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A non-perturbative orbifold gauge theory

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Abstract. *We construct a Z_2 orbifold projection of $SU(N)$ gauge theories formulated in five dimensions with a compact fifth dimension. We show through a non–perturbative argument that no boundary terms diverging with powers of the five–dimensional ultraviolet cutoff are generated. This opens the possibility of studying these theories non–perturbatively in order to establish if they can be used as effective weakly interacting theories at low energies.*

Key words: *Extra Dimensions*

We would like to investigate here if one could construct a four–dimensional non–supersymmetric effective gauge theory coupled to a Higgs field without a hierarchy problem. We consider $SU(N)$ pure gauge theories in five dimensions, compactified on the orbifold $R^4 \times S^1/Z_2$. In this scenario, the gauge symmetry is generically broken at the orbifold fixed points and the Higgs field corresponds to those extra dimensional components of the five dimensional gauge field that point along the broken gauge directions.

A 5D gauge theory is non-renormalizable. To make sense out of such a theory, an UV cutoff Λ can be introduced and the theory can be treated as an effective low–energy theory. One is however not guaranteed that this is a consistent program unless there exists a range of the cutoff Λ where the low–energy physical properties depend only weakly on Λ (this is called the scaling region) and the theory is weakly interacting.

In an orbifold compactification of a field theory there is an additional problem that appears due to the presence of the boundaries: fields acquire Dirichlet or Neumann boundary conditions at the fixpoints of the orbifold.

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The formulation of a field theory with prescribed boundary values for some of the field components requires in general additional renormalization. The presence of boundaries introduces additional divergences and these induce boundary counterterms with renormalization factors calculable in perturbation theory. Of particular relevance to the hierarchy problem is the mass of the Higgs field. It will receive corrections from the bulk, from the boundary and from mixing effects between the bulk and the boundary. To tame the pure bulk effects a perturbative formulation is not sufficient since it is not guaranteed that the non-renormalizability of the higher dimensional gauge theory will allow the Higgs mass to remain finite to all orders, as some low order perturbative calculations seem to suggest [1], [2]. The only way to explore the cutoff dependence of the bulk Higgs mass is to define and regularize the theory non-perturbatively on the lattice [3] and perform a numerical study. Similarly, the pure boundary contributions to the Higgs mass must be controlled non-perurbatively. In this case however, an analytical method is possible to construct and show that such contributions vanish identically non-perturbatively:

The orbifold theory can be formulated on the strip

$$I_0 \quad = \quad \{x_\mu, 0 \le x_5 \le \pi R\} \tag{1}$$

as follows [3]. One starts with an $SU(N)$ gauge theory defined on the open set $I_\epsilon = \{x_\mu, x_5 \in (-\epsilon, \pi R + \epsilon)\}$ with a gauge field $A_M(z)$ defined everywhere on I_ϵ and a *spurion field* $\mathcal{G}(z) \in SU(N)$ defined in the neighborhoods $O_1 = \{x_\mu, x_5 \in (-\epsilon, \epsilon)\}$ and $O_2 = \{x_\mu, x_5 \in (\pi R - \epsilon, \pi R + \epsilon)\}$ that satisfies

$$(\mathcal{R}\,\mathcal{G})\,\mathcal{G} \quad = \quad \pm 1\,, \tag{2}$$

with $\mathcal{R}$ the reflection operator $\mathcal{R} : x_5 \longrightarrow -x_5$. The gauge field on O_i is constrained by

$$\mathcal{R}\,A_M \quad = \quad \mathcal{G}\,A_M\,\mathcal{G}^{-1} + \mathcal{G}\,\partial_M \mathcal{G}^{-1}\,, \tag{3}$$

which implies $\mathcal{R}\,F_{MN} = \mathcal{G}\,F_{MN}\,\mathcal{G}^{-1}$. The transformation property of the spurion field under a gauge transformation is

$$\mathcal{G} \quad \longrightarrow \quad (\mathcal{R}\,\Omega)\,\mathcal{G}\,\Omega^{-1}\,. \tag{4}$$

The covariant derivative of $\mathcal{G}$ can be defined on the neighborhoods O_i by requiring that it transforms like $\mathcal{G}$. Such a covariant derivative is

$$D_M \mathcal{G} \quad = \quad \partial_M \mathcal{G} + (\mathcal{R}\,A_M)\,\mathcal{G} - \mathcal{G}\,A_M \tag{5}$$

and in fact it is easy to see that

$$D_M \mathcal{G} \equiv 0. \tag{6}$$

For any $\epsilon \neq 0$ the theories are gauge invariant and equivalent. The breaking of the gauge symmetry is realized by taking the limit $\epsilon \longrightarrow 0$. In this limit the neighborhoods O_i shrink to single points and one is left with boundaries at $x_5 = 0$ and $x_5 = \pi R$. We approach the limit $\epsilon \longrightarrow 0$ so that (in the limit), the spurion field and its derivatives take the value

$$\mathcal{G}(0) = \mathcal{G}(\pi R) = g\,, \tag{7}$$
$$\partial_5^p \mathcal{G}(0) = \partial_5^p \mathcal{G}(\pi R) = 0,\ p > 0 \tag{8}$$

for a constant matrix g obeying $g^2 = \pm 1$. Since g is constant all derivatives ∂_μ of $\mathcal{G}$ vanish as $\epsilon \longrightarrow 0$. Only gauge transformations for which

$$\Omega = g\,\Omega\, g \qquad \text{at } x_5 = 0 \text{ and } x_5 = \pi R \tag{9}$$

are still a symmetry of the theory. Taking the limit $\epsilon \longrightarrow 0$ yields the Dirichlet boundary conditions

$$\alpha_M\, A_M = g\, A_M\, g \qquad \text{at } x_5 = 0 \text{ and } x_5 = \pi R\,, \tag{10}$$

where no sum on M is implied on the left hand side. Similarly, all Neumann boundary conditions can be obtained.

Notice that the term

$$tr\{[A_M(z), g][A_M(z), g]\}\,, \tag{11}$$

is invariant under eq. (9). This term is proportional to $A_5^{\hat{a}} A_5^{\hat{a}}$, a would be quadratically divergent boundary mass term for the Higgs. An operator of the $\epsilon \neq 0$ effective action that could give rise to such a term is

$$\mathrm{tr}\{D_M \mathcal{G}\, D_M \mathcal{G}\}\,, \tag{12}$$

which is however identically zero, by eq. (6). In fact, it is not hard to check that none of the operators of the $\epsilon \neq 0$ effective action can induce a boundary Higgs mass.

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