FACTA UNIVERSITATIS
Series: **Physics, Chemistry and Technology** Vol. 4, N^o 2, 2006, pp 435 -449

Constrained generalized supersymmetries and their classification.

Francesco Toppan

CBPF, Rio d. J., Brazil

Abstract. *Generalized superymmetries going beyond the HLS scheme and admitting the presence of bosonic tensorial central charges are constructed and classified in terms of the division algebras* **R**, **C**, **H** *and* **O**. *The eleven-dimensional M-algebra falls into this class of supersymmetries. Division-algebra compatible constraints can be introduced and fully classified. They can be used to construct and analyze various dynamical systems, the simplest examples being the superparticles with tensorial central charges which generalize the Rudychev-Sezgin and the Bandos-Lukierski models.*

Key words: *Supersymmetry, M-Theory, Tensorial Central Charges*

1. INTRODUCTION

The generalized supersymmetries going beyond the Haag, Łopuszański and Sohnius classification [1] were first introduced by D'Auria and Fré in 1982 [2]. The fermionic supersymmetry generators are, essentially, square roots operators. Their anticommutators produce a r.h.s. which is totally saturated and has to be expanded in terms of higher-rank bosonic tensors. It was recognized, see e.g. [3] and [4], that such supersymmetries are related with the dynamics of extended objects like branes.

The eleven-dimensional M-algebra, given by

$$\{Q_a, Q_b\} = (A\Gamma_\mu)_{ab} P^\mu + \left(A\Gamma_{[\mu\nu]}\right)_{ab} Z^{[\mu\nu]} + \left(A\Gamma_{[\mu_1\dots\mu_5]}\right)_{ab} Z^{[\mu_1\dots\mu_5]}. \tag{1}$$

is an example of such a generalized supersymmetry. We recall that, in Minkowskian eleven dimensions, the fundamental spinors are 32-component,

Received: 20 August 2005

real (Majorana) spinors. The saturated bosonic r.h.s. is in this case given by the most general 32×32 symmetric matrix with 528 bosonic real components, expressed in terms of 11 vectors, 55 rank-two and 462 rank-5 antisymmetric tensors ($11+55+462=528$).

The "generalized momenta" P_μ, $P_{\mu\nu}$, $P_{\mu_1\ldots\mu_5}$ entering the (1) r.h.s. can be associated to "generalized coordinates" X^μ, $X^{\mu\nu}$, $X^{\mu_1\ldots\mu_5}$, while the supersymmetry generators Q_α should be associated to the Grassmann superspace coordinates θ^α. This kind of structure can be used to introduce a class of models, first produced by Rudychev and Sezgin in [5], known as "superparticles with tensorial central charges". These models realize a generalization of the Brink-Schwarz superparticle since they allow the presence of bosonic tensorial coordinates. It was later proved by Bandos and Lukierski in [6], see also [7], that a different formulation of the Rudychev-Sezgin models in terms of complex generalized superalgebras (the basic ingredients being complex supersymmetric charges), describes towers of massless particles with higher helicities. A previous proposal by Fronsdal [8] of using tensorial coordinates to produce a tower of higher spin particles was concretely implemented in the Bandos-Lukierski framework. In this talk we discuss the following items. We point out at first that, since spinors can be introduced in association with each one of the four division algebras (real and complex numbers, quaternions and even the non-associative algebra of the octonions), see e.g. [9], generalized supersymmetries can be introduced in association with each one of the above division-algebras in each space-time supporting the corresponding spinors. Later, division-algebras compatible constraints on the bosonic r.h.s. can be produced [10] and classified [11]. An immediate application of the classification of the (constrained) generalized supersymmetry concerns the construction of generalizations of the Bandos-Lukierski models and the analysis of their equations of motion, with the possibility of introducing dynamically-compatible constraints. In the Conclusions some other examples of dynamical systems currently under investigation, which are based on the present classification of (constrained) generalized supersymmetries, will be mentioned.

2. Division algebras and generalized supersymmetries

The four division algebra of real (**R**) and complex (**C**) numbers, quaternions (**H**) and octonions (**O**) possess respectively 0, 1, 3 and 7 imaginary elements e_i satisfying the relations

$$e_i \cdot e_j \quad = \quad -\delta_{ij} + C_{ijk} e_k, \tag{2}$$

(i, j, k are restricted to take the value 1 in the complex case, $1, 2, 3$ in the quaternionic case and $1, 2, \ldots, 7$ in the octonionic case; furthermore, the sum over repeated indices is understood).

