# THE SECOND HARMONIC GENERATION IN A COLLISIONAL MAGNETOACTIVE PLASMA 

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#### Abstract

Characteristics for the generation of the second harmonic in homogeneous, collisional and magnetized plasma are investigated theoretically by solving the system of nonlinear equations for the fundamental and second harmonic extraordinary waves. The dependence of the efficiency of the wave frequency doubling on the distance from the plasma boundary and on the collisional frequency has been calculated.


## Introduction

The nonlinear process of frequency doubling of an electromagnetic wave has been widely investigated by many authors [1-7]. Reasons for it range from applications in plasma diagnostics techniques to purely theoretical interests in studies of nonlinear phenomena in plasmas.

In this paper, we treat the problem of the second harmonic generation of the extraordinary mode propagating in a homogeneous, magnetoactive, and collisional plasma. We focus our attention on how the second harmonic amplitude and the efficiency of the doubling process depend on the plasma slab thickness and on the collisional frequency at the eee-phase synchronism, i.e. when the resonant condition $\operatorname{Re}\left\{N_{e}^{(1)}\right\}=\operatorname{Re}\left\{N_{e}^{(2)}\right\}$ for the extraordinary wave is satisfied. Here, $N_{e}^{(1)}$ and $N_{e}^{(2)}$ are the reflection indices for the fundamental and second harmonic extraordinary wave respectively.

## Basic equations

The initial unperturbed state is defined by a cold plasma and vacuum domain at $\mathrm{x}>0$ and $\mathrm{x}<0$ respectively, separated by a plane boundary $\mathrm{x}=0$, and in presence of an external uniform and stationary magnetic field $\mathbf{B}_{0}=B_{0} \mathbf{e}_{z}$ i.e. $B_{0}=$ const .

A driving pump electromagnetic wave originates in vacuum and is taken to propagate along the x -axis direction toward the vacuum-plasma boundary $\mathrm{x}=0$ where the double refraction occurs giving rise to the ordinary and extraordinary modes. In this paper, we shall consider only the extraordinary mode and a resonant generation of its second har-
monic when the phase synchronism condition is fulfilled. It will be shown that the second harmonic amplitude grows in space as the wave propagates through the plasma medium at $x>0$ which results into a simultaneous decrease of the amplitude of the initial fundamental wave.

We start from the standard nonlinear equations describing the electric field perturbation amplitudes for the first and the second harmonic waves:

$$
\begin{equation*}
\nabla^{(\eta)} \times\left[\nabla^{(\eta)} \times \mathbf{E}^{(\eta)}\right]-\frac{\eta^{2} \omega^{2}}{\mathrm{c}^{2}} \stackrel{(\eta)}{\varepsilon} \mathbf{E}^{(\eta)}=\frac{i \eta \omega}{\mathrm{c}^{2} \varepsilon_{0}} \mathbf{j}_{\mathrm{nl}}^{(\eta)} \tag{1}
\end{equation*}
$$

where $\eta=1,2$ refer to the first and the induced second harmonic wave, respectively. The ${ }^{(\eta)}$. dielectric tensor $\hat{\varepsilon}^{(n)}$ is given by

$$
\hat{\varepsilon}^{(\eta)}=\left(\begin{array}{lll}
\varepsilon_{1}^{(\eta)} & i \varepsilon_{2}^{(\eta)} & 0  \tag{2}\\
-i \varepsilon_{2}^{(\eta)} & \varepsilon_{1}^{(\eta)} & 0 \\
0 & 0 & \varepsilon_{3}^{(\eta)}
\end{array}\right)
$$

where

$$
\varepsilon_{1}^{(\eta)}=1-\frac{\mathrm{v}(\eta-i s)}{\eta\left[(\eta-i s)^{2}-u\right]}, \varepsilon_{2}^{(\eta)}=-\frac{\mathrm{v}(\mathrm{u})}{\eta\left[(\eta-i s)^{2}-\mathrm{u}\right]}, \varepsilon_{3}^{(\eta)}=1-\frac{\mathrm{v}}{\eta(\eta-i s)}
$$

and

$$
\mathrm{v}=\frac{\omega_{p}^{2}}{\omega^{2}}, \quad \mathrm{u}=\frac{\omega_{c}^{2}}{\omega^{2}}, \quad \mathrm{~s}=\frac{v_{e i}}{\omega} .
$$

