

Black holes and extra dimensions

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Abstract. *In models with large extra dimensions that accommodate a TeV-scale gravity Einstein's equations can exist. We study interaction of such black holes with our world — a brane embedded in a higher dimensional space. In such a setup there exist a host of new phenomena that do not have analogs in usual 3 + 1-dim models. We concentrate on the case where the black hole is located on the brane.*

Key words: *black holes, extra dimensions*

Recently, the TeV-scale gravity models have attracted much of interest. It is basically the idea that our (3 + 1)-dimensional universe is only a sub-manifold on which the standard model fields are confined inside a higher dimensional space. The original ADD (Arkani-Hamed, Dimopoulos and Dvali) proposal [1] implements extra space as a multi-dimensional compact manifold, so that our universe is a direct product of an ordinary (3 + 1)-dimensional FRW (Freedman, Robertson and Walker) universe and an extra space. This construction was primarily motivated by attractive particle physics feature — namely a solution to the hierarchy problem (large difference between the Planck scale, $M_{Pl} \sim 10^{16}\text{TeV}$ and the electroweak scale, $M_{EW} \sim 1\text{TeV}$). By allowing only geometrical degrees of freedom to propagate in extra dimensions and making the volume of the extra space large, we can lower a fundamental quantum gravity scale, M_* , down to the electroweak scale ($\sim \text{TeV}$). The size of extra dimensional manifold is then limited from above only by short distance gravity experiments (current experiments do not probe any deviations from a four-dimensional Newton's

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gravity law on distances smaller than 0.2mm). Thus, for different numbers of extra dimensions compactified on a flat manifold (for an alternative way of compactification see [2]) the compactification radius can vary from the fundamental length scale M_*^{-1} to the macroscopic dimensions of order 0.2mm.

The other option, exercised in [3], uses a non-factorizable geometry with a single extra dimension. In the so-called RSI (Randall, Sundrum) scenario extra dimension is made compact by introducing two branes (one with positive and one with negative tension) with a piece of anti-de Sitter space between them. If we put all the standard model fields on the negative tension brane, due to exponential scaling properties of masses in this background, we can solve the hierarchy problem by setting the distance between the two branes only one or two orders of magnitude larger than the anti-de Sitter radius. In order to make a model selfconsistent, one has to impose a Z_2 symmetry around both branes. Alternatively, we can put all the standard model fields on the positive tension brane and make the extra dimension infinite by moving the negative tension brane to infinity (so called RSII scenario [4]). Since this model does not yield a TeV strength gravity we will confine our discussion (except for the comment at the end) to the ADD and RSI models. Large black holes in these two scenarios whose gravitational radius in brane directions is much larger than the size of extra dimensions should have properties similar to those of ordinary $(3+1)$ -dimensional black holes. Intermediate size black holes whose gravitational radius is of the order of the characteristic length of extra dimensions (compactification radius in ADD or AdS radius in RS model) can have quite different properties due to the “edge effects” and different geometry of the extra space. We will not discuss these two regimes.

Finally, if a gravitational radius of a black hole is much smaller than the characteristic length of extra dimensions, then the black hole can be very well described by asymptotically flat solutions, i.e. Tangherlini [5] or Myers-Perry [6] solutions for higher dimensional static and rotating black holes respectively. Thus from this point of view, one might conclude that there should not be any practical difference between the small black holes in these two scenarios. The aim of this paper is to point out that important differences, most of them concerning the experimental signatures in near future accelerator experiments, still exist. They stem from the fact that RS models usually have a discrete Z_2 symmetry that fixes the brane and makes it the boundary of space-time. Such brane behaves as if it were an infinite tension brane for the processes of interest here. Probably the most

