FACTA UNIVERSITATIS
Series: **Physics, Chemistry and Technology** Vol. 4, N^o 2, 2006, pp 287 -301

A TOY MODEL FOR THE LANDSCAPE

Emilian Dudas [1,2]

[1] CPhT, Ecole Polytechnique-CPHT, 91128 Palaiseau Cedex, France.
[2] LPT , Bat. 210, Univ. de Paris-Sud, F-91405 Orsay, France

Abstract. *Motivated by recent discussions of the string-theory landscape, we propose field-theoretic realizations of models with large numbers of vacua. These models contain multiple $U(1)$ gauge groups, and can be interpreted as deconstructed versions of higher-dimensional gauge theory models with fluxes in the compact space. We find that the vacuum structure of these models is very rich, defined by parameter-space regions with different classes of stable vacua separated by boundaries. We find that this landscape picture evolves with energy, allowing vacua to undergo phase transitions as they cross the boundaries between different regions in the landscape.*

Key words: *Landscape*

1. INTRODUCTION

Recent developments in the study of string-theory compactifications suggest that there exist huge numbers of string vacua, with different cosmological constants and different low-energy phenomenological properties [1]. The resulting picture, dubbed the "landscape" [2], has stimulated a statistical analysis of the number of string vacua, the supersymmetry-breaking scale, and other phenomenological features. Anthropic principles have even been advanced to resolve difficult issues such as the cosmological constant problem [3].

One of the problems facing this landscape picture of string theory is that of calculating physical parameters. This is, to a large extent, due to the limited technology for performing string calculations in the relevant flux vacua. It is therefore useful to present field-theoretic counterparts of such constructions, *i.e.*, field-theoretic models which naturally give rise to very

Received: 20 November 2005

large numbers of vacua, and to be able to quantitatively determine statistical distributions of relevant physical quantities such as the cosmological constant, the scale of supersymmetry breaking, the Higgs mass, gauge and Yukawa couplings, and the like.

The purpose of this talk is to present such field-theoretic examples based on multiple Abelian gauge groups and multiple charged scalar fields. As we shall see, such models naturally lead to large numbers of vacua, and allow us to quantitatively address many of the pressing questions that such pictures raise.

2 Constructing a toy model.

We shall now turn to a simple model which, as we shall see, will exhibit almost all of the relevant features which we shall encounter when we proceed to consider more complicated situations in subsequent sections. In particular, we shall see that this toy model gives rise to a non-trivial "landscape" consisting of multiple stable (or metastable) vacua with different low-energy phenomenologies, unstable extrema, phase transitions, etc. Moreover, even though this toy model is relatively simple, we expect that the resulting landscape is literally a component of the full string-theory landscape in cases of string models with multiple $U(1)$ gauge factors.

Our model consists of two $U(1)$ gauge symmetries, denoted $U(1)_1$ and $U(1)_2$, and three $\mathcal{N} = 1$ chiral superfields, $\Phi_{i=1,2,3}$. The charges of these chiral superfields under the $U(1)$ gauge symmetries are chosen as in Table 1. In a string-theory context, such $U(1)$ gauge factors can be imagined as arising from different D-branes, and the Φ_i fields can arise as strings stretched between these branes. We shall also assume that the $\mathcal{N} = 1$ supersymmetry is broken by Fayet-Iliopoulos D-term coefficients ξ_1 and ξ_2, and by a renormalizable Wilson-line superpotential of the form

$$W \;=\; \lambda\, \Phi_1 \Phi_2 \Phi_3 \;. \tag{1}$$

Our model is thus defined by three parameters, $\{\xi_1, \xi_2, \lambda\}$, and our goal will be to study the vacuum structure of this model as a function of these parameters. Of course, the resulting physics is unchanged if $\lambda \to -\lambda$. We shall therefore restrict ourselves to $\lambda \geq 0$ for simplicity.