Here $\omega_{p}$ and $\omega_{c}$ are the electric plasma and electron cyclotron frequencies, respectively and $\nu_{e i}$ is the electron-ion collision frequency. The operator $\nabla^{(\eta)}$ has the following form

$$
\nabla^{(1)}=\left(i k^{(1)}+\frac{\partial}{\partial \mathrm{x}}\right) \mathbf{e}_{\mathbf{x}}, \quad \nabla^{(2)}=\left(i k^{(2)}+\frac{\partial}{\partial \mathrm{x}}\right) \mathbf{e}_{\mathbf{x}} .
$$

The expression $\mathbf{j}_{\mathbf{n l}}{ }^{(\mathfrak{\eta})}$ at the right-hand side of the equation (1) represents the nonlinear second order electric currents [4] which depend on the electric field $\mathbf{E}^{(\mathfrak{\eta})}$, and $\mathbf{j}_{\mathrm{nl}}^{(1)}=\mathbf{j}_{\mathrm{nl}}^{(\mathbf{1})}(\omega)$ and $\mathbf{j}_{\mathrm{nl}}^{(2)}=\mathbf{j}_{\mathrm{nl}}^{(2)}(2 \omega)$.

Supposing small spatial variations of wave amplitude the following set of coupled equations is obtained from $\mathrm{Eq}(1)$ :

$$
\begin{align*}
& \frac{\partial \mathrm{A}^{(1)}}{\partial \mathrm{x}}=\left(\mathrm{C}_{11} \cos \Psi-\mathrm{C}_{12} \sin \Psi\right) \mathrm{A}^{(1)} \mathrm{A}^{(2)}, \\
& \frac{\partial \mathrm{A}^{(2)}}{\partial \mathrm{x}}=\left(\mathrm{C}_{21} \cos \Psi+\mathrm{C}_{22} \sin \Psi\right) \mathrm{A}^{(1)^{2}},  \tag{3}\\
& \frac{\partial \Psi}{\partial \mathrm{x}}=\left(\mathrm{C}_{22} \frac{\mathrm{~A}^{(1)^{2}}}{\mathrm{~A}^{(2)}}+2 \mathrm{C}_{12} \mathrm{~A}^{(2)}\right) \cos \Psi-\left(\mathrm{C}_{21} \frac{\mathrm{~A}^{(1)^{2}}}{\mathrm{~A}^{(2)}}+\mathrm{C}_{11} \mathrm{~A}^{(2)}\right) \sin \Psi
\end{align*}
$$

where $\mathrm{C}_{11}, \mathrm{C}_{12}, \mathrm{C}_{21}$ and $\mathrm{C}_{22}$ are the coupling constants, depending on $\mathrm{u}, \mathrm{v}$ and s . The phase $\psi$ is given by the relation:

$$
\psi=\varphi_{2}-2 \varphi_{1}
$$

where $\varphi_{1}$ and $\varphi_{2}$ are the phases of the complex amplitudes given by:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{y}}{ }^{(1)}=\mathrm{A}_{\mathrm{y}}^{(1)} \exp \left(\mathrm{i} \varphi_{1}\right) \text { and } \mathrm{E}_{\mathrm{y}}^{(2)}=\mathrm{A}_{\mathrm{y}}^{(2)} \exp \left(\mathrm{i} \varphi_{2}\right) \tag{4}
\end{equation*}
$$

