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THREE-PION CORRELATIONS IN HEAVY-ION COLLISIONS: A TEST FOR q,p -BOSE GAS MODEL

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Abstract.

We perform, within the q-Bose gas model as well as its two-parameter extended q,p-Bose gas model, the combined study of two- and three-pion correlations using our formulas for the (q- or q,p-dependent) correlation functions intercepts $C^{(2)}(\mathbf{K},\mathbf{K})$, $C^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$, *their combination* $r^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$, *and then confront them with the existing data on pion correlations from the experiments at CERN SPS and RHIC. We exhibit the peculiar explicit dependence of* $C^{(2)}(\mathbf{K},\mathbf{K})$, $C^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$, *and* $r^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$ *on pions' mean momenta, especially interesting for low momenta. The approach is based on the assumption of complete chaoticity of the emitting sources, in contrast with other approaches to simultaneous description of two- and three-pion correlations, in terms of effective parameters (including index of partial non-chaoticity).*

Key words: *Two-, three-pion correlations; n-particle correlations; intercept; q,p-oscillators; q,p-Bose gas model; heavy ion collisions; source chaoticity*

1. INTRODUCTION

The approach based on multimode system of q-deformed oscillators and the related model of ideal gas of q-bosons, aimed to effectively describe in experiments on relativistic heavy-ion collisions the observed non-Bose type behaviour of the intercept ("strength") $\lambda^{(2)} \equiv C^{(2)}(K,K)-1$ of two-pion correlation function $C^{(2)}(p_1,p_2)$, has been proposed in [1, 2]. Its real efficiency, for both pions and kaons, was shown in [3]. Main point of the approach is the assumed perfect chaoticity of emitting sources; so, $\lambda^{(2)}$ implies other, than the exploited in some treatments partial coherence (non-chaoticity),

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reason(s) for deviating from pure Bose-type values. Three-particle correlation functions $C^{(3)}(p_1,p_2,p_3)$ of identical pions (or kaons) are also important, encoding more information [4] on the space-time geometry and dynamics of the emitting sources. A special combination $r^{(3)}(p_1,p_2,p_3)$ of the two- and three-pion correlation functions, proposed in [4], is very useful for the analysis of data since, in it, the effects of long-lived resonances cancel out.

In [5], the results of q-Bose gas based approach were further extended: (i) for q-Bose gas model, in both the Arik-Coon (AC) and the Biedenharn-Macfarlane (BM) versions, the intercepts of n-particle correlation functions have been derived; (ii) for the two-parameter extended q,p-Bose gas model, a closed form of n-particle correlation function intercepts has been obtained. This general expression yields AC and BM case formulas as particular cases.

Below, within the q-Bose gas model and its extension (q,p-Bose gas model), we present the combined analysis of two- and three-pion correlation functions intercepts using our formulas for $C^{(2)}(\mathbf{K},\mathbf{K})$, $C^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$, and the combination $r^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$. A comparison with the existing data on pion correlations drawn in the SPS and RHIC experiments is made. We stress the *utmost importance* of the dependence of $C^{(2)}(\mathbf{K},\mathbf{K})$, $C^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$, and $r^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K})$ on pions' mean momenta, for both the small and the asymptotically large values of momenta.

The principal feature of our models is that these are based on the assumption of *complete chaoticity* of the emitting sources, in contrast with other approaches to the description of two- and three-pion correlations by means of the effective parameters, such as *partial non-chaoticity* (to which the intercepts are related) and a core-halo fraction [6].