C_{ijk} are the totally antisymmetric division-algebra structure constants. The octonionic division algebra is the maximal, since quaternions, complex and real numbers can be obtained as its restriction. The totally antisymmetric octonionic structure constants can be expressed as

$$C_{123} = C_{147} = C_{165} = C_{246} = C_{257} = C_{354} = C_{367} = 1 \tag{3}$$

(and vanishing otherwise). The fact that the structure constants are antisymmetric implies that the anticommutators between imaginary elements is a specific realization of the basic relation of the Euclidean Clifford algebra $\Gamma_i\Gamma_j + \Gamma_j\Gamma_i = -2\delta_{ij}$. As a result, matrices with division-algebra valued entries satisfying the basic relations of Clifford algebras in different space-times can be produced [9]. They act on corresponding, division-algebra valued, spinors. The generalized supersymmetries are known as "real", "complex", "quaternionic" or "octonionic" according to the nature of the supersymmetry charges.

If the real spinors Q_a have n components, the most general supersymmetry algebra is represented by

$$\{Q_a, Q_b\} \quad = \quad \mathcal{Z}_{ab}, \tag{4}$$

where the matrix $\mathcal{Z}$ appearing in the r.h.s. is the most general $n \times n$ symmetric matrix with total number of $\frac{n(n+1)}{2}$ components. For any given spacetime we can easily compute its associated decomposition of $\mathcal{Z}$ in terms of the antisymmetrized products of k-Gamma matrices, namely

$$\mathcal{Z}_{ab} \quad = \quad \sum_k (A\Gamma_{[\mu_1 \ldots \mu_k]})_{ab} Z^{[\mu_1 \ldots \mu_k]}, \tag{5}$$

where the values k entering the sum in the r.h.s. are restricted by the symmetry requirement for the $a \leftrightarrow b$ exchange and are specific for the given spacetime. The coefficients $Z^{[\mu_1 \cdots \mu_k]}$ are the rank-k abelian tensorial central charges.

In the above formula the matrix A is the generalization of Γ^0, needed to introduce barred spinors. Another useful matrix is the charge conjugation matrix C, which is used in order to construct rank-k antisymmetric tensors which are all hermitian or antihermitian in the $a \leftrightarrow b$ exchange (see [9] for details). When the fundamental spinors are complex or quaternionic (let us limit ourselves to discuss this associative case, but the generalization to octonionic spinors can be made, see [9]) they can be organized in complex (for the **C** and **H** cases) and quaternionic (for the **H** case) multiplets, whose entries are respectively complex numbers or quaternions.

The real generalized supersymmetry algebra (4) can now be replaced by the most general complex or quaternionic supersymmetry algebras, given by the anticommutators among the fundamental spinors Q_a and their conjugate ${Q^*}_{\dot{a}}$ (where the conjugation refers to the principal conjugation in the given division algebra. We have in this case

$$\{Q_a, Q_b\} = \mathcal{P}_{ab} \qquad , \qquad \{{Q^*}_{\dot{a}}, {Q^*}_{\dot{b}}\} = {\mathcal{P}^*}_{\dot{a}\dot{b}}, \tag{6}$$

together with

$$\{Q_a, {Q^*}_{\dot{b}}\} \quad = \quad \mathcal{R}_{a\dot{b}}, \tag{7}$$

where the matrix $\mathcal{P}_{ab}$ (${\mathcal{P}^*}_{\dot{a}\dot{b}}$ is its conjugate and does not contain new degrees of freedom) is symmetric, while $\mathcal{R}_{a\dot{b}}$ is hermitian.

The maximal number of allowed components in the r.h.s. is given, for complex fundamental spinors with n complex components, by
ia) $n(n+1)$ (real) bosonic components entering the symmetric $n \times n$ complex matrix $\mathcal{P}_{ab}$ plus
iia) n^2 (real) bosonic components entering the hermitian $n \times n$ complex matrix $\mathcal{R}_{a\dot{b}}$.