$\mathrm{A}_{\mathrm{y}}{ }^{(1)}$ and $\mathrm{A}_{\mathrm{y}}{ }^{(2)}$ are the real amplitudes which are the functions of $\mathrm{x}, \mathrm{s}, \mathrm{u}$ and v . The system (3) can be solved by using the Runge-Kutt method [8] taking:

$$
\begin{equation*}
\mathrm{A}(1)(0)=\sqrt{\frac{2 S_{i}}{c \varepsilon_{0}\left(N_{o}^{(1)}+N_{e}^{(1)}\right)}}, \mathrm{A}^{(2)}(0)=0, \Psi(0)=\frac{\pi}{2}, \tag{5}
\end{equation*}
$$

where $N_{o}^{(1)}$ and $N_{e}^{(1)}$ are the real parts of the reflection indices given by

$$
\begin{align*}
& N_{o}^{(1)}=\sqrt{\frac{1}{2}\left[1-\frac{\mathrm{v}-\sqrt{\left(1+\mathrm{s}^{2}-\mathrm{v}\right)^{2}+\mathrm{s}^{2} \mathrm{v}^{2}}}{1+\mathrm{s}^{2}}\right]}  \tag{6}\\
& N_{e}^{(1)}=\sqrt{1-\frac{\mathrm{v}\left(1-2 \mathrm{v}-\mathrm{u}+\mathrm{s}^{2}+\mathrm{uv}+\mathrm{v}^{2}\right)}{(1-\mathrm{v}-\mathrm{u})^{2}+\mathrm{s}^{2}}} \tag{7}
\end{align*}
$$

We introduce the second harmonic generation efficiency parameter W as

$$
\begin{equation*}
\mathrm{W}(\mathrm{x})=\frac{S^{(2)}(\mathrm{x})}{S_{i}}=\frac{c \varepsilon_{0} N_{e}^{(2)} \mathrm{A}_{\mathrm{y}}^{(2)^{2}}}{2 S_{i}} \tag{8}
\end{equation*}
$$

where $S_{i}$ is the incident energy flux and $S^{(2)}(\mathrm{x})$ is the second harmonic energy flux. The real part of the index reflection of the second harmonic is given by

$$
\begin{equation*}
N_{e}^{(2)}=\sqrt{1-\frac{v\left(16-8 v-4 u+4 s^{2}+u v+v^{2}\right)}{4(4-v-u)^{2}+16 s^{2}}} . \tag{9}
\end{equation*}
$$



Fig. 1. Resonant domain for $\mathrm{s}=10^{-2}($ series 1$)$ and $\mathrm{s}=10^{-1}($ series 2$)$.

The resonant condition for the extraordinary modes $N_{e}^{(1)}=N_{e}^{(2)}$ yields the relation connecting the parameters $u, v$ and $s$ as shown in Figure 1. This figure differs from the corresponding one in [4] in the domain of weak magnetic fields where $u \approx 0$, and in the domain where the cyclotron frequency is close to fundamental wave frequency i.e. at $u \rightarrow 1$. This means the curve of the resonant values of parameters $u$ and $v$ for the case of a collisional plasma with rare collisions ( $\mathrm{s}<10^{-2}$ ) is close to the corresponding curve for collisionless plasma. Figure 1 thus shows that the region where the second harmonic emission can be resonantly excited is narrower than in the case of collisionless plasma.

## Numerical results

The second harmonic amplitude $\mathrm{A}^{(2)}$ and the generation efficiency parameter W are investigated numerically for various values of wave parameters and the results are plotted in Figures 2-4 for the resonant domain shown in Fig. 1.

According to Fig. 2 one can see that the second harmonic amplitude of the extraordinary wave $\mathrm{A}^{(2)}$ grows with distance x from the vacuum-plasma boundary. If the plasma collisional frequency is lower (smaller s) this growth is more pronounced. The collisions therefore decrease the effect of the resonant second harmonic generation in the case of the eee-phase synchronism.


