gradient and state equation involved in pressure correction method.

Main solving procedure for obtaining results is SIMPLE iterative procedure, line by line, and then TDM algoritam. Connection between pressure and velocity is used for transformation continuity equation into the pressure equation. Procedure is iterative, line by line with left to right direction. In order to obtain results we made our own computer code in FORTRAN named PROSTOR, which contains some routines for preparing, initializing and solving.

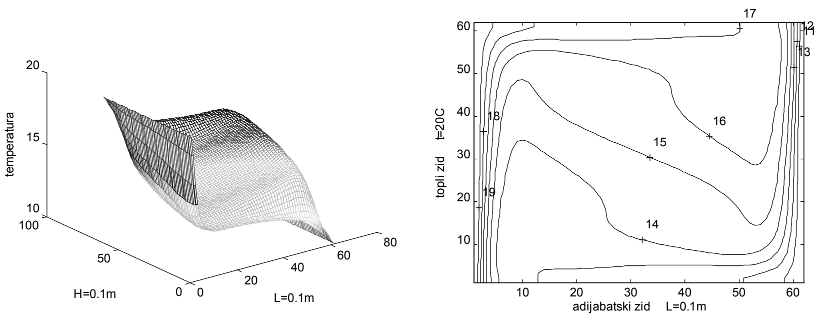
As result, we obtain matrix for p,u,v and T as depended variables in all points of discretization.

For visual presentation, we used additional software because of it's useful graphic and some other routines for some other solving. We obtain a number of results, graphic presentation, results for temperature gradients, local N_u numbers and average N_u numbers.

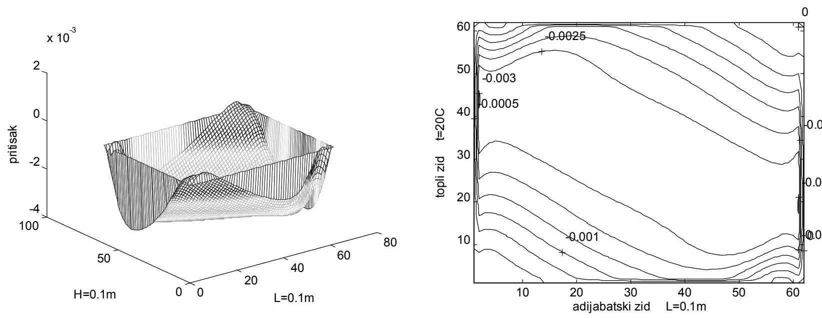
Those results represent good enough base for comparation of for each other and with ones from literature.

We also obtain all maximum and minimum values of variables, and their places in domain of investigation.

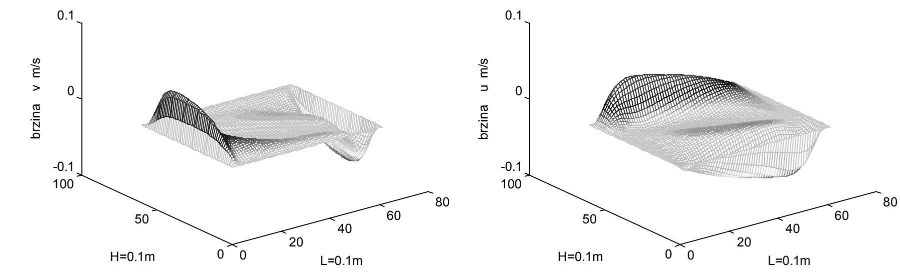
Some of the graphic results we are presenting in this paper on the some next figures:



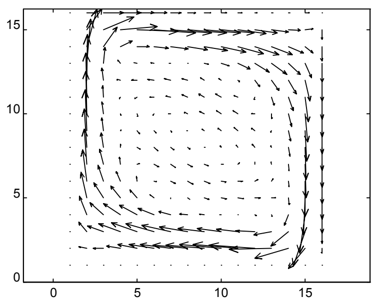
Temperature (mesh, contour): angle 60°; ΔT=10°C; Ra=1,28×10^6, 60×60 control volumes