Similarly, the maximal number of allowed components in the r.h.s. for quaternionic fundamental spinors with n quaternionic components is given by
ib) $2n(n+1)$ (real) bosonic components entering the symmetric $n \times n$ quaternionic matrix $\mathcal{P}_{ab}$ plus
iib) $2n^2 - n$ (real) bosonic components entering the hermitian $n \times n$ quaternionic matrix $\mathcal{R}_{a\dot{b}}$.

The previous numbers do not necessarily mean that the corresponding generalized supersymmetry is indeed saturated. This is in particular true in the quaternionic case. Some further remarks are in order. We can expand the r.h.s. of (6) and (7) in terms of the antisymmetrized product of Gamma matrices only when the division-algebra character of the Gamma matrices coincides with the division-algebra character of spinors.

3. Constrained generalized supersymmetries

In this section we investigate and classify the set of consistent constraints that can be imposed on the complex generalized supersymmetries.

Saturated complex generalized supersymmetries (i.e. the ones admitting as bosonic r.h.s. both the most general symmetric matrix $\mathcal{P}$ entering (6) and the most general hermitian matrix $\mathcal{R}$ entering (7)) contain the same number of bosonic degrees of freedom as the corresponding saturated generalized supersymmetries realized with *real* spinors. In this respect the big advantage of the introduction of the complex formalism, whenever this is indeed possible, consists in the implementation of some constraint that cannot be otherwise imposed within the real framework.

In [10] the two big classes of hermitian and holomorphic generalized supersymmetries were introduced and discussed. This result was further extended [11] with a presentation of a whole new class of division-algebra related constraints that can be consistently imposed. The bosonic r.h.s. can be expressed in terms of the rank-k totally antisymmetric tensors (denoted as M_k). It is clear that any restriction on the saturated bosonic generators which allows all possible combinations of the rank-k antisymmetric tensors entering the r.h.s. is in principle admissible by a Lorentz-covariant requirement. It is worth noticing that we are limiting our discussion on the generalized supersymmetries which can be loosely denoted as "generalized supertranslations", see [10]. Supersymmetries of this kind present no Lorentz generators. However, they can be regarded as building blocks to construct superconformal algebras, out of which the generalized super-Poincaré algebras, admitting Lorentz subalgebras, can be recovered through an Inonü-Wigner type of contraction. It requires the introduction of two separated copies of "generalized supertranslations". The implementation of super-Jacobi identities is sufficient to detect the remaining generators and close the whole set of algebraic relations defining the associated superconformal algebra. Therefore, all the information about such superconformal algebras is already contained in the generalized supertranslations, the subject of the present investigation and classification. On the other hand few particular combinations of the rank-k antisymmetric tensors have more compelling reasons to appear than just arising as a hand-imposed restriction on the saturated bosonic r.h.s. They can indeed be present due to a division-algebra constraint based on an underlying symmetry. It is expected that restrictions of this type offer a protecting mechanism towards the arising of anomalous terms, in application to the supersymmetries realized by certain classes of dynamical systems. This is an important reason to analyze and classify these constraints. Their whole class is presented in the table below.

It consists of all possible combinations of restrictions on the $\mathcal{P}$, $\mathcal{R}$ matrices of (6) and (7) (e.g. whether both of them are present or just one of them, if a reality or an imaginary condition is applied). The entries in the table below specify the number of bosonic components (in the real counting) associated with the given constrained supersymmetry realized by n-component complex spinors. The columns represent the restrictions on $\mathcal{R}$, the rows the restrictions on $\mathcal{P}$ (an imaginary condition on $\mathcal{P}$ is equivalent to the reality condition and therefore is not reported in the table below). We have

$\mathcal{P}\backslash\mathcal{R}$	1) *Full*	2) *Real*	3) *Imag.*	4) *Abs.*
a) *Full*	$2n^2+n$	$\frac{3}{2}(n^2+n)$	$\frac{1}{2}(3n^2+n)$	n^2+n
b) *Real*	$\frac{1}{2}(3n^2+n)$	n^2+n	n^2	$\frac{1}{2}(n^2+n)$
c) *Abs.*	n^2	$\frac{1}{2}(n^2+n)$	$\frac{1}{2}(n^2-n)$	0

(8)

Some comments are in order. The above list of constraints is not necessarily implemented for any given supersymmetric dynamical system. One should check, e.g., that the above restrictions are indeed compatible with the equations of motion. On a purely algebraic basis, however, they are admissible restrictions which require a careful investigation.