Fig. 2. The second harmonic amplitude as a function of distance from the vacuum-plasma boundary for $\mathrm{s}=10^{-3}($ series 1$), \mathrm{s}=5 \times 10^{-3}($ series 2$)$ and $\mathrm{s}=10^{-2}($ series 3$)$. Other parameters: $\omega=2.092 \mathrm{GHz}, \frac{\omega_{p}^{2}}{\omega^{2}}=1.13$, resonant value of $\frac{\omega_{c}^{2}}{\omega^{2}}, S_{i}=10^{4} \frac{\mathrm{~W}}{\mathrm{~m}^{2}}$


Fig. 3. The second harmonic generation efficiency as a function of distance from the va- vacuumplasma boundary for $\mathrm{s}=10^{-3}$ (series 1$), \mathrm{s}=5 \times 10^{-3}($ series 2$)$ and $\mathrm{s}=10^{-2}$ (series 3). Other parameters: $\omega=2.092 \mathrm{GHz}, \frac{\omega_{p}^{2}}{\omega^{2}}=1.13$, resonant value of $\frac{\omega_{c}^{2}}{\omega^{2}}, S_{i}=10^{4} \frac{\mathrm{~W}}{m^{2}}$

Fig. 3 shows the second harmonic efficiency parameter W as a function of distance x . This parameter increases with $x$ and it saturates at the distance $x=15 \mathrm{~km}$. Figures 2 and 3 show that $W$ grows slower with $x$ in domain $(0-5) k m$ than $\mathrm{A}^{(2)}$.


Fig. 4. The second harmonic generation efficiency as a function of collisional parameter s for distances from the plasma boundary $\mathrm{x}=1 \mathrm{~km}$ (series 1), $\mathrm{x}=5 \mathrm{~km}$ (series 2) and $\mathrm{x}=25 \mathrm{~km}$ (series 3). Other parameters: $\omega=2.092 \mathrm{GHz}, \frac{\omega_{p}^{2}}{\omega^{2}}=1.13$, resonant value of $\frac{\omega_{c}^{2}}{\omega^{2}}, S_{i}=10^{4} \frac{\mathrm{~W}}{m^{2}}$

Finally, Fig. 4 gives conversion efficiency parameter W against the collision parameter s for various distances x . For some typical values of $\mathrm{s}\left(10^{-4}\right.$ laser plasma, $10^{-5}$ ionospheric plasma, $10^{-5}$ tokamak plasma) the conversion efficiency parameter W is constant and takes bigger values at larger distances x . Only for plasmas with frequent collisions ( $\mathrm{s}>10^{-3}$ ) W decreases with s .

## Conclusion

In this paper it has been shown that collisions in the magnetoactive plasma exert influence on the resonant generation of the second harmonic when collisions are sufficiently frequent ( $\mathrm{s}>10^{-3}$ ). The influence of collisions on the process is negative, because the second harmonic amplitude and conversion efficiency decrease with the increase of the collisional frequency. This result is similar to the one obtained for the ooe - synchronism in [8] but the order of quantity of amplitude $\mathrm{A}^{(2)}$ and W is lower in the case of eee phase synchronism. If collisions in a plasma are sufficiently rare ( $\mathrm{s}<10^{-3}$ ) the process of the second harmonic generation is the same as in the collisionless magnetoactive plasma.

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# GENERACIJA DRUGOG HARMONIKA U SUDARNOJ MAGNETOAKTIVNOJ PLAZMI 

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Teorijski su istraživane osobine generacije drugog harmonika u homogenoj, sudarnoj i manetizovanoj plazmi rešavanjem sistema nelinearnih jednačina za osnovni i drugi harmonik neredovnih talasa. Izračunavana je zavisnost efikasnosti dupliranja talasne frekvencije od rastojanja od granice plazme i od sudarne frekvencije.