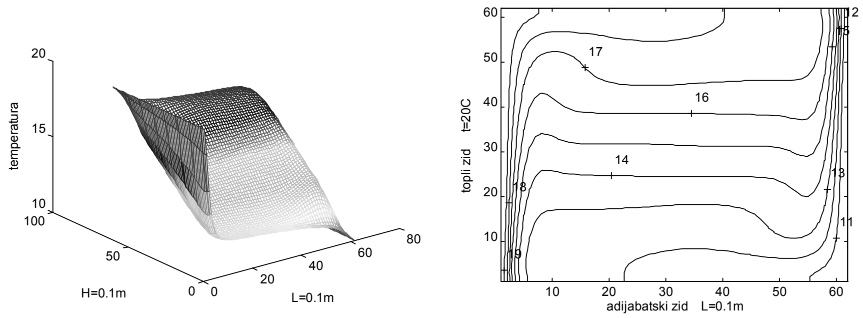
Pressure (mesh, contour): angle 60°; ΔT=10°C; Ra=1,28×10^6



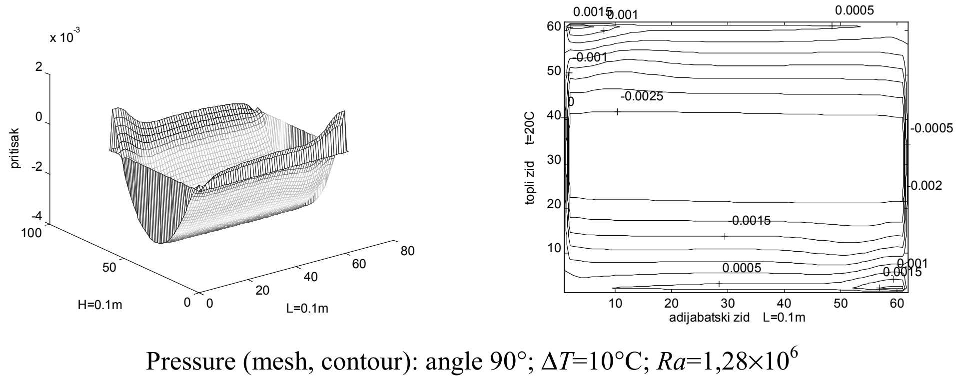
Velocity (x direction, y direction, mesh): angle 60°; ΔT=10°C; Ra=1,28×10^6



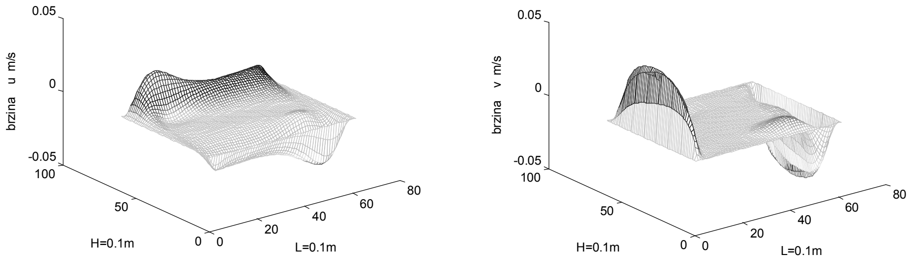
Velocity vectors: angle 60°; scale 1:3; Geometric scale (1:4:62)



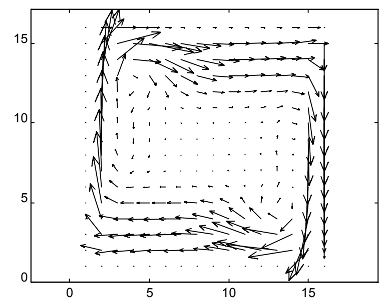
Temperature (mesh, contour): angle 90°; ΔT=10°C; Ra=1,28×10^6



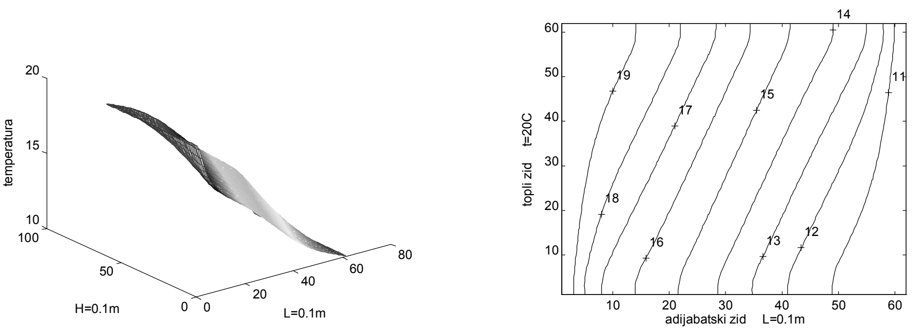
Pressure (mesh, contour): angle 90°; ΔT=10°C; Ra=1,28×10^6



