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KINETIC THEORY OF ELECTROSTATIC ION CYCLOTRON WAVES (*QPESIC*) IN MULTICOMPONENT PLASMAS WITH NEGATIVE IONS

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Abstract. *The instabilities of the quasi-perpendicular electrostatic* ($\delta \boldsymbol{B} = 0$) *ion-cyclotron waves (QPESIC) are investigated. The kinetic theory with BGK model collision integrals is used to estimate the critical electron drift velocity in the presence of positively or negatively charged resonant ions in multi-component plasma. Analytical evaluation for the ion-cyclotron modes and instabilities in the long-wave range in a weakly-ionized Maxwellian plasma with two positive ion species, one negative ion species and with electrons, drifting along magnetic lines of force is demonstrated. The spectrum in these situations is also given. It is shown that the critical drift decreases as the state of plasma approaches the isothermic state.*

Key words: *kinetic theory, ion cyclotron waves, negative ions, multi-component plasmas*

1. Introduction

Analyzing the waves in weakly ionized plasmas received much attention in the last decades due to the technical simplicity of obtaining this kind of plasma and corresponding low relative temperatures. Density of such plasmas is rather low, as well as the collision frequencies. For analysis we used a linear kinetic theory, giving the possibility of studying thermal effects, collision processes, excitation of higher harmonic ion cyclotron waves, etc.

In a majority of cases, new waves are created when the angle θ, between the wave vector $\boldsymbol{k}$ and the external magnetic field $\boldsymbol{B_0}$, is near π/2 [10]. Frequencies in these free waves for $\boldsymbol{k} \to 0$ and $\boldsymbol{k} \to \infty$ are close to $n\,|\omega_{B\alpha}|$, where n is an integer, and $\omega_{B\alpha}$ are gyro frequencies for each ionic species α. Those waves are called ion cyclotron, and their existence is the consequence of the resident magnetic field.

Potential cyclotron waves are not followed by perturbations of magnetic field (just by an electric field oscillating) and because of that they are often called electrostatic. For the first time potential electron cyclotron waves were studied [1]. The existence of ion cy-

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clotron oscillations is the result of superposition of electromagnetic fields which emit single ions in the process of cyclotron rotation in external magnetic field. Electrostatic cyclotron waves appear in plasma for the small value of $k\rho_{Li}$. If $k\rho_{Li}$ is too big, then ion cyclotron waves become non-potential.

This study is based on the assumption of infinite, weakly ionized, low-temperature, collision plasma, placed in a mutually parallel field $\boldsymbol{E_0}$ and $\boldsymbol{B_0}$ [11-14]**.** The multi-component plasma contains electrons, neutrals, one kind of negative ions (h), and two kinds of positive ions (l_1 and l_2). Plasmas with negative ions are encountered both in laboratory devices [4,7], and in astrophysical situations [5, 8].

2. Theory

The condition of macroscopic quasi-neutrality for such plasma is:

$$n_e + zn_h = z_1 n_{l_1} + z_2 n_{l_2}, \tag{1}$$

where n_e, n_h, n_{l1}, n_{l2} are number densities for electrons, negative ions and two species of positive ions, respectively, and z, z_1, z_2 are their charges. We use the subscripts e for electrons, h for negative ions, and l_1 and l_2 for positive ions.

It is convenient to introduce the following parameters:

$$\delta_1 = \frac{n_{l_1}}{n_e},\ \delta_2 = \frac{n_{l_2}}{n_e}, \tag{2}$$

$$T = \frac{T_h}{T_e},\ T_1 = \frac{T_h}{T_{l_1}},\ T_2 = \frac{T_h}{T_{l_2}}, \tag{3}$$

$$M = \frac{m_h}{m_e},\ M_1 = \frac{m_h}{m_{l_1}},\ M_2 = \frac{m_h}{m_{l_2}}, \tag{4}$$

$$\mu_\alpha = \frac{k_\perp^2 v_{T\alpha}^2}{\omega_{B\alpha}^2}, \tag{5}$$

$$\mu_{l_1} = \frac{z^2\mu_h}{z_1^2 T_1 M_1},\ \mu_{l_2} = \frac{z^2\mu_h}{z_2^2 T_2 M_2}. \tag{6}$$

Parameters in Eq. (2) are the ratios of the number densities of positive ions and electrons. Equations (3) and (4) represent the ratios of the relative temperatures and masses for negative ions refer to electrons and positive ions.