One can notice that certain numbers appear twice as entries in the above table. This is related with the fact that the same constrained superalgebra can admit a different, but equivalent, presentation. We refer to these equivalent presentations as "dual formulations" of the constrained supersymmetries. Dual formulations are expected in correspondence of the constraints

$$\begin{aligned} a3 &\leftrightarrow b1, \\ a4 &\leftrightarrow b2, \\ b3 &\leftrightarrow c1, \\ b4 &\leftrightarrow c2. \end{aligned} \tag{9}$$

It is worth stressing that in application to dynamical systems, which need more data than just superalgebraic data, one should explicitly verify whether the above related constraints indeed lead to equivalent theories.

The inequivalent constrained generalized supersymmetries can be listed as follows

I	$(a1)$	$2n^2+n,$	$k=3,$	$l=1$
II	$(a2)$	$\frac{3}{2}(n^2+n),$	$k=3,$	$l=0$
III	$(a3\,\&\,b1)$	$\frac{1}{2}(3n^2+n),$	$k=2,$	$l=1$
IV	$(a4\,\&\,b2)$	$n^2+n,$	$k=2,$	$l=0$
V	$(b3\,\&\,c1)$	$n^2,$	$k=1,$	$l=1$
VI	$(b4\,\&\,c2)$	$\frac{1}{2}(n^2+n),$	$k=1,$	$l=0$
VII	$(c3)$	$\frac{1}{2}(n^2-n),$	$k=0,$	$l=1$

(10)

The integral numbers k,l have the following meaning. For the given constrained supersymmetry the bosonic r.h.s. can be presented in the following form

$$Z \quad = \quad kX+lY, \quad k=0,1,2,3, \quad l=0,1, \tag{11}$$

where X and Y denote the bosonic sectors associated with the VI and respectively VII constrained supersymmetry.

In association with the maximal Clifford algebras in D-dimensional spacetimes (with no dependence on their signature), the X and Y bosonic sectors are given by the following set of rank-k antisymmetric tensors

	X	Y
$D=3$	M_1	M_0
$D=5$	M_2	M_0+M_1
$D=7$	M_0+M_3	M_1+M_2
$D=9$	$M_0+M_1+M_4$	M_2+M_3
$D=11$	$M_1+M_2+M_5$	$M_0+M_3+M_4$
$D=13$	$M_2+M_3+M_6$	$M_0+M_1+M_4+M_5$

(12)

Formula (11) specifies the admissible class of division-algebra related, constrained bosonic sectors.

4. Real superparticles with tensorial central charges

Let us at first introduce the superparticle models with tensorial central charges, based on real generalized supersymmetries. It consists of an extension of the first-order formalism of Brink-Schwarz used to formulate the ordinary massless superparticles.

The most general action S involving real spinors is constructed as follows [5] in terms of the real superspace coordinates X^{ab}, Θ^a conjugated to the superalgebra generators $\mathcal{Z}_{ab}$ and Q_a of (4) (X^{ab} is symmetric in the $a \leftrightarrow b$ exchange). We have

$$S = \frac{1}{2}\int d\tau tr\left[\mathcal{Z}\cdot\Pi - e(\mathcal{Z})^2\right], \tag{13}$$

where

$$\Pi^{ab} = dX^{ab} - \Theta^{(a}d\Theta^{b)}, \tag{14}$$

while e^{ab} denotes the Lagrange multipliers whose (anti)symmetry property is the same as the one of the charge conjugation matrix C^{ab}, i.e.

$$e^T = \varepsilon e \qquad for \qquad C^T = \varepsilon C. \tag{15}$$

By construction

$$(\mathcal{Z})^2{}_{ab} = \mathcal{Z}_{ac}C^{cd}\mathcal{Z}_{db}, \tag{16}$$

namely the charge conjugation matrix is used as a metric to raise and lower spinorial indices.

The massless constraint

$$(\mathcal{Z})^2_{ab} = 0 \tag{17}$$

is obtained from the variation δe^{ab} of the Lagrange multipliers.

A symmetric charge conjugation matrix ($\varepsilon = 1$) allows us [5] to construct a massive model by simply performing a shift $\mathcal{Z} \rightarrow \mathcal{Z} + mC$ in the action (13).