2. q-DEFORMED AND q,p-DEFORMED OSCILLATORS

We first give a necessary setup on the two-parameter qp-deformed oscillator and its best known particular cases of q-oscillators. Set of (independent modes of) q,p-deformed oscillators obey the relations [7]

$$A_iA_j^\dagger - q^{\delta_{ij}}A_j^\dagger A_i = \delta_{ij}p^{N_i^{(qp)}}, \qquad A_iA_j^\dagger - p^{\delta_{ij}}A_j^\dagger A_i = \delta_{ij}q^{N_i^{(qp)}}, \qquad (1)$$

and those with $N_i^{(qp)}$. Besides, with X a number or an operator,

$$A_i^\dagger A_i = [\![N_i^{(qp)}]\!]_{qp}\ , \qquad [\![X]\!]_{qp} \equiv (q^X - p^X)/(q-p)\ . \qquad (2)$$

For an operator X, the q,p-bracket $[\![X]\!]_{qp}$ is meant as formal series.

The $p=1$ case yields the AC-type q-oscillators with the relations [8]

$$a_ia_j^\dagger - q^{\delta_{ij}}a_j^\dagger a_i = \delta_{ij}\ , \qquad [a_i,a_j] = [a_i^\dagger, a_j^\dagger] = 0\ , \qquad (3)$$

and those for $\mathcal{N}_i$. Here $-1 \le q \le 1$; differing modes are independent. Vectors $|n_1, \ldots, n_i, \ldots\rangle$ are got from $|0, 0, \ldots\rangle$ as usual, and $a_i^\dagger$ act as

$$\langle \ldots, n_i+1, \ldots | a_i^\dagger | \ldots, n_i, \ldots \rangle = \sqrt{\lfloor n_i+1 \rfloor}$$

where for the AC case the "basic numbers" $\lfloor r \rfloor \equiv (1-q^r)/(1-q)$, the $p=1$ limit of $[\![r]\!]_{qp}$, appear. As $q \to 1$, the $\lfloor r \rfloor$ resp. $\lfloor A \rfloor$ yields r resp. A. For $-1 \le q \le 1$ the operators $a_i^\dagger$, a_i are mutual conjugates. Note that $a_i^\dagger a_i$ depends on *the number operator* $\mathcal{N}_i$ *nonlinearly* (compare with (2)):

$$a_i^\dagger a_i = \lfloor \mathcal{N}_i \rfloor \ . \tag{4}$$

Only at $q = 1$ the familiar equality $a_i^\dagger a_i = \mathcal{N}_i$ is recovered.

At $p = q^{-1}$, eq. (1) yields the q-oscillators of BM type [9] such that

$$b_i b_j^\dagger - q^{\delta_{ij}} b_j^\dagger b_i = \delta_{ij} q^{-N_j} \ , \qquad [b_i, b_j] = [b_i^\dagger, b_j^\dagger] = 0 \ . \tag{5}$$

The q-Fock space is constructed likewise; now, instead of basic numbers, the "q-bracket" and "q-numbers" are used:

$$b_i^\dagger b_i = [N_i]_q \ , \qquad [r]_q \equiv (q^r - q^{-r})/(q - q^{-1}) \ . \tag{6}$$

At $q = 1$, equality $b_i^\dagger b_i = N_i$ is recovered. For the BM case (5) we set

$$q = \exp(i\theta) \ , \qquad 0 \le \theta < \pi/2 \ . \tag{7}$$

3. STATISTICAL q-DEFORMED DISTRIBUTIONS

For the dynamical multi-particle (multi-pion, multi-kaon, ...) system, we adopt the model of ideal gas of q- or q,p-bosons. The Hamiltonian is

$$H = \sum \omega_i \mathcal{N}_i \ , \qquad \omega_i = \sqrt{m^2 + \mathbf{K}_i^2} \ , \tag{8}$$

where $\mathcal{N}_i$ is one of the above three versions of the number operator and $'i'$ labels different modes. The (unique, non-interacting) choice (8) of Hamiltonian implies additive spectrum [10]. The 3-momenta of particles take discrete values (the system is in a large finite box of volume $\sim L^3$). To evaluate thermal averages (with $\beta = 1/T$, Boltzmann constant $k = 1$), use

$$\langle A \rangle = \mathrm{Sp}(A\rho)/\mathrm{Sp}(\rho) \ , \qquad \rho = e^{-\beta H}.$$

For the AC q-bosons, the q-distribution is found as [10]:

$$\langle a_i^\dagger a_i \rangle = \left(e^{\beta\omega_i} - q\right)^{-1} \ . \tag{9}$$

This is the usual Bose-Einstein one if $q \to 1$. At $q = -1$ ($q = 0$) the q-distribution (9) reduces, formally, to Fermi-Dirac (Boltzmann) case.