From the condition of macroscopic quasi-neutrality we have:

$$\frac{n_h}{n_e} = \frac{1}{z}(z_1\delta_1 + z_2\delta_2 - 1). \tag{7}$$

Dispersion equation for the electrostatic waves is in general [3]:

$$1 + \sum_\alpha \delta\varepsilon_\alpha = 0, \tag{8}$$

where $\delta\varepsilon_\alpha$ ($\alpha = e, h, l_1, l_2$) designate the contributions of the plasma constituents to the longitudinal permeability [9,10]. We can reject figure one because it is very small compared to the mentioned contributions, so equation (8) became:

$$\delta\varepsilon_e + \delta\varepsilon_h + \delta\varepsilon_{l_1} + \delta\varepsilon_{l_2} = 0. \tag{9}$$

Dynamics for this kind of plasma is described by the kinetic theory in which collision processes are calculated with BGK model collision integrals.

In the long wave domain of the modes considered ($\nu_e >> k_{||}\nu_{Te}$ and $\omega\nu_e << k_{||}^2\nu_{Te}^2$), the electron contribution to (8) is given by [3]:

$$\delta\varepsilon_e = \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left(1 + i\frac{\nu_e}{k_{||}v_{Te}}\frac{\omega - k_{||}u}{k_{||}v_{Te}}\right). \tag{10}$$

Here u is the electron drift velocity (assumed constant), ν_e is the effective collision frequency for electron-neutral collisions, $\nu_{Te} = (\kappa T_e/m_e)^{1/2}$ is the thermal velocity of electrons (κ is the Boltzmann constant), and $\omega_{pe} = (e^2 n_e / \varepsilon_0 m_e)^{1/2}$ is the electron plasma frequency.

The simplifications in $\delta\varepsilon_i$ ($i = h, l_1, l_2$) are based on the conditions of quasi-perpendicular propagation, i.e. ω, $|\omega - n\omega_{Bi}| >> k_{||}\nu_{Ti}, \nu_i$. For Maxwellian ion steady-state distribution functions, one thus arrives at:

$$\delta\varepsilon_i = \frac{\omega_{pi}^2}{k^2 v_{Ti}}\left\{1 - \sum_{n=-\infty}^{\infty}\left[\frac{\omega A_n(\mu_i)}{\omega - n\omega_{Bi}} - i\frac{\nu_i\omega A_n(\mu_i)}{(\omega - n\omega_{Bi})^2}\right]\right\}. \tag{11}$$

The quantities ω_{pi}, ω_{Bi}, ν_{Ti} and ν_i have the same meaning as the analogous quantities in (10), and $A_n(z) = I_n(z)\exp(-z)$ (I_n is the modified Bessel function of order n). We must accept that ionic drift velocity is irrelevant. In concrete calculations for resonant ions, in the sum (11), we take, as the biggest ones, those members [12] with $n = 0, 1$, and for non-resonant ions the member $n = 0$. Resonant ions have cyclotron frequency close to modal frequency of the analyzed waves.

In this case:

$$\frac{\mu_h}{\mu_{l_1}} = \frac{z_1^2}{z^2}T_1 M_1,\ \frac{\mu_h}{\mu_{l_2}} = \frac{z_2^2}{z^2}T_2 M_2\ . \tag{12}$$

The spectra of studied wave modes are determined when we equalize the real part of (9) with zero. Drift velocity for electron u along magnetic field $\boldsymbol{B_0}$, leading to spontaneous excitation of potential ion-cyclotron waves, follows from the condition marginal instability, obtained by equating the imaginary part of (9) with zero. The critical drift velocity would be evaluated relating to thermal velocity of negative ions.