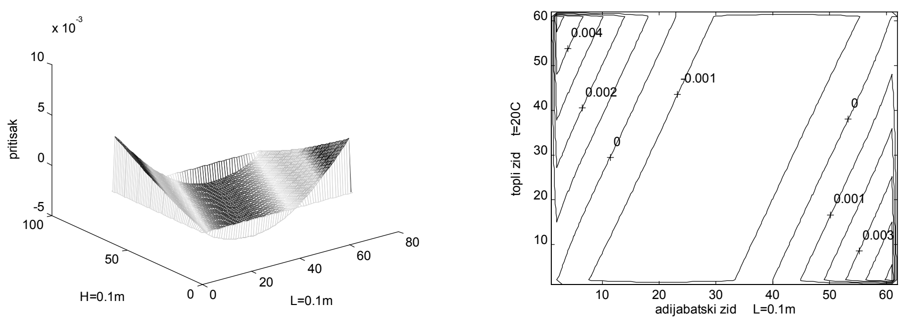
Velocity (x direction, y direction, mesh): angle 90°; ΔT=10°C; Ra=1,28×10^6



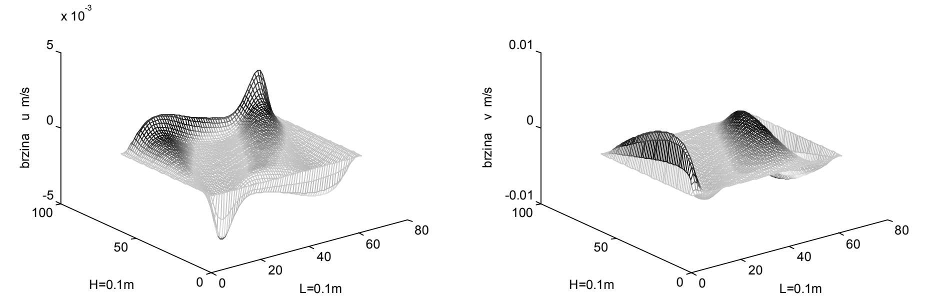
Velocity vectors: angle 90°; scale 1:3; Geometric scale (1:4:62)



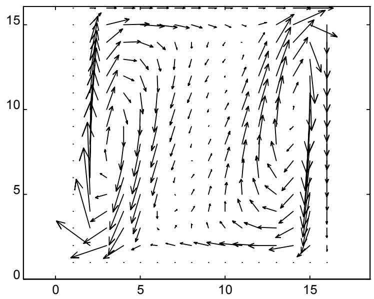
Temperature (mesh, contour): angle 160°; ΔT=10°C; Ra=1,28×10^6



Pressure (mesh, contour): angle 160°; ΔT=10°C; Ra=1,28×10^6



Velocity (x direction, y direction, mesh): angle 160°; ΔT=10°C; Ra=1,28×10^6



Velocity vectors: angle 160°; scale 1:3; Geometric scale (1:4:62)

4. Conclusion

Laminar natural convection in square enclosure is successfully simulated, and the results are predicted by the numerical cheme developed. The study shows complex flow patterns heat transfer rates (represented by averaged N_u numbers on the isothermal walls), with different orientation of the enclosure. Orientation above 90° adversely affects the convection process particularly if we reach to 180°. Orientation under 20°, in fact 17° and lower, shows some inconsistent flow pattern, we may say time-dependent patterns of variables, so we consider angles of rotation only between 20° and 180°.

Angle of rotation about 65°÷75° maximize Nu number value which represent maximum heat transfer rate for named conditions.

References

1. Bejan A., Tien C. L., "Laminar Natural Convection Heat Transfer in a Horizontal Cavity with Different end Temperatures", Journal of Heat Transfer, vol. 100, pp 641-647, 1978.
2. Catton I., "Natural Convection in Enclosures", Proceeding of 6th International Heat Transfer Conference, vol. 6, pp 13-31, 1978.
3. Graham de Vahl Davis, "Finite Difference Methods for Natural and Mixed Convection in Enclosures", Proceeding of the Eighth International Heat Transfer Conference, Vol. 4, pp 101-109, San Francisco, 1986.
4. Hoogendoorn C. J., "Natural Convection in Enclosures", Proceeding of the Eighth International Heat Transfer Conference, Vol. 1, pp 111-121, San Francisco, 1986.