It is straightforward to analyze the vacuum states of this theory. As usual, the scalar potential is given by

$$V \;=\; \frac{1}{2} \sum_a g_a^2 D_a^2 \;+\; \sum_i |F_i|^2 \tag{2}$$

	$U(1)_1$	$U(1)_2$
Φ_1	-1	0
Φ_2	1	-1
Φ_3	0	1

Table 1: $U(1)$ charge assignments for chiral superfields in our toy model.

where the D- and F-terms are given by

$$D_a \;=\; \sum_i q_i^{(a)} |\phi_i|^2 \;+\; \xi_a \;, \qquad F_i \;=\; \frac{\partial W}{\partial \phi_i} \;. \tag{3}$$

Here $a = 1, 2$ is the index of the gauge group $U(1)$ factor, g_a is the gauge coupling corresponding to the $U(1)_a$ factor, and $i = 1, 2, 3$ is the index of the chiral superfield. Thus, $q_i^{(a)}$ denotes the $U(1)_a$ charge of Φ_i. In most of our considerations the gauge couplings g_a will not be important, so we will henceforth consider $g_1 = g_2 = 1$ for simplicity. We will reinstall gauge couplings whenever relevant for our analysis. Our task is to determine the extrema of V by seeking solutions to the simultaneous equations

$$\frac{\partial V}{\partial \phi_i} \;=\; \frac{\partial V}{\partial \phi_i^*} \;=\; 0 \;, \tag{4}$$

and then to determine whether these extrema represent stable (or metastable) vacua by calculating the eigenvalues of the corresponding mass matrix

$$\mathcal{M}^2 \;\equiv\; \begin{pmatrix} \dfrac{\partial^2 V}{\partial \phi_i^* \partial \phi_j} & \dfrac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} \\ \dfrac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \dfrac{\partial^2 V}{\partial \phi_i \partial \phi_j^*} \end{pmatrix} \tag{5}$$

evaluated at the extrema. Note that in general, there will be a zero eigenvalue for each spontaneously broken $U(1)$; these eigenvalues correspond to the resulting Nambu-Goldstone bosons. The extrema defined by Eq. (4) represent stable (or metastable) vacua only if each of the remaining eigenvalues is positive.

It will prove convenient to group the extrema of V into classes depending on which combinations of chiral superfields receive non-zero vacuum expectation values. This classification will help in determining such features as whether the sources of SUSY-breaking are primarily D-terms or F-terms, and whether they are likely to lead to R-symmetry breaking when incorporated into a supergravity framework. Since there are three chiral superfields in this example, there are correspondingly $8 = 2^3$ classes of extrema of V. Denoting $\langle \Phi_i \rangle \equiv v_i$, we shall define our classes of extrema according to their values of v_i, using the notation {ijk...} to indicate the class of vacua in which v_i, v_j, v_k, ... are all non-zero (with $\{0\}$ denoting the vacua in which all v_i vanish). Note that in this toy model, we can choose all v_i to be real without loss of generality.

2.1 The case $\lambda < 1$

We consider here only the case $\lambda < 1$, whereas the general case can be found in [4]. We find that class $\{0\}$ extrema exist for all (ξ_1, ξ_2) and continue to be unstable everywhere. Class $\{1\}$ extrema, by contrast, exist for all $\xi_1 > 0$, but are now stable only within the smaller region $|\xi_2| < \lambda^2 \xi_1$. Similarly, Class $\{2\}$ extrema exist for all $\xi_2 > \xi_1$ and are stable only within the region defined by

$$(1 - \lambda^2)\, \xi_2 \;<\; -(\lambda^2 + 1)\, \xi_1 \;, \qquad (\lambda^2 + 1)\, \xi_2 \;>\; (\lambda^2 - 1)\, \xi_1 \;, \tag{6}$$

while Class $\{3\}$ extrema exist for all $\xi_2 < 0$ but are stable only within the smaller region $|\xi_1| < \lambda^2 |\xi_2|$. All of these results reduce to the previous case as $\lambda \to 1$. In general, these results indicate that the $\{1\}$, $\{2\}$, and $\{3\}$ regions become smaller as $\lambda \to 0$, occupying narrower and narrower "pie-slices" in the (ξ_1, ξ_2) landscape plane. Specifically, each of these "pie-slices" has total angle $\theta_{\{1\},\{2\},\{3\}} = 2\theta_\lambda$, where