In the next part, we study situations:

A. Negative ions are resonant ($\omega \approx \omega_{Bh}$)

B. Positive ions of first kind are resonant ($\omega \approx \omega_{Bl1}$)

C. Positive ions of second kind are resonant ($\omega \approx \omega_{Bl2}$).

A. Negative ions are resonant ($\omega \approx \omega_{Bh}$)

Dispersion equation in this case is:

$$\frac{\omega_{Pe}^2}{k^2 v_{Te}^2} + \frac{\omega_{Ph}^2}{k^2 v_{Th}^2}\left\{1 - \left[A_0(\mu_h) + \frac{\omega A_1(\mu_h)}{\omega - |\omega_{Bh}|}\right]\right\} + \frac{\omega_{Pl1}^2}{k^2 v_{Tl_1}^2}[1 - A_0(\mu_{l_1})] + \frac{\omega_{Pl_2}^2}{k^2 v_{Tl_2}^2}[1 - A_0(\mu_{l_2})] = 0. \quad (13)$$

From this equation, we get the expression for the modal frequency:

$$\omega = \omega_{Bh}\frac{V_1}{V_2}, \quad (14)$$

where

$$V_1 = T + z(z_1\delta_1 + z_2\delta_2 - 1)[1 - A_0(\mu_h)] + z_1^2\delta_1 T_1[1 - A_0(\mu_{l_1})] + z_2^2\delta_2 T_2[1 - A_0(\mu_{l_2})], \quad (15)$$

$$V_2 = V_1 - z(z_1\delta_1 + z_2\delta_2 - 1)A_1(\mu_h). \quad (16)$$

The process of spontaneous excitation of electrostatic ion-cyclotron waves is determined by the critical velocity of electron drift.

The ratio of the critical drift velocity and thermal velocity of negative ions is given by the equation:

$$\left(\frac{u^-}{v_{Th}}\right)_1 = 2\left(\frac{M}{T^3}\right)^{1/4}(Y + Y_1 + Y_2)^{1/2}, \quad (17)$$

where

$$Y = z(z_1\delta_1 + z_2\delta_2 - 1)\frac{\nu_h}{\nu_e}\left[\frac{A_1(\mu_h)}{W^2} + A_0(\mu_h)\right], \quad (18)$$

$$Y_1 = z_1^2\delta_1\frac{\nu_{l_1}}{\nu_e}T_1 A_0(\mu_{l_1}), \quad (19)$$

$$Y_2 = z_2^2\delta_2\frac{\nu_{l_2}}{\nu_e}T_2 A_0(\mu_{l_2}). \quad (20)$$

Here, $W = (\omega - \omega_{Bh}) / \omega$ is the relative deviation of ω from ω_{Bh}.

Collision of charged particles with neutrals plays the most important role in the weakly ionized plasma. The collision frequencies can be calculated according to the billiard-ball model and they are given by the following relation:

$$\nu_\alpha = n_n \sigma_{\alpha n} v_{T\alpha}, \quad (21)$$

where n_n is the number densities of neutrals and $\sigma_{\alpha n}$ is the effective cross-section for collision of charged particles with neutrals.

According to the billiard-ball model ($\sigma_{en} = \sigma_{\alpha n}$) used for the collisions herewith, one has:

$$\frac{\nu_e}{\nu_h} = \frac{v_{Te}}{v_{Th}}, \quad \frac{\nu_e}{\nu_{l_1}} = \frac{v_{Te}}{v_{Tl_1}}, \quad \frac{\nu_e}{\nu_{l_2}} = \frac{v_{Te}}{v_{Tl_2}}, \quad (22)$$

thus, the drift equation becomes:

$$\left(\frac{u^-}{v_{Th}}\right)_1 = 2\left(\frac{M}{T^3}\right)^{1/4}(Y' + Y_1' + Y_2')^{1/2}, \tag{23}$$