For BM-type q-bosons, the Hamiltonian again is (8), now with N_i. We get $\langle q^{\pm N_i} \rangle = (e^{\beta\omega_i} - 1)/(e^{\beta\omega_i} - q^{\pm 1})$. The q-distribution is found [1, 10] as

$$\langle b_i^\dagger b_i \rangle = \frac{e^{\beta\omega_i} - 1}{e^{2\beta\omega_i} - 2\cos\theta \; e^{\beta\omega_i} + 1} \tag{10}$$

which is real: $q + q^{-1} = [2]_q = 2\cos\theta$. It is such that at $q \neq 1$ the function $f(\mathbf{K}) \equiv \langle b^\dagger b \rangle(\mathbf{K})$ in (10) lies in between the Bose-Einstein curve and the classical Boltzmann one. The same is true of the AC q-distribution (9).

The one-particle *momentum space distribution* of q, p-bosons is

$$\langle A_i^\dagger A_i \rangle = (e^{\beta\omega_i} - 1)/\{(e^{\beta\omega_i} - p)(e^{\beta\omega_i} - q)\} \; , \tag{11}$$

see [11]. Clearly, (9) or (10) are its $p = 1$ or $p = q^{-1}$ particular cases.

4. n-PARTICLE CORRELATIONS OF qp-BOSONS

Most general result for the q,p-Bose gas model (of q,p-oscillators) is given in [5]. With $H = \sum \omega_i N_i^{(qp)}$, the n-particle monomode distribution

$$\langle (A_i^\dagger)^n (A_i)^n \rangle = \frac{[\![n]\!]_{qp}! \; (e^{\beta\omega_i} - 1)}{\prod_{r=o}^{n} (e^{\beta\omega_i} - q^r p^{n-r})} \; , \tag{12}$$

$$[\![m]\!]_{qp}! = [\![1]\!]_{qp} [\![2]\!]_{qp} \cdots [\![m-1]\!]_{qp} [\![m]\!]_{qp} \; ,$$

results in the n-th order intercept $\lambda_{q,p}^{(n)} \equiv \frac{\langle A^{\dagger n} A^n \rangle}{\langle A^\dagger A \rangle^n} - 1$ (omitting 'i'):

$$\lambda_{q,p}^{(n)} = [\![n]\!]_{qp}! \; \frac{(e^{\beta\omega} - p)^n (e^{\beta\omega} - q)^n}{(e^{\beta\omega} - 1)^{n-1} \prod_{k=0}^{n} (e^{\beta\omega} - q^{n-k} p^k)} - 1 \; . \tag{13}$$

This provides generalization both to the n-th order of correlations and to the two-parameter (q,p-)deformation [5].

If $\beta\omega \to \infty$ (*asymptotics of large momenta*), the intercepts $\lambda_{q,p}^{(n)}$ simplify:

$$\lambda_{q,p}^{(n),\text{ asympt}} = -1 + [\![n]\!]_{qp}! = -1 + \prod_{k=1}^{n-1} \left(\sum_{r=0}^{k} q^r p^{k-r} \right) . \tag{14}$$

For each case (AC, BM, and q,p-generalization) the asymptotics of n-th order intercept is nothing but the corresponding deformation of n-factorial (the usual $n!$ yields the intercept of pure Bose-Einstein n-particle correlation).