$$\theta_\lambda \;\equiv\; \tan^{-1} \lambda^2 \;, \tag{7}$$

and differ only in their orientations in the (ξ_1, ξ_2) plane, with the Class $\{1\}$ pie-slice centered around the positive ξ_1-axis, the Class $\{2\}$ pie-slice centered around the $\xi_2 = -\xi_1 > 0$ diagonal axis, and the Class $\{3\}$ pie-slice centered around the negative ξ_2-axis. Thus, as $\lambda \to 0$, the $\{1\}$, $\{2\}$, and $\{3\}$ regions disappear entirely.

Just as in the $\lambda = 1$ case, extrema in Classes $\{12\}$ and $\{23\}$ continue to exist in the gaps between the $\{1\}$ and $\{2\}$ regions, and $\{2\}$ and $\{3\}$ regions, respectively. Each of these regions has angle $\theta_{\{12\},\{23\}} = 3\pi/4 - 2\theta_\lambda$. Moreover, these extrema are stable everywhere within these regions. The new feature for $\lambda < 1$ compared to $\lambda = 1$ is the emergence of new Class $\{13\}$ extrema which populate the gap that has opened up between the $\{1\}$ and $\{3\}$ regions, with angle $\theta_{\{13\}} = \pi/2 - 2\theta_\lambda$. Just as with the extrema in Classes $\{12\}$ and $\{23\}$, extrema in Class $\{13\}$ are stable wherever they exist.

2.2 Renormalization-group flow and boundary crossings

One of the interesting features of the vacuum structure we are seeing in this toy model is the fact that all of the boundaries between different vacuum regions are actually energy-dependent (or temperature-dependent in the early universe). Therefore, it is possible that a vacuum can cross a boundary between regions as the result of renormalization group evolution, either because the landscape location of the vacuum changes, because the boundary changes, or both.

In order to understand this observation, let us now analyze the evolution of our toy model under renormalization-group (RG) flow. As we have seen, this model actually contains several quantities which are potentially renormalized: these include the FI coefficients ξ_i and the Yukawa coupling λ. However, we must remember that our toy model also depends on the $U(1)_i$ gauge couplings g_i that were implicitly dropped from Eq. (2). In this model, the RGEs for the two gauge couplings g_1, g_2, and the Yukawa coupling λ are given by

$$\begin{aligned} \mu\frac{d}{d\mu}g_i &= \frac{g_i^3}{16\pi^2}\mathrm{Tr}\,Q_i^2 , \\ \mu\frac{d}{d\mu}\lambda &= \frac{\lambda}{16\pi^2}\left(3\lambda^2-4g_1^2-4g_2^2\right) , \end{aligned} \tag{8}$$

where Tr Q_i^2 is the sum of (squared) charges under $U(1)_i$.

We will assume for simplicity that the entire matter content of our theory consists of the three chiral superfields Φ_1, Φ_2, and Φ_3, and that the FI terms (ξ_1, ξ_2) are introduced at tree level. We therefore find that $\mathrm{Tr}\,Q_1^2 = \mathrm{Tr}\,Q_2^2 = 2$. Moreover, since $\mathrm{Tr}\,Q_1 = \mathrm{Tr}\,Q_2 = 0$ in this case, the one-loop induced FI coefficients are zero, and therefore the tree-level FI coefficients (ξ_1, ξ_2) are not renormalized. Thus, we see that in this toy model, the location of an individual vacuum in the landscape is invariant, and the only changes that can occur are those which change the topography of the landscape itself (moving boundaries, growing or shrinking regions, *etc.*) with respect to that fixed location.

Let us consider initial boundary values $g_i^2(\Lambda) = g_{i,0}^2$ and $\lambda^2(\Lambda) = \lambda_0^2$, where Λ is some initial reference ultraviolet (UV) scale. In order to make this calculation tractable, let us further assume that our gauge couplings are originally equal at the UV scale: $g_{1,0} = g_{2,0} \equiv g_0$. We therefore find from Eq. (8) that our gauge couplings remain equal at all subsequent energy scales: $g_1(\mu) = g_2(\mu) \equiv g(\mu)$.