interesting and intriguing feature of theories with TeV-scale gravity is the possibility of production of mini black holes in future collider experiments (for recent reviews see [7]). Calculations [8] indicate that the probability for creation of a mini black hole in near future hadron colliders such as the LHC (Large Hadron Collider) is so high that they can be called “black hole factories”. Consider two particles (partons in the case of the LHC) moving in opposite direction with the center of mass energy $\sqrt{\hat{s}}$. If the impact parameter is less than the gravitational radius r_g of a $(N + 1)$ -dimensional black hole, then a black hole with a mass of the order of $\sqrt{\hat{s}}$ will form. For high energy scattering of two particles with a non-zero impact parameter, the formation of a rotating black hole is much more probable than the formation of a non-rotating black hole [13]. One may expect that mainly highly rotating mini black holes are to be formed in such scattering. To simplify calculations of cross section of mini black hole production, effects connected with rotation of the black hole are usually neglected. The same simplification is usually made when quantum decay of mini black holes is discussed. However, we argue that rotation is of crucial interest if we want to distinguish between the small ADD and RS black holes.

After the black hole is formed it decays by emitting Hawking radiation with temperature $T \sim 1/r_g$. Thermal Hawking radiation consists of two parts: (1) particles propagating along the brane, and (2) bulk radiation. The bulk radiation includes bulk gravitons. Usually the bulk radiation is neglected. The reason is as following. The wavelength of emitted radiation is larger than the size of the black hole, so the black hole will behave as a point radiator radiating mostly in s-wave. Thus, the radiation for each particle mode will be equally probable in every direction (brane or bulk). For each particle that can propagate in the bulk there is a whole tower of bulk Kaluza-Klein excitations, but since they are only weakly coupled (due to small wave function overlap) to the small black hole, the whole tower counts only as one particle. Since the total number of species which are living on the brane is quite large (~ 60) and there is only one graviton, radiation along the brane should be dominant (see e.g. [9]). This reasoning works very well if the black hole is not rotating. Rotation can significantly modify the conclusion. Indeed, the number of degrees of freedom of gravitons in the $(N + 1)$ -dimensional space-time is $\mathcal{N} = (N + 1)(N - 2)/2$. For example, for $N + 1 = 10$ we have $\mathcal{N} = 35$. One may expect that if a black hole is non-rotating, emission of particles with non-zero spin (e.g. gravitons) is suppressed with respect to emission of scalar quanta as it happens in $(3 + 1)$ -dimensional space-time [10] (see also Section 10.5 [11] and references

therein). However, due to existence of the ergosphere (region between the infinite redshift surface and the event horizon), a rotating black hole exhibits an interesting effect known as super-radiance. Some of the modes of radiation get amplified taking away the rotational energy of the black hole. The effect of super-radiance is strongly spin-dependent, and emission of higher spin particles is strongly favored. For extremely rotating black hole the emission of gravitons is a dominant effect. For example, $(3+1)$ -dimensional numerical calculations done by Don Page [10] (see also [11]) show that the probability of emission of a graviton by an extremely rotating black hole is about 100 times higher than the probability of emission of a photon or neutrino. In [12], it was shown that super-radiance also exist in higher dimensional space-times. Mini black holes created in the high energy scattering are expected to have high angular momentum. In the highly non-linear, time-dependent and violent process of a black hole creation, up to 30% of the initial center of mass energy is lost to gravitational radiation (this percentage may be even larger in higher dimensional scenarios due to larger number of gravitational degrees of freedom). Since gravitons are not bound to the brane, most of them would be radiated in the bulk giving the black hole a non-zero bulk component of the angular momentum. For such a black hole, the bulk radiation may dominate the radiation along the brane, at least in the first stages of evaporation [14].

The first signature of bulk graviton emission is virtual energy non-conservation for an observer located on the brane. Also, as a result of the emission of the graviton into the bulk space, the black hole recoil can move the black hole out of the brane. After the black hole leaves the brane, it cannot emit brane-confined particles anymore. Black hole radiation would be abruptly terminated for an observer located on the brane. Probability for something like this to happen depends on many factors (mass of the black hole, brane tension...) and it was studied in [14].