where:

$$Y' = z(z_1\delta_1 + z_2\delta_2 - 1)\left[\frac{A_1(\mu_h)}{W^2} + A_0(\mu_h)\right], \tag{24}$$

$$Y_1' = z_1^2\delta_1 T_1\sqrt{M_1 T_1}\,A_0(\mu_{l_1}), \tag{25}$$

$$Y_2' = z_2^2\delta_2 T_2\sqrt{M_2 T_2}\,A_0(\mu_{l_2}). \tag{26}$$

B. Positive ions of first kind are resonant ($\omega \approx \omega_{Bl_1}$)

In this case dispersion equation is:

$$\frac{\omega_{Pe}^2}{k^2 v_{Te}^2} + \frac{\omega_{Ph}^2}{k^2 v_{Th}^2}[1 - A_0(\mu_h)] + \frac{\omega_{Pl_1}^2}{k^2 v_{Tl_1}^2}\left[1 - A_0(\mu_{l_1}) - \frac{\omega\ \ A_1(\mu_{l_1})}{\omega - \omega_{Bl_1}}\right] + \frac{\omega_{Pl_2}^2}{k^2 v_{Tl_2}^2}[1 - A_0(\mu_{l_2})] = 0 \tag{27}$$

From this equation, the expression for the spectrum is obtained:

$$\omega = \omega_{Bh}\frac{V_3}{V_4}, \tag{28}$$

where

$$V_3 = T + z(z_1\delta_1 + z_2\delta_2 - 1)[1 - A_0(\mu_h)] + z_1^2\delta_1 T_1[1 - A_0(\mu_{l_1})] + z_2^2\delta_2 T_2[1 - A_0(\mu_{l_2})], \tag{29}$$

$$V_4 = V_3 - z_1^2\delta_1 T_1 A_1(\mu_{l_1}). \tag{30}$$

Critical drift velocity for the first term in the billiard-ball model is defined by:

$$\left(\frac{u_1^+}{v_{Tl_1}}\right)_1 = 2\left(\frac{M}{T^3}\right)^{1/4}\left(\frac{T_1}{M_1}\right)^{1/2}(Y'' + Y_1'' + Y_2'')^{1/2}, \tag{31}$$

with parameters:

$$Y'' = z(z_1\delta_1 + z_2\delta_2 - 1)A_0(\mu_h), \tag{32}$$

$$Y_1'' = z_1^2\delta_1 T_1\sqrt{M_1 T_1}\left[A_0(\mu_{l_1}) + \frac{A_1(\mu_{l_1})}{W_1^2}\right], \tag{33}$$

$$Y_2'' = z_2^2\delta_2 T_2\sqrt{M_2 T_2}\,A_0(\mu_{l_2}). \tag{34}$$

Here, $W = (\omega - \omega_{l1}) / \omega$ is the relative deviation of ω from ω_{Bl1}.

Equation can be expressed in the following way:

$$\frac{u_1^+}{v_{Tl_1}} = \frac{u_1^+}{v_{Th}}\left(\frac{T_1}{M_1}\right)^{1/2}. \tag{35}$$

Finally, critical drift velocity, when positive ions are resonant, in respect to the thermal velocity of negative ions, has the form:

$$\left(\frac{u_1^+}{v_{Th}}\right)_1 = 2\left(\frac{M}{T^3}\right)^{1/4}(Y'' + Y_1'' + Y_2'')^{1/2}. \tag{36}$$

C. Positive ions of second kind are resonant $(\omega \approx \omega_{Bl_2})$

In this case dispersion equation is:

$$\frac{\omega_{Pe}^2}{k^2 v_{Te}^2} + \frac{\omega_{Ph}^2}{k^2 v_{Th}^2}[1 - A_0(\mu_h)] + \frac{\omega_{P_{l1}}^2}{k^2 v_{Tl_1}^2}[1 - A_0(\mu_{l_1})] + \frac{\omega_{Pl_2}^2}{k^2 v_{Tl_2}^2}\left[1 - A_0(\mu_{l_2}) - \frac{\omega \;\; A_1(\mu_{l_2})}{\omega - \omega_{Bl_2}}\right] = 0 \tag{37}$$