5. Complex superparticles with tensorial central charges

As discussed before, constrained generalized supersymmetries can be introduced for spinors which are at least complex. In order to introduce the action for the superparticle with complex spinors we should mimick, as much as possible, the real formulation. The bosonic matrix $\mathcal{Z}_{ab}$ is now replaced by the pair of matrices $\mathcal{P}_{ab}$ and $\mathcal{R}_{a\dot{b}}$ (respectively symmetric and hermitian) entering (6) and (7). They can be accommodated in a symmetric matrix $\mathbf{P}$ ($\mathbf{P}^T = \mathbf{P}$) as follows

$$\mathbf{P} = \begin{pmatrix} \mathcal{P} & \mathcal{R} \\ \mathcal{R}^* & \mathcal{P}^* \end{pmatrix}. \tag{18}$$

The supercoordinates conjugated to $\mathcal{P}_{ab}$, $\mathcal{R}_{a\dot{b}}$, Q_a and $Q^*{}_{\dot{a}}$ are given by X^{ab}, $Y^{a\dot{b}}$, Θ^a and $\Theta^{*\dot{a}}$.

It is convenient to use the notation

$$\mathbf{\Pi} = \begin{pmatrix} dX - \Theta d\Theta & dY - \Theta d\Theta^* \\ dY^* - \Theta^* d\Theta & dX^* - \Theta^* d\Theta^* \end{pmatrix}. \tag{19}$$

We will also need the matrix

$$\mathbf{P}^2 = \mathbf{P}\mathcal{C}\mathbf{P}, \tag{20}$$

whose indices are raised by the metric $\mathcal{C}$. There are three inequivalent available specific choices for $\mathcal{C}$ which are discussed below. The (anti)-symmetry property of $\mathbf{P}^2$ coincides with the (anti)-symmetry property of $\mathcal{C}$.

The Lagrange multipliers enter a matrix

$$\mathbf{E} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}. \tag{21}$$

In general, for any $\mathbf{U}$ (for our purposes $\mathbf{U} \equiv \mathbf{P}^2$) s.t.

$$\mathbf{U} = \begin{pmatrix} U & V \\ \lambda\mu V^* & U^* \end{pmatrix} \tag{22}$$

with $U^T = \lambda U$, $V^\dagger = \mu V$ (therefore $\mathbf{U}^T = \lambda \mathbf{U}$), the reality of the term $tr(\mathbf{EU})$ requires

$$\begin{aligned} g &= \lambda\mu f^*, \\ h &= e^*. \end{aligned} \tag{23}$$

A reality (imaginary) condition imposed on either $\mathbf{U}$ or $\mathbf{V}$ implies a reality (imaginary) condition for the lagrange multipliers e and f respectively.

We are now in the position to write the action S for the superparticle with bosonic tensorial central charges and complex spinors as

$$S = \frac{1}{2}\int d\tau tr \left[\mathbf{P\Pi} - \mathbf{E}(\mathbf{P})^2\right]. \tag{24}$$

As in the real case, a massive model can be introduced in correspondence of a symmetric $\mathcal{C}$ through the shift $\mathbf{P} \to \mathbf{P} + m\mathcal{C}$ in the action (24). For what concerns the metric $\mathcal{C}$, it has to be of the same form as $\mathbf{P}$ (see (18)) entering the action (24), with an upper-left (anti)symmetric block and an upper-right (anti)hermitian block. More specifically, $\mathcal{C}$ should be presented as in formula (22), in terms of two (an (anti)symmetric and an (anti)hermitian) scalar matrices respectively denoted as U and V. Since U and V are both scalars, their available choices are therefore given by $U \equiv \widetilde{C}$, $V \equiv \widetilde{A}$, where, essentially, see [11] for details, $\widetilde{C}$ denotes the charge-conjugation matrix C and $\widetilde{A}$ the generalization of Γ^0.

It is convenient to denote with $\epsilon, \delta = \pm 1$ ($\widetilde{C}^T = \epsilon \widetilde{C}$, $\widetilde{A}^\dagger = \delta \widetilde{A}$) the (anti)symmetry and (anti)hermitian properties of $\widetilde{C}$, $\widetilde{A}$ respectively.