5. Kuyper R. A., "Transport Phenomena in Underground Coal Gasification Channels", ", Ph.D. Thesis, Technishe Universitet Delft, The Netherlands, 1994.
6. Maekawa T., Tanasawa I., "Natural Convection Heat Transfer in Parallelogrammic Enclosures", Proceeding of the Seventh International Heat Transfer Conference, Vol. 2, pp 227-233, Munchen, 1982.
7. Mossel M. D., "Thermal Stratification of Turbulent air Flows in a Rectangular Channel", Phd Thesis, Technishe Universitet Delft, The Netherlands, 1995.
8. Ostrach S., "Natural Convection Heat Transfer in Cavities and Cells", Proceeding of the 7th International Heat Transfer Conference, vol. 1, pp 365-379, Munchen, 1982.
9. Ostrach S., "Advances in Heat Transfer", Academic Press, New York 1972.
10. Patankar S., "Numerical Heat Transfer and Fluid Flow", Hemisphere Publishing Co., New York, 1980.
11. Patankar S. V., "Numerical Methods in Heat Transfer", Proceeding of the Seventh International Heat Transfer Conference, Vol. 1, pp 83-90, Munchen, 1982.
12. Raos M. 1999, Laminar Natural Convection in Enclosures. M.Sc. thesis, University of Nis, Faculty of Mechanical Engineering, Nis, Yugoslavia
13. Reddy C. S., "Numerical Simulation of Laminar Natural Convection in Shallow Inclined Enclosures", Proceeding of the International Heat Transfer Conference, Vol. 2, pp 263-269, Munchen, 1982.
14. Warrington Jr. R. O., Brown P. K., Powe R. E., "Natural Convection Heat Transfer Within Enclosures at Reduced Pressures" Proceeding of the International Heat Transfer Conference, Vol. 4, pp 1483-1488, San Francisco, 1986.
15. Wang L. W., Sun D. J., "Convective Flows in Enclosures with Vertical Temperature or Concentration Gradients", ", Journal of Thermophysics and Heat Transfer Index, vol. 6, No2, pp 379-381, 1992.
16. Yang H. Q., Yang K. T., Lloyd J. R., "Flow Transition in Laminar Buoyant Flow in Three Dimensional Tilted Rectangular Enclosure", Proceeding of the International Heat Transfer Conference, Vol. 4, pp 1495-1500, San Francisco, 1986.
17. Yang K. T., "Numerical Modeling of Natural Convection Radiation Interactions in Enclosures", Proceeding of the International Heat Transfer Conference, Vol. 1, pp 131-140, San Francisco, 1986.

NUMERIČKO ISTRAŽIVANJE LAMINARNE PRIRODNE KONVEKCIJE U ZATVORENOM PROSTORU OBLIKA KVADRATA POD NAGIBOM

Miomir Raos

U ovom radu je prikazan pregled dela istraživanja u oblasti prenosa toplote prirodno konvektivnih strujanja u zatvorenim prostorima, a odnosi se na laminarnu prirodnu konvekciju. Istraživanje takvog fenomena izvršeno je u više faza koje treba da bolje objasne strukturu polja temperatura, brzina i pritisaka u prostoru. Fizički model prostora predstavljen je dvodimenzionalnim paralelogramom sa različito zagrevanim bočnim zidovima i adijabatskim gornjim i donjim zidom. Jedna od bitnih karakteristika numeričkog eksperimenta je i rotacija prostora u odnosu na horizontalnu ravan. Takav fizički model predstavljen je odgovarajućim matematičkim modelom, a njegovo rešavanje izvrešeno je numeričkom procedurom i primenom originalnog softvera. Rezultati istraživanja slažu se sa rezultatima iz literature i opravdavaju postavljene predpostavke i metodologiju. Istraživanja mogu predstavljati dobru osnovu za objašnjenje različitih fenomena koji u osnovi sadrže fenomen prirodne konvekcije u prostorima.