Therefore, we obtain:

$$\omega = \omega_{Bh}\frac{V_5}{V_6}, \tag{38}$$

where:

$$V_5 = T + z(z_1\delta_1 + z_2\delta_2 - 1)[1 - A_0(\mu_h)] + z_1^2\delta_1 T_1[1 - A_0(\mu_{l_1})] + z_2^2\delta_2 T_2[1 - A_0(\mu_{l_2})] \tag{39}$$

$$V_6 = V_5 - z_2^2\delta_2 T_2 A_1(\mu_{l_2}). \tag{40}$$

The critical drift velocity for the first term in the billiard-ball model is:

$$\left(\frac{u_2^+}{v_{Tl_2}}\right)_1 = 2\left(\frac{M}{T^3}\right)^{1/4}\left(\frac{T_2}{M_2}\right)^{1/2}(Y''' + Y_1''' + Y_2''')^{1/2}, \tag{41}$$

with parameters:

$$Y''' = z(z_1\delta_1 + z_2\delta_2 - 1)A_0(\mu_h), \tag{42}$$

$$Y_1''' = z_1^2\delta_1 T_1\sqrt{M_1 T_1}\,A_0(\mu_{l_1}), \tag{43}$$

$$Y_2''' = z_2^2\delta_2 T_2\sqrt{M_2 T_2}\left[A_0(\mu_{l_2}) + \frac{A_1(\mu_{l_2})}{W_2^2}\right]. \tag{44}$$

Here, $W_2 = (\omega - \omega_{l2}) / \omega$ is the relative deviation of ω from ω_{l2}.

Now we have:

$$\frac{u_2^+}{v_{Tl_2}} = \frac{u_2^+}{v_{Th}}\left(\frac{T_2}{M_2}\right)^{1/2}. \tag{45}$$

Hence, in this case, we arrive to the following expression:

$$\left(\frac{u_2^+}{v_{Th}}\right) = 2\left(\frac{M}{T^3}\right)^{1/4} (Y''' + Y_1''' + Y_2''')^{1/2} . \tag{46}$$

Secondly, we shall analyze the mentioned modes.

3. NUMERICAL APPLICATIONS

In previous laboratory investigation [4], SF_6^- negative ion was taken together with one kind of positive ions. In this paper we shall consider the plasma containing SF_6^- and two kinds of positive ions K^+ and H_2^+ as well as electrons and neutrals. Taking into account that the number densities of the negative, both kinds of positive ions and electrons are approximately equal, the parameters δ_1 and δ_2 become equal to one. Also, if we take that the temperatures of the ions are equal, we have that $T_1 = T_2 = 1$. For such a case, one obtains $M = 268056$, $M_1 = 3.73$ and $M_2 = 73$.

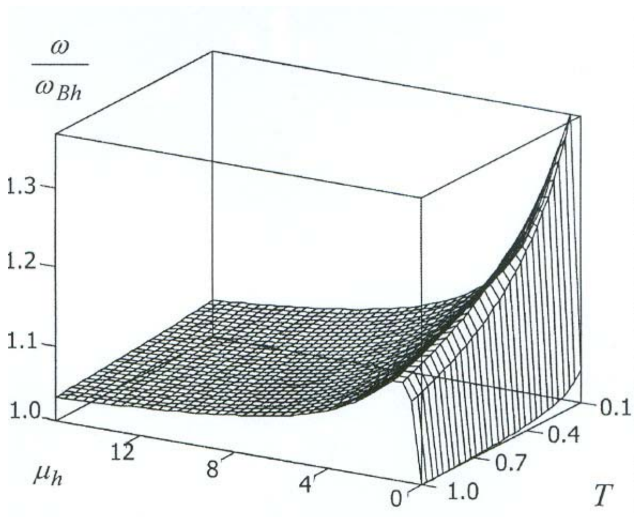

Fig. 1. Dependence of ion-cyclotron waves on μ_h and T in case negative ions SF_6^- are resonant.