The question that we wish to pursue, then, is that of determining the evolution of the entire landscape structure as a function of the energy scale μ. We shall do this in several steps.

First, since $g_1(\mu) = g_2(\mu) \equiv g(\mu)$, we see that we can easily restore the gauge couplings to all of our previous landscape calculations simply by rescaling $\lambda(\mu) \to \lambda(\mu)/g(\mu)$. Of course, such a universal rescaling of λ would not be possible if we did not assume that $g_{1,0} = g_{2,0}$.

If we wish to understand the RG flow in the landscape, we simply need to determine the flow of the single quantity λ/g which parametrizes which landscape sketch is appropriate at which energy scale. Our original location on the landscape doesn't change with energy, but we simply have to look at the correct figure corresponding to the appropriate value of λ/g.

For example, when $g_1(\mu) = g_2(\mu) \equiv g(\mu)$, the boundaries demarcating the $\{1\} + \{3\}$ overlap region for $Y(\mu) > 1$ can be specified directly in terms of $Y(\mu)$:

$$-Y^2(\mu)\,\xi_1 \;<\; \xi_2 \;<\; -\frac{1}{Y^2(\mu)}\,\xi_1 \;. \tag{9}$$

Given Eq. (8), it is relatively straightforward to determine the RG equation for $Y(\mu)$. We find

$$\mu\frac{d}{d\mu}Y(\mu) \;=\; \frac{Y}{16\pi^2}\,(3Y^2 - 10)\,g^2(\mu) \;, \tag{10}$$

and substituting the explicit solution for $g^2(\mu)$ from Eq. (8), we can integrate Eq. (10) to obtain the solution

$$3 - \frac{10}{Y^2(\mu)} \;=\; \left(3 - \frac{10}{Y_0^2}\right)\left[\frac{g(\mu)}{g_0}\right]^{10} \;. \tag{11}$$

Thus, we see that quantity $3 - 10/Y^2$ scales according to the ratio of gauge couplings $[g(\mu)/g_0]^{10}$. Regardless of the initial value Y_0, our theory always flows towards an *infrared fixed point*

$$\overline{Y} \;\equiv\; \sqrt{\frac{10}{3}} \;\approx\; 1.826 \;. \tag{12}$$

Note that a generic feature of the boundaries separating two different vacuum stability regions is the presence of a massless scalar in the spectrum. Thus, crossing a boundary is in some sense equivalent to a phase transition, with the appearance of long-range order as the new massless state appears in the spectrum.

Note that in general, there are only two generic classes of boundaries that appear in our landscape diagrams for this toy model:

- Boundaries separating single-vev regions, such as $\{1\}$, and two-vev regions, such as $\{12\}$, in which one of the two vev's is the same as that in the single-vev region.
- Boundaries separating single-vev regions, such as $\{1\}$, and overlap regions, such as $\{1\} + \{2\}$, in which one of the overlapping vacua is the same as the vacuum in the single-vev region. Note that near the boundary, it is the common vacuum that has the lower vacuum energy V in the overlap region, while the other vacuum in the overlap region is only metastable.

It is easy to verify that the vacuum energy V is continuous across the first class of boundaries, while for the second class, the stable and the metastable vacua are not degenerate in energy on the boundary.

Given these classes of boundaries, let us therefore consider the kinds of phase transitions which can result as a consequence of RG flow in our toy model.

If $Y_0 > \overline{Y}$, then we find ourselves in a landscape containing only single-vev and overlap regions. Thus, we have only boundaries of the second type. Moreover, since RG flow pushes us towards landscapes with smaller values of Y, we can only have situations in which our overlap regions are getting smaller rather than larger. Thus, depending on our original (fixed) landscape location, the only type of boundary crossing that may occur in this case is one in which our location changes from being within an overlap region to within a single-vev region.