Another important question, often neglected in discussion, is the interaction between the black hole and the brane. In [15] the rate of the loss of the angular momentum of the black hole which interacts with a stationary brane was calculated. It was shown that a black hole in its final stationary state can have only those components of the angular momenta which are connected with Killing vectors generating transformations preserving a position of the brane (see Fig. 1.). This is a direct consequence of the “friction” between the black hole and the brane. As a result of this friction the black hole loses all of the bulk components of its angular momentum to the brane. The only components of angular momentum which survive are those

along the brane. The characteristic time when a rotating black hole with the gravitational radius r_g reaches this final state is

$$(\Delta t)_F \sim r_g^{k-1} / (G_* \sigma), \tag{1}$$

where G_* is the higher dimensional gravitational coupling constant, σ is the brane tension, and k is the number of extra dimensions. The rotating black hole can also lose its bulk components of the rotation by emitting Hawking quanta in the bulk. The characteristic time of this process is $(\Delta t)_H \sim t_*(r_g/L_*)^{3+k}$, where t_* and L_* are the fundamental time and length. For black holes which can be treated classically $r_g \gg L_*$, so that we have $(\Delta t)_H \gg (\Delta t)_F$. Thus the friction effect induced by the brane is the dominant one. Finally, let us see how previous discussion can help us distinguish between the small ADD and RS black holes in accelerator experiments. A characteristic property of RS models is the existence of a discrete Z_2 symmetry with respect to $y \rightarrow -y$ (where y is the extra dimension). Under this Z_2 transformation the brane remains unchanged, while the components of any vector orthogonal to the brane change their sign. Thus Z_2 symmetry implies that any bulk components of the angular momenta of the small black hole attached on the brane are strictly zero (see Fig. 1.). Hence a stationary black hole attached to the brane in the RS-model can rotate only within the brane. Exact solutions describing rotating black holes on $(2+1)$ -dimensional branes [16] possess this property. This implies that for a small RS black hole emission of bulk gravitons would be heavily suppressed since they carry a non-zero bulk angular momentum. This is in strong contrast with a small ADD black hole.

It is not difficult to show formally the absence of bulk rotation for a small RS black hole on the brane. Consider a small black hole in RS scenario that can be described by a Myers-Perry solution with two rotational parameters. The metric is [6]:

$$ds^2 = -dt^2 + \frac{r^2 \rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2 \tag{2}$$

$$+ (r^2 + b^2) \cos^2 \theta d\psi^2 + \frac{r_0^2}{\rho^2} \left[dt + a \sin^2 \theta d\phi + b \cos^2 \theta d\psi \right]^2.$$

Here,

$$\rho^2 = r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta, \tag{3}$$

$$\Delta = (r^2 + a^2)(r^2 + b^2) - r_0^2 r^2.$$

Angles ϕ and ψ take values from the interval $[0, 2\pi]$, while angle θ takes values from $[0, \pi/2]$. We specify the position of the brane in the equatorial plane of the black hole at $\psi = 0$. Then, a is the rotational parameter describing rotations within the brane, while b is the bulk rotational parameter. The general form of a 5-dimensional metric is:

$$ds^2 = g_{MN} dx^M dx^N \quad (4)$$

where indices M and N go over all 5 dimensions. We can decompose it into:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + 2g_{\mu y} dx^\mu dy + g_{yy} dy^2, \quad (5)$$

where $\mu = 0, 1, 2, 3$, while y is the extra coordinate.

We can now impose a Z_2 symmetry under the transformation $y \rightarrow -y$. Then, the metric components must satisfy this:

$$g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad g_{\mu y} \rightarrow -g_{\mu y} \quad \text{and} \quad g_{yy} \rightarrow g_{yy}. \quad (6)$$