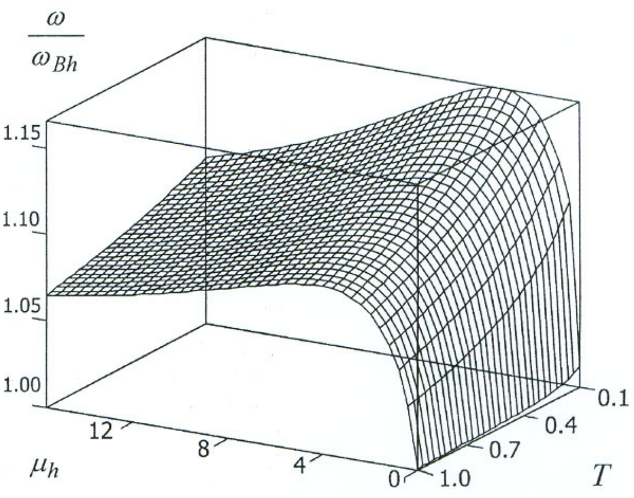

Fig. 2. Dependence of ion-cyclotron waves on μ_h and T in case positive ions K^+ are resonant.

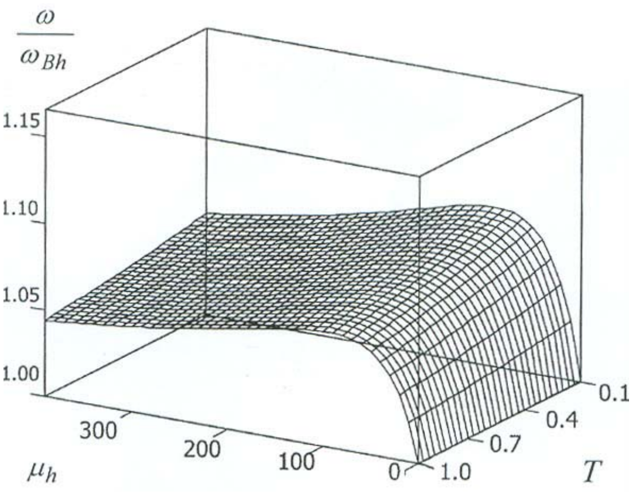

Fig. 3. Dependence of ion-cyclotron waves on μ_h and T in case positive ions H_2^+ are resonant.

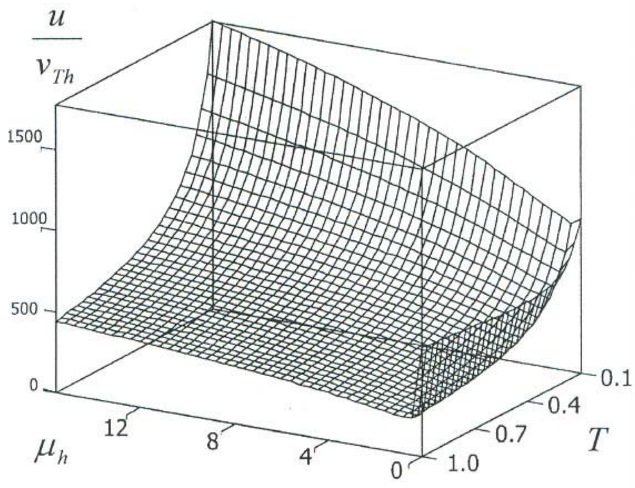

Fig. 4. Dependence of electron critical drift velocity on μ_h and T in case negative ions SF_6^- are resonant.