The analysis is slightly more complicated for $Y_0 < \overline{Y}$. If $Y_0 < 1$, then our original landscape has only one-vev and two-vev regions, with the two-vev regions shrinking as a result of the RG flow towards larger Y-values. If we are originally located in a two-vev region of this landscape, then we will necessarily eventually experience a second-order phase transition into a one-vev vacuum as a result of RG flow. However, if $1 < Y_0 < Y_*$, then our landscape consists of a mixture of one-vev, two-vev, and overlap regions. Two different types of transitions are possible: either we can be located in a two-vev region and experience a second-order phase transition into a one-vev region, as described above, or we can be originally located in a single-vev region next to a growing overlap region. In the latter case, our vacuum state in the single-vev region continues to be the truly stable vacuum state in the overlap region, so there is no phase transition. Finally, if $Y_* < Y_0 < \overline{Y}$, we find ourselves in a landscape in which there are only single-vev regions and overlap regions, with the overlap regions growing as a result of RG flow. In such a case, as above, no phase transitions are possible: if we pass from a single-vev region into an overlap region, our original vacuum state continues to be the truly stable vacuum state in the overlap region, and no phase transition occurs.

One important consequence of the infrared fixed-point behavior towards $Y = \overline{Y} \approx 1.826$ is that *our theory always flows in the infrared to one in which R-symmetry is preserved*. This observation is true in our toy model regardless of the original landscape location or Yukawa/gauge couplings.

Indeed, the emergence of an infrared fixed point has an even more significant consequence: in such cases, the low-energy phenomenology becomes *insensitive* to the plethora of (ultimately string-theoretic) variables that define the ultraviolet landscape physics. If such infrared fixed points are generic features of the string-theoretic landscapes, their existence suggests that it may not be necessary to understand the full ultraviolet string theory in order to extract physically testable predictions from the landscape.

The RG evolution of the gauge couplings must clearly stop below the scale of gauge symmetry breaking; likewise, the beta-function coefficients depend on the scale of supersymmetry breaking in the sense that the matter spectrum is supersymmetric above this scale and non-supersymmetric below it. As already emphased, we stress that the possible vacuum-structure phase transitions that we have discussed in this section must be understood in terms of temperature phase transitions in the early universe.

3 Adding more U(1) gauge groups

In this section we shall consider generalizing the model in Sect. 3 by adding more $U(1)$ gauge groups. This will significantly increase the number of vacua and the complexity of the corresponding landscape. More importantly, since the Wilson-line superpotential can in principle contain more fields, we see from dimensional analysis that the F-terms will generically be suppressed. Thus, R-symmetry will be tend to be broken only at very low energies and only for relatively few vacua.

Let us consider a generalization of the two-$U(1)$ model of Sect. 3 to the case of n different $U(1)$ gauge group factors, with $n+1$ chiral superfields. Inspired by deconstruction models of extra dimensions, we shall take our charge assignments to follow the pattern indicated in Table 2. Thus, as evident from Table 2, we are only considering bi-fundamental "nearest-neighbor" charges; other configurations will be briefly discussed in Sect. 6.

	$U(1)_1$	$U(1)_2$	$U(1)_3$	...	$U(1)_n$
Φ_1	-1				
Φ_2	1	-1			
Φ_3		1	-1		
$\vdots$			$\ddots$	$\ddots$	
Φ_n				1	-1
Φ_{n+1}					1

Table 2: $U(1)$ charge assignments for chiral superfields, inspired by "deconstruction" models of extra dimensions.

Likewise, we shall assume for simplicity that only the boundary $U(1)$ gauge-group factors, *i.e.*, $U(1)_1$ and $U(1)_n$, have Fayet-Iliopoulos coefficients ξ_1 and ξ_n respectively. Given these charges, we can in general write down a Wilson-line superpotential of the form[1]

$$W = \lambda \prod_{i=1}^{n+1} \Phi_i \ . \tag{13}$$

We can continue to study the landscape of this model as a function of the Fayet-Iliopoulos coefficients (ξ_1, ξ_n) for arbitrary values of the Yukawa parameter λ.