For AC case, set $p = 1$ in the n-particle monomode distribution (12) and get the intercept $\lambda^{(n)} \equiv \frac{\langle a^{\dagger n}\ a^n\rangle}{\langle a^\dagger a\rangle^n} - 1$ of n-particle correlation function:

$$\lambda_{\rm AC}^{(n)} = -1 + \frac{\lfloor n\rfloor!\ (e^{\beta\omega} - q)^{n-1}}{\prod_{r=2}^{n}(e^{\beta\omega} - q^r)}\ . \tag{15}$$

Asymptotically at $\beta\omega \to \infty$ the $\lambda_{\rm AC}^{(n)}$ depends on the q-parameter only, i.e.

$$\lambda_{\rm AC}^{(n),\ \rm asympt.} = -1 + \lfloor n\rfloor! = -1 + \prod_{k=1}^{n-1}\left(\sum_{r=0}^{k} q^r\right)\ . \tag{16}$$

This is remarkable, and should be tested using numerical data for pions' and kaons' intercepts extracted in the experiments on heavy ion collisions.

5. TWO- AND THREE-PARTICLE (q-BOSON) CORRELATIONS

AC-type q-bosons. From (15), the intercepts of order 2 and 3 read:

$$\lambda_{\rm AC}^{(2)} \equiv \frac{\langle a^\dagger a^\dagger aa\rangle}{\langle a^\dagger a\rangle^2} - 1 = -1 + \frac{(1+q)(e^{\beta\omega} - q)}{e^{\beta\omega} - q^2} = q\frac{e^{\beta\omega} - 1}{e^{\beta\omega} - q^2}\ , \tag{17}$$

$$\lambda_{\rm AC}^{(3)} \equiv \frac{\langle a^{\dagger 3} a^3\rangle}{\langle a^\dagger a\rangle^3} - 1 = -1 + \frac{(1+q)(1+q+q^2)(e^{\beta\omega} - q)^2}{(e^{\beta\omega} - q^2)(e^{\beta\omega} - q^3)}\ , \tag{18}$$

with respective $\beta\omega \to \infty$ asymptotics (compare with (16)):

$$\lambda_{\rm AC}^{(2),\ asympt.} = q\ , \qquad \lambda_{\rm AC}^{(3),\ asympt.} = q(2 + 2q + q^2)\ . \tag{19}$$

BM-type q-bosons. The intercepts of 2nd and 3rd order correlations are

$$\lambda_{\rm BM}^{(2)} = -1 + \frac{\langle b^\dagger b^\dagger bb\rangle}{\langle b^\dagger b\rangle^2} = -1 + \frac{2\cos\theta(t + 1 - \cos\theta)^2}{t^2 + 2(1 - \cos^2\theta)t}\ , \tag{20}$$

$$\lambda_{\rm BM}^{(3)} = -1 + \frac{[2]_q[3]_q\ (e^{2\beta\omega} - 2e^{\beta\omega}\cos\theta + 1)^2}{(e^{\beta\omega} - 1)^2(e^{2\beta\omega} - 2e^{\beta\omega}\cos(3\theta) + 1)} \tag{21}$$

where $t \equiv \cosh(\beta\omega) - 1$. Asymptotics $\beta\omega \to \infty$ looks as

$$\lambda_{\rm BM}^{(2),\ asympt.} = -1 + [2]_q = -1 + 2\cos\theta\ , \tag{22}$$

$$\lambda_{\rm BM}^{(3),\ \rm asympt.} = -1 + [2]_q[3]_q = -1 + 2\cos\theta\ (4\cos^2\theta - 1)\ . \tag{23}$$

6. COMPARISON WITH EXPERIMENTAL DATA

The properties of the two-particle correlation intercepts (17) and (20) of AC and BM q-bosons were analyzed in detail in [1, 2]. Moreover, in [3] it was shown that the intercept $\lambda^{(2)}_{\rm AC}$ with fixed $q = 0.63$ ($\lambda^{(2)}_{\rm BM}$ with fixed $\theta = 28.5°$) nicely fit the experimental data for positive (negative) pions.