Without loss of generality, three possible choices for $\mathcal{C}$ are at disposal. They are given by

i)

$$\mathcal{C} = \begin{pmatrix} \widetilde{C} & 0 \\ 0 & \widetilde{C}^* \end{pmatrix}, \tag{25}$$

in this case $\mathcal{C}$ is (anti)symmetric in accordance with the sign of ϵ;

ii)

$$\mathcal{C} = \begin{pmatrix} 0 & \widetilde{A} \\ \xi \widetilde{A}^* & 0 \end{pmatrix}, \tag{26}$$

where ξ is an arbitrary sign ($\xi = \pm 1$); in this case the (anti)symmetry property of $\mathcal{C}$ is specified by the sign of $\delta\xi$;

iii)

$$\mathcal{C} = \begin{pmatrix} \widetilde{C} & \widetilde{A} \\ \epsilon\delta \widetilde{A}^* & \widetilde{C}^* \end{pmatrix}, \tag{27}$$

the (anti)symmetry property of $\mathcal{C}$ is specified by the sign of ϵ. It should be noticed that in this last case an (anti)symmetric matrix $\mathbf{P}^2$ ($\mathbf{P}^2 = \mathbf{P}\mathcal{C}\mathbf{P}$) is only possible, for both non-vanishing $\mathcal{P}$, $\mathcal{R}$ entering $\mathbf{P}$, if the condition

$$\epsilon = \delta \tag{28}$$

is matched.

The above three sets of choices for $\mathcal{C}$ completely specify the available actions for the superparticles with tensorial central charges and complex spinors.

6. Constrained superparticles with tensorial central charges

The analysis of the constrained generalized supersymmetries can be applied to the dynamics of the complex superparticles. The equations of motion of this class of models can be easily derived from the action (24). It is sufficient here to present the constraints arising from the variations δe, δf of the lagrange multipliers entering (24). Such constraints will be denoted with the symbols "X"and "Y", respectively. In correspondence with the three above choices for $\mathcal{C}$ we get the following constraints
i)

$$\begin{aligned} X &= \mathcal{P}\widetilde{C}\mathcal{P} + \mathcal{R}\widetilde{C}^*\mathcal{R}^* = 0, \\ Y &= \mathcal{P}\widetilde{C}\mathcal{R} + \mathcal{R}\widetilde{C}^*\mathcal{P}^* = 0; \end{aligned} \tag{29}$$

ii)

$$\begin{aligned} X &= \xi\mathcal{R}\widetilde{A}^*\mathcal{P} + \mathcal{P}\widetilde{A}\mathcal{R}^* = 0, \\ Y &= \xi\mathcal{R}\widetilde{A}^*\mathcal{R} + \mathcal{P}\widetilde{A}\mathcal{P}^* = 0; \end{aligned} \tag{30}$$

iii)

$$\begin{aligned} X &= \mathcal{P}\widetilde{C}\mathcal{P} + \epsilon\delta\mathcal{R}\widetilde{A}^*\mathcal{P} + \mathcal{P}\widetilde{A}\mathcal{R}^* + \mathcal{R}\widetilde{C}^*\mathcal{R}^* = 0, \\ Y &= \mathcal{P}\widetilde{C}\mathcal{R} + \epsilon\delta\mathcal{R}\widetilde{A}^*\mathcal{R} + \mathcal{P}\widetilde{A}\mathcal{P}^* + \mathcal{R}\widetilde{C}^*\mathcal{P}^* = 0. \end{aligned} \tag{31}$$

The analysis now goes as follows. One can check whether the constrained generalized supersmmetries indeed apply to the the different classes of complex superparticles models (i.e., whether the constraints are compatible with the equations of motion) and whether the duality relations between constrained generalized supersymmetries are indeed satisfied in the dynamical setting. The detailed list of results is rather complicated and has been presented in [11].We will not report it here in full generality. Instead, we are limiting ourselves to furnish a table specifying, in association with the given choices of $\mathcal{C}$ the dynamical compatibility of the constraints discussed and introduced above for *generic* choices of the spacetimes. We get

	i	ii	iii
I	yes	yes	yes
$IV\,(a4)$	yes	yes	no
$IV\,(b2)$	yes	yes	$yes^*\,(\epsilon = 1)$
$V\,(b3)$	yes	yes	$yes^*\,(\epsilon = 1)$
$V\,(c1)$	yes	yes	no
$VI\,(b4)$	$yes^*\,(\epsilon = -1)$	yes	no
$VI\,(c2)$	$yes^*\,(\epsilon = -1)$	yes	no
VII	$yes^*\,(\epsilon = -1)$	yes	no

(32)

The "$*$" denotes which choices are consistent only for a specific value of ϵ.