3.1 Arbitrary n and large-n limit

We now turn our attention to the general-n case, with particular interest in the large-n limit. In the general case, the D- and F-terms are now given by

$$D_a = \sum_{i=1}^{n+1} q_i^{(a)} |\phi_i|^2 + \xi_1 \delta_{a1} + \xi_n \delta_{an} \ , \qquad F_i = \frac{\partial W}{\partial \phi_i} \ , \tag{14}$$

where $a = 1, \ldots, n$. As we already remarked for the $n = 3$ case, the F-term contributions coming from the Wilson line superpotential (13) in the large-n case have a negligible effect on the vacuum structure for FI terms smaller than the Planck (or string) scale. The field equations each have two solutions: $v_k \equiv \langle \phi_k \rangle = 0$, and a solution obtained by setting the term in parentheses to zero. This gives a large number $\sim \mathcal{O}(2^{n+1})$ of different

[1] This implies that the Φ_i superfields have R-charge $2/(n+1)$, giving the F-terms R-charge $-2n/(n+1)$. Thus, as claimed earlier, F-term breaking will correspond to R-symmetry breaking in this model.

extrema which were explicitly analyzed in Sect. 3 for $n = 2$ and Sect. 4.1 for $n = 3$.

Since $\lambda/M_P^{n-2} \to 0$, we expect that the stable vacua are dominated by the three classes $\{1, n+1\}$, $\{1, ..., n\}$, and $\{2, ..., n+1\}$. Specifically, we find that the $\{1, n+1\}$ solution exists for $\xi_1 > 0$, $\xi_n < 0$, and is given by

$$\{1, n+1\} : \qquad |v_1|^2 = \xi_1 \ , \quad |v_{n+1}|^2 = -\xi_n \ , \quad v_2 = ... = v_n = 0 \ . \tag{15}$$

Similarly, the $\{1, ..., n\}$ solution exists for $\xi_n > 0$, $\xi_1 + \xi_n > 0$, and is given by

$$\begin{aligned} \{1, ..., n\} : \qquad & |v_1|^2 = (\xi_1 + \xi_n)(1 + \mathcal{O}(\epsilon_n^2)) \\ \{1, ..., n\} : \qquad & |v_2|^2, ..., |v_n|^2 = \xi_n(1 + \mathcal{O}(\epsilon_n^2)) \ , \quad v_{n+1} = 0 \end{aligned} \tag{16}$$

where $\epsilon_n^2 \sim (\lambda/M_P^{n-2})^2 (\xi_n/M_P^2)^{n-2}$, while the $\{2, ..., n+1\}$ solution exists for $\xi_1 < 0$, $\xi_1 + \xi_n < 0$, and is given by

$$\begin{aligned} \{2, ..., n+1\} : \qquad & v_1 = 0 \ , \quad |v_{n+1}|^2 = -(\xi_1 + \xi_n)(1 + \mathcal{O}(\epsilon_1^2)) \\ \{2, ..., n+1\} : \qquad & |v_2|^2, ..., |v_n|^2 = -\xi_1(1 + \mathcal{O}(\epsilon_1^2)) \end{aligned} \tag{17}$$

where $\epsilon_1^2 \sim (\lambda/M_P^{n-2})^2 (\xi_1/M_P^2)^{n-2}$. Note that ϵ_1 and ϵ_n are very small numbers for $n \gg 1$ and $\xi_i \ll M_P^2$. The vacuum in Eq. (16) was discussed in detail in [4]. Each of these classes of vacua occupy non-overlapping regions in the two-dimensional (ξ,ξ_n) parameter space. Note that the $\{1, 2, ..., n\}$ and $\{2, 3, ..., n+1\}$ vacua completely break all of the $U(1)$ gauge factors, while the $\{1, n+1\}$ vacuum is supersymmetric and λ-independent for all $n \geq 3$.