Imposing a symmetry under $y \rightarrow -y$ is equivalent to restricting the interval for ψ to $[0, \pi]$ and requiring a symmetry under $\psi \rightarrow -\psi$. From the explicit form of the metric (2) we see that in this case the following must be true:

$$a \rightarrow a, \quad b \rightarrow -b. \quad (7)$$

If the RS black hole is in the bulk, this would imply that the (Z_2 -symmetric) “mirror” image black hole on the other side of the brane must be spinning in the opposite direction as far as the bulk angular momentum is concerned. However, if the black hole is located on the brane, the Z_2 transformed metric must describe the same object as the original one, and we can conclude that b must be zero. Note that this argument is valid only for a small black hole. If the extent of the black hole horizon into the extra dimension is larger than the AdS radius, the symmetry group that describes rotations in 4 spatial dimensions is not $SO(4)$ anymore since the fourth dimension is not equivalent to the first three dimensions. It is still possible that the same conclusion remains but it requires more careful arguments. If an ADD black hole emits any particle with a bulk angular momentum, it would acquire a bulk angular momentum itself. The same would happen if a black hole collides with a particle from the bulk. But we saw that RS black hole can not have any bulk component of angular momentum. This implies that a Z_2 symmetric brane behaves as an infinite tension brane absorbing all the incoming bulk angular momentum.

As a consequence of a virtual absence of the bulk radiation, a small RS black hole that is attached to the brane can not recoil and leave the brane. This is not (quite) surprising since the Z_2 symmetry anyway prevents the black hole from leaving the brane. The process in which a small RS black hole leaves the brane in the Z_2 symmetric space where the two sides of the bulk space are not identified reminds of a black hole splitting into two symmetric black holes (see Fig. 2.). Classically this process is forbidden in a higher dimensional space-time for the same reason as in $(3+1)$ -dimensional space-time in connection with non-decreasing property of the entropy. The entropy of a black hole in fundamental units is $S \sim M_{BH}^{\frac{2+k}{1+k}}$, and the entropy of the final state (two black holes of mass $M_{BH}/2$) is lower than that of the initial one. If the two sides of the bulk space are identified, the “image” black hole is identified with the original one and we do not count degrees of freedom of two black holes separately. However, even in this case the recoil effect is suppressed due to highly suppressed bulk radiation. This is again in strong contrast with a small ADD black hole. One may argue that the process of the extraction of the black hole from the brane is time dependent and that there will be some energy flux through the horizon during the process. The black hole would grow in mass and thus we could go around the argument of the smaller entropy in the final state. In [17], energy fluxes in time-dependent configurations of a black hole-brane system were calculated. It was shown that if the process of extraction is adiabatic (quasi static) the energy flux through the horizon is negligible. Thus, in general, we can not use counter-arguments of this type. Therefore, even if a black hole emits a particle in the bulk direction (or gets hit by a particle from the bulk), it will not recoil off the brane. Again, it looks as if a Z_2 symmetric brane behaves as an infinite tension brane absorbing all the incoming bulk linear momentum. We note here that there is no paradox in these statements. In a Z_2 symmetric space, it is implicitly assumed that two identical particles are emitted (absorbed) by a black hole in a Z_2 symmetric way, thus canceling out any bulk components of momentum. In a representation where only one half of the space is shown (the two sides are identified) and the brane is a boundary of the space, it looks like the bulk component of (linear or angular) momentum is absorbed by the brane since there is no “other side” of the space. Also, a brane in a Z_2 symmetric space is fixed and can not vibrate, thus behaving as if it were a infinite tension brane though its physical tension can be finite.

In conclusion, we analyzed a small higher dimensional black hole located on the brane. In quantum gravity energy scale is really as low as 1 TeV as

proposed in brane world models (see [19] for some important issues), small black holes could be produced in near future accelerator experiments. In high energy scattering of two particles with a non-zero impact parameter, the formation of a rotating black hole is much more probable than the formation of a non-rotating black hole. One may expect that mainly highly rotating mini black holes are to be formed in such scattering.

Small black holes whose gravitational radius is much smaller than the characteristic size of extra dimensions (compactification radius in flat or AdS radius in warped extra dimensions) can be very well described by asymptotically flat solutions, thus losing the information about the global geometry of the extra manifold. One might conclude that such small black holes would be indistinguishable in different scenarios. We argue that important differences still exist, especially regarding experimental signature in colliders, which may help us distinguish between the various extra dimensional scenarios. The main differences come from the fact that most of the models with the warped extra dimension have an additional discrete Z_2 symmetry that makes the brane behave as if it were an infinite tension brane.

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