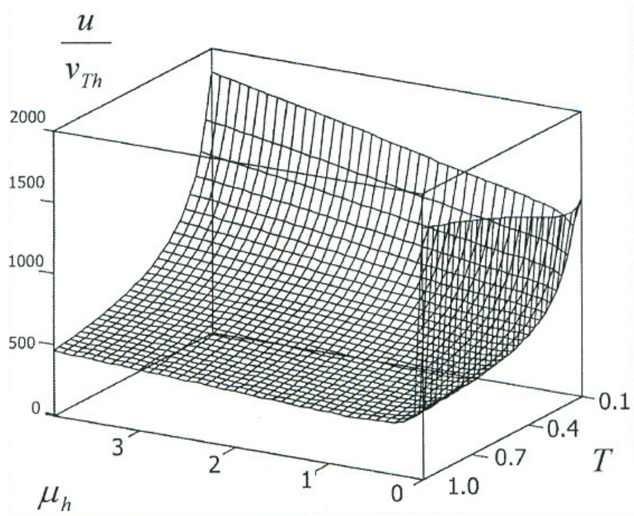

Fig. 5. Dependence of electron critical drift velocity on μ_h and T in case negative ions K^+ are resonant.

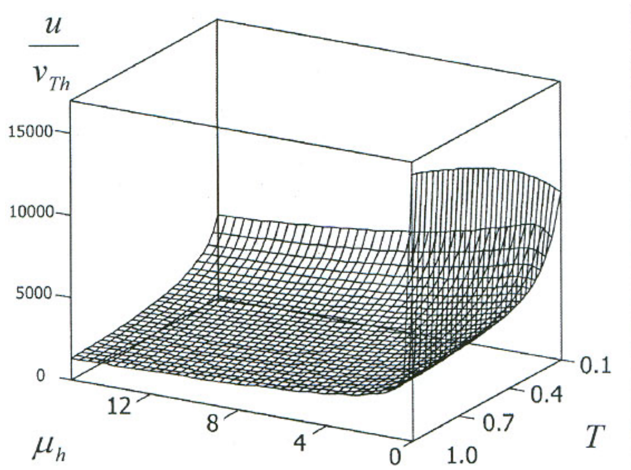

Fig. 6. Dependence of electron critical drift velocity on μ_h and T in case negative ions H_2^+ are resonant.

4. Conclusions

We notice that the deviation of ω from ω_{Bh} greatly depends on the relation between the masses of the positive resonant ion and the negative ion. The bigger amount of negative ions, the less the deviation. This is in accordance with the observation in paper [6], where the potential ion cyclotron waves are treated by non-collision theoretical approach. The dependence of the spectrum on T is such that the discrepancy of ω from ω_{Bh} decreases as the value of T approaches to one (isothermal plasma).

It is shown that it is more difficult to excite the potential ion cyclotron waves with positive resonant ions. We have also shown that the resonant *QPEIC* is easier to excite as more charges is carried [4,6]. The results of analysis of two kinds of ions in plasmas cannot be accurately compared with experiment, because collisions need to be considered.

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KINETIČKA TEORIJA ELEKTROSTATIČKIH JONO-CIKLOTRONSKIH TALASA (*QPEIC*) U MULTI-KOMPONENTNIM PLAZMAMA SA NEGATIVNIM JONIMA

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Razmatrane su nestabilnosti kvazi-perpendikularnih elektrostatičkih ($\delta\boldsymbol{B}$=0) *jono-ciklotronskih talasa.Pomoću kinetičke teorije sa BGK kolizionim integralom izračunavana je brzina kritičnog elektronskog drifta u prisustvu pozitivnih ili negativnih rezonantnih jona u multi-komponentnim plazmama. Prezentovano je analitičko izračunavanje za jon-ciklotronske mode i nestabilnosti u dugotalasnoj oblasti za slučaj slabo jonizovane maksvelovske plazme sa dve vrste pozitivnih jona, jednom vrstom negativnih jona i elektronima koji driftuju u odnosu na linije sile magnetnog polja. Izračunavan je i spektar u ovim situacijama. Pokazano je da brzina kritičnog drifta elektrona opada sa približavanjem plazme izotermnom stanju.*