We shall also be interested in several other explicit solutions for general n. All of the following solutions are λ-independent. For example, the $\{2, 3, ..., n\}$ solution is given by

$$\begin{aligned} \{2, 3, ..., n\} : \qquad & v_1 = v_{n+1} = 0 \ , \\ \{2, 3, ..., n\} : \qquad & |v_k|^2 = \frac{1}{n} \left[(\xi_1 + \xi_n)(k-1) - n\xi_1 \right] \quad \text{for } k = 2, ..., n \ . \end{aligned} \tag{18}$$

This solution has an unbroken $U(1)$ generator $Q_1 + ... + Q_n$, where Q_i are the generators of $U(1)_i$, and gives rise to the D-terms

$$\langle D_1 \rangle \ = \ ... \ = \ \langle D_n \rangle \ = \ \frac{\xi_1 + \xi_n}{n} \ . \tag{19}$$

Note that this solution has a linear "profile", in the sense that the sequence of non-zero vacuum expectation values $|v_k|^2$ in Eq. (18) grows linearly with k.

Needless to say, there are numerous other solutions which can be generated for general n. However, the above solutions will be sufficient for our purposes.

Note that the last vacum, as well as all vacua with smaller numbers of non-zero vev's, are unstable at the level of our discussion (in which we are taking $\lambda/M_P^{n-2} \to 0$ and $\tilde{m} = 0$).

3.2 Higher-dimensional flux interpretation

We shall now demonstrate that many of the above general-n solutions have natural interpretations in terms of higher-dimensional flux compactifications.

First, let us consider the $\{1, 2, ..., n\}$ and $\{2, 3, ..., n+1\}$ vacua. These clearly can be interpreted as emerging from a five-dimensional supersymmetric $U(1)$ gauge theory compactified on the orbifold S^1/Z_2, with compactification radii

$$R \ \sim \ n/\sqrt{\xi_n} \ \text{ and } \ \sim \ n/\sqrt{|\xi_1|} \ , \tag{20}$$

respectively. In each case, the four-dimensional zero-mode gauge field receives a mass from the four-dimensional FI term $\xi_1+\xi_n$. The supersymmetry-breaking scale in these two cases is controlled by the Wilson-line superpotential, and all soft masses are $\sim \mathcal{O}(\epsilon_n)$ in the first case and $\sim \mathcal{O}(\epsilon_1)$ in the second case. By contrast, the third vacuum $\{1, n+1\}$ is supersymmetric and probably does not have an extra-dimensional interpretation.

We now turn to the $\{2, 3, ..., n\}$ solution in Eq. (18). We shall now argue that this vacuum can be given the higher-dimensional interpretation of having magnetic flux on a torus in a six-dimensional Abelian gauge theory. Indeed, as we will see, the smoking gun for such a magnetic flux interpretation is the presence of a linear profile in the associated vacuum expectation values.

From a five-dimensional $R^4 \times S^1/Z_2$ perspective, where the fifth dimension is the interval $0 < y < \pi R$, a supersymmetric Abelian gauge theory contains a gauge field and a Z_2-odd real scalar Σ. The D-term from the four-dimensional point of view in the continuous limit of the deconstruction setup discussed above is given by

$$D \ = \ \partial_5 \Sigma \ + \ (-|\phi_1|^2 + \xi_1)\, \delta(y) \ + \ (|\phi_{n+1}|^2 + \xi_n)\, \delta(y - \pi R) \ . \tag{21}$$

The standard profile for the scalar Σ, largely discussed in the literature, is of the form $\langle \Sigma \rangle = \epsilon(y) \xi_n/2$, which in the case $\xi_1 + \xi_n = 0$ is the needed profile for preserving supersymmetry and the gauge symmetry. Notice, however, that the field equations

$$\delta\Sigma \left\{ \partial_4^2 \Sigma + \partial_5 \left[\partial_5 \Sigma + (\xi_1 - |v_1|^2)\delta(y) + (\xi_n + |v_{n+1}|^2)\delta(y - \pi R) \right] \right\} \ = \ 0 \tag{22}$$

have another solution on the orbifold S_1/Z_2, namely

$$\langle \Sigma \rangle \ = \ \frac{\xi_1 + \xi_n}{2\pi R}\, y \ - \ \frac{\xi_1}{2}\, \epsilon(y) \ , \quad \langle D \rangle \ = \ \frac{\xi_1 + \xi_n}{2\pi R} \ . \tag{23}$$