7. Conclusions

The understanding of generalized supersymmetry is of preliminary, capital importance, for investigating the dynamical content of the M-theory, which should be based on a particular example of generalized supersymmetry, the so-called M algebra. We pointed out that generalized supersymmetries can be classified according to their division-algebra property. We can therefore speak of real, complex, quaternionic and even octonionic generalized supersymmetries. In the real case the conjugation acts trivially as the identity operator. In the remaining cases, however, the conjugation acts non-trivially and allows to reexpress the single generalized superalgebra relation as three separated relations (two of them mutually conjugated). On these relations we saw that we can impose on the bosonic r.h.s. division-algebra compatible constraints. We are therefore allowed to speak of constrained generalized supersymmetries. We have further seen that, in several cases which have been listed, the one and the same constrained generalized supersymmetry can be presented in different, dual, formulations. The classification of these superalgebras, presented in [11], was reviewed. Generalized supersymmetries should appear as symmetry algebras of the M-theory, see e.g. [12]. It is therefore quite important to analyze some given examples of dynamical models based on generalized supersymmetries. Within our framework we reformulate the superparticles with tensorial central charges, first introduced by Rudychev-Sezgin and, in a complex formalism case, by Bandos-Lukierski. We proved that complex tensorial superparticles admit three different inequivalent formulations associated with the choices of the metric used to raise and lower the spinorial indices. We finally investigated under which conditions the constraints on generalized supersymmetries can be consistently applied on the equations of motion of the associated tensorial superparticle models. We should mention that there are at least two other classes of systems, which are currently under investigation, that can be analyzed in the framework here presented. The first class of dynamical systems corresponds to the tensionless strings and branes, see [13]. The division-algebra framework for generalized supersymmetries can provide the consistency conditions for the existence of these models (the so-called brane-scans). Another very important class of models, somehow "orthogonal" to the tensorial superparticles (since they they admit only particles with spin less or equal than two) is given by the higher-dimensional Chern-Simon supergravitiesin any given odd dimension, whose possible relation with the M-theory has been pointed out in [14].

Acknowledgments.

The main results here reported concern a work done in collaboration with Z. Kuznetsova.

References

[1] R. Haag, J. Łopuszański and M. Sohnius, Nucl.Phys. **B 88** (1975) 257.

[2] R. D'Auria and P. Fré, Nucl. Phys. **B 201** (1982), 101; erratum *ibid.* **B 206** (1982) 496.

[3] J.A. de Azcarraga, J.P. Gauntlett, J.M. Izquierdo and P.K. Townsend, Phys. Rev. Lett. **63** (1989) 2443.

[4] P. Townsend, hep-th/9712004, Cargese Lectures, 1997.

[5] E. Sezgin and I. Rudychev, "Superparticles, p-form coordinates and the BPS condition", hep-th/9711128.

[6] I. Bandos and J. Lukierski, Mod. Phys. Lett. **A 14** (1999) 1257.

[7] I. Bandos, J. Lukierski and D. Sorokin, Phys. Rev. **D 61** (2000) 045002.

[8] C. Fronsdal, "Massless Particles, Ortosymplectic Symmetry and Another Type of Kaluza-Klein Theory", in "Essays on Supersymmetry", Reidel (1986) (Math. Phys. Studies, v. 8).

[9] H.L. Carrion, M. Rojas and F. Toppan, JHEP04 (2003) 040.

[10] F. Toppan, JHEP09 (2004) 016.

[11] Z. Kuznetsova and F. Toppan, JHEP05 (2005) 060.

[12] P. West, JHEP 0008 (2000) 007.

[13] A.A. Zheltukhin and U. Lindström, Nucl. Phys. Proc. Suppl. **102** (2002) 126; JHEP 0201 (2002) 034; A.A. Zheltukhin and D. V. Uvarov, JHEP 0208 (2002) 008; Phys. Lett. **B 565** (2003) 229.

[14] M. Hassaine, R. Troncoso and J. Zanelli, "Eleven-dimensional supergravity as a gauge-theory for the M-algebra", hep-th/0306258.