Here in Fig. 1, the shape of the intercepts $\lambda^{(3)}_{\rm AC}$, $\lambda^{(3)}_{\rm BM}$ from (18), (21) is shown. For large $|\mathbf{K}|$, the asymptotical saturation given by the q (AC case (19)) or by the θ (BM case (23)) is manifest, at any fixed temperature.

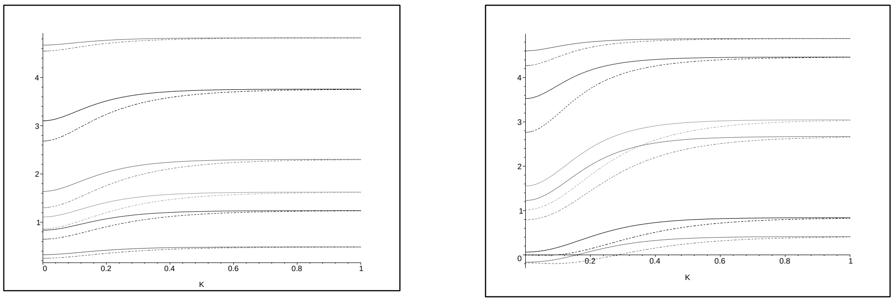

Figure 1: Shape of $\lambda^{(3)}_{\rm AC}$ (left) and $\lambda^{(3)}_{\rm BM}$ (right) versus pions' mean momentum $|\mathbf{K}|$ (GeV/c). Of each two merging curves, $T\!=\!120$ MeV for the upper and $T\!=\!180$ MeV for the lower. The values $q\!=\!0.98, 0.8, 0.63, 0.5, 0.4142, 0.2$ label pairs of AC-curves (from top down), and $\theta\!=\!\pi/30, \pi/14, \pi/7, \pi/6, \pi/5, \pi/4$ label pairs of BM-curves (from top down).

Now confront the q,p-Bose gas (q,p real) correlation intercepts of order 2 and 3, i.e. $n = 2, 3$ in (13), with available data on pions [12]. If we equate $\lambda^{(2)}_{qp}$ to 0.46, $\lambda^{(3)}_{q,p}$ to 4.35 (note 4.35 is unrealistic for pions), we get the two distant surfaces in Fig. 2 (left). However, equating the intercepts to (central values of) the NA44 data [12] $\lambda^{(2),exp.} = 0.57{\pm}0.04$ and $\lambda^{(3),exp.} = 1.92{\pm}0.49$, we arrive at Fig. 2 (right), where the resulting two surfaces almost coincide. Thus, we see that for a whole range of $w \equiv \beta\omega$ the q,p-Bose gas model, due to q and p, *is able to jointly describe* the data on $\lambda^{(2),exp.}$ and $\lambda^{(3),exp.}$.

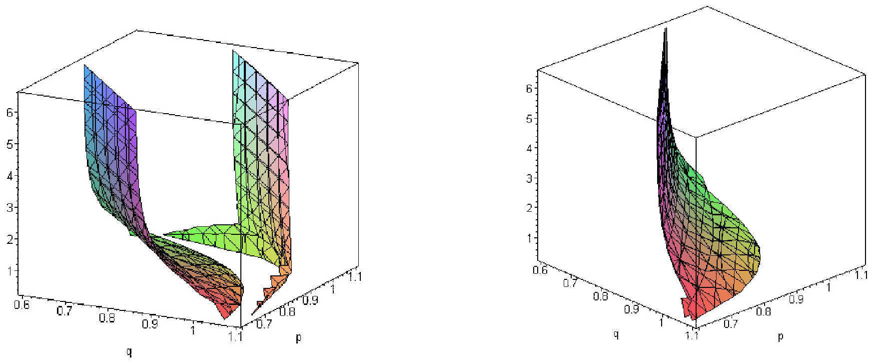

Figure 2: The functions $w \equiv \beta\omega$ of variables q, p, given implicitly by equating $\lambda^{(2)}_{p,q} = 0.46$, $\lambda^{(3)}_{pq} = 4.35$ (left) and by $\lambda^{(2)}_{p,q} = 0.57$, $\lambda^{(3)}_{pq} = 1.92$ (right).