This solution does not describe the absolute minimum of the theory, since it has a large positive vacuum energy, but it is an extremum of the theory. By using $R \sim n$, it is clear that Eq. (23) matches the deconstructed result Eq. (18). On the other hand, from a six-dimensional perspective, Σ corresponds to the sixth-component A_6 of the gauge field. Then the flux in the two-dimensional compact space is given by

$$F_{56} \ \equiv \ \langle \partial_5 \Sigma \rangle \ = \ \frac{\xi_1 + \xi_n}{2\pi R} \ - \ \xi_1\, \delta(y) \ - \ \xi_n\, \delta(y - \pi R) \ . \tag{24}$$

The first term is the magnetic flux we were searching for, whereas the localized terms, already discussed in the literature, have the interpretation of fluxes localized at the orbifold fixed points. Note that the integrated flux in the compact space is actually zero,

$$\int_{-\pi R}^{\pi R} dy \ \langle \partial_5 \Sigma \rangle \ = \ 0 \ , \tag{25}$$

the magnetic flux cancelling the localized contributions at the fixed points.

4 Discussion

In [4] we proposed a field-theoretic framework giving rise to models containing large numbers of vacuum solutions. The field-theoretic nature of these models therefore allowed us to explicitly calculate quantities as the ratio of stable versus total numbers of vacua, the number of R-symmetry preserving vacua, and the supersymmetry-breaking scale. Our examples have the advantage of describing large classes of string compactifications with Abelian gauge groups and FI terms. Moreover, within this large class, we presented specific examples involving discretized versions of magnetic fluxes in the internal space, as obtained by deconstructing (supersymmetric) models with $U(1)$ gauge fields on the orbifold S^1/Z_2. By examining the extrema involving vanishing vev's for bi-fundamental fields, we found that these solutions correspond to profiles which are linear in x_5 for the odd-scalar $\Sigma = A_6$ in the five-dimensional vector multiplet. These solutions therefore correspond to constant magnetic fluxes $\partial_5\Sigma = F_{56}$ from a six-dimensional perspective. This therefore generates a field-theory landscape which is similar in spirit and closely related to the landscapes currently under discussion in string-theory contexts [2]. One of the interesting results of the landscape picture emerging in the class of models we considered is the possibility of passing from one vacuum to another by renormalization group flow. This possibility arises because our fundamental defining parameters, such as ξ_i and λ, can change with the energy scale. We showed in an explicit toy model that this renormalization group flow can indeed induce boundary crossings. A general feature of the boundary separating two different vacua is the presence of a massless scalar in the spectrum, which suggests an interpretation in terms of phase transitions. There are clearly more general examples that can arise in string models with D-branes. String models of all sorts generically give rise to multiple $U(1)$ gauge factors. We therefore believe that the models we presented in this paper will emerge naturally in realistic string contexts and should be viewed, quite literally, as at least one component of the full string landscape.

Acknowledgments

I would like to thank the organizers of BW2005 for their kind invitation. Work supported in part by the CNRS PICS #2530 and the RTN grants MRTN-CT-2004-503369 and MRTN-CT-2004-005104.

REFERENCES

[1] R. Bousso and J. Polchinski, JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134]; J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, Nucl. Phys. B **602**, 307 (2001) [arXiv:hep-th/0005276]; for a general review, see R. Bousso and J. Polchinski, Sci. Am. **291**, 60 (2004).

[2] L. Susskind, arXiv:hep-th/0302219; M. R. Douglas, JHEP **0305**, 046 (2003) [arXiv:hep-th/0303194]; M. Dine, arXiv:hep-th/0410201.

[3] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987); A. Vilenkin, Phys. Rev. Lett. **74**, 846 (1995) [arXiv:gr-qc/9406010]; V. Agrawal, S. M. Barr, J. F. Donoghue and D. Seckel, Phys. Rev. Lett. **80**, 1822 (1998) [arXiv:hep-ph/9801253]; M. L. Graesser, S. D. H. Hsu, A. Jenkins and M. B. Wise, Phys. Lett. B **600**, 15 (2004) [arXiv:hep-th/0407174]; A. Vilenkin, arXiv:astro-ph/0407586.

[4] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Rev. D **72** (2005) 026005 [arXiv:hep-th/0412185].