The order 2 and 3 correlations are involved in the expression [4]

$$r^{(3)}(p_1,p_2,p_3) \equiv \frac{C^{(3)}(p_1,p_2,p_3) - C^{(2)}(p_1,p_2) - C^{(2)}(p_2,p_3) - C^{(2)}(p_3,p_1) + 2}{2\sqrt{(C^{(2)}(p_1,p_2)-1)\,(C^{(2)}(p_2,p_3)-1)\,(C^{(2)}(p_3,p_1)-1)}}$$

and (set $\mathbf{p}_1 = \mathbf{p}_2 = \mathbf{p}_3 = \mathbf{K}$) in the quantity

$$r_0(\mathbf{K}) \equiv r^{(3)}(\mathbf{K},\mathbf{K},\mathbf{K}) = \frac{1}{2}\,\frac{\lambda^{(3)}(\mathbf{K}) - 3\lambda^{(2)}(\mathbf{K})}{\left(\lambda^{(2)}(\mathbf{K})\right)^{3/2}} \tag{24}$$

made of intercepts $\lambda^{(3)}(\mathbf{K}) \equiv C^{(3)}(\mathbf{K}) - 1$ and $\lambda^{(2)}(\mathbf{K}) \equiv C^{(2)}(\mathbf{K}) - 1$.

Fig. 3 in its left (right) panel shows the behavior of $r_0(\mathbf{K})$ with the AC (BM) type intercepts. As seen, very peculiar is the behaviour of $r_0(\mathbf{K})$ *for small mean momenta* in the BM case (right). It would be extremely interesting to reveal such a behavior in the future experiments, especially, possible negative (or close to zero) values of $r_0(\mathbf{K})$ *at small* $|\mathbf{K}|$. Present data [12] on the r_0 values can be placed on our curves, but it is unclear to which values of the momentum $|\mathbf{K}|$ those $r_0(\mathbf{K})$ actually correspond.

To conclude, we believe the q,p-Bose gas model is capable to describe the peculiar features of 2- and 3-pion correlations in relativistic heavy ion collisions. To draw further conclusions about adequacy of the model, about physical meaning and values of the parameters q, p, more data are needed.

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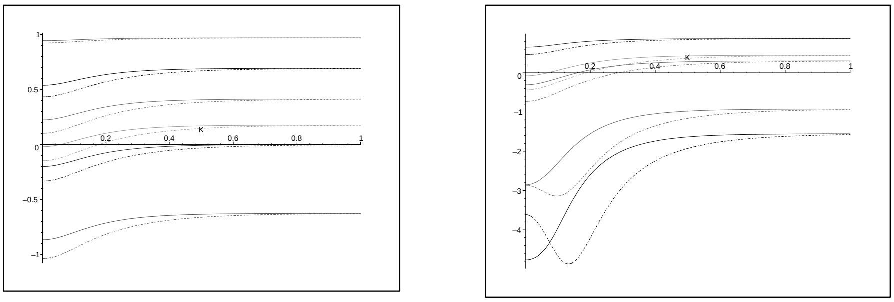

Figure 3: Shape of $r_0(\mathbf{K})$ versus pions' mean momentum $|\mathbf{K}|$ (GeV/c) for the AC (left) and BM (right) cases of intercepts $\lambda^{(2)}$, $\lambda^{(3)}$ in (24). Values of the temperature and of the parameter q or θ are like in Fig. 1.

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