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ULTRAMETRIC EXTRADIMENSIONS AND TACHYONIC COSMOLOGY

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Abstract. *We consider (4+D)-dimensional Kaluza-Klein cosmological model with two scaling factors. One of the scaling factors corresponds to the D-dimensional internal space, and second one to the 4-dimensional universe. In standard quantum cosmology, i.e. over the field of real numbers R it leads to dynamical compactification of additional dimensions and to the accelerating evolution of* 4*-dimensional universe We construct corresponding p-adic quantum model and explore existence of its p-adic ground state. In addition, we explore evolution of this model and a possibility for its adelic generalization. It is necessary for the further investigation of space-time discreteness at very short distances, i.e. in very early phase of the universe. The corresponding propagators are calculated. Tachyon matter from classical and quantum point of view is examined.*

Key words: *Quantum cosmology, tachyons, p-adic numbers*

1. INTRODUCTION

There are many consideration in quantum gravity (e.g. [1]) suggesting theoretical uncertainty of measuring distances to be greater or equal to the Planck distance. It could be concluded that on very short distances Archimedean axiom is not valid, i.e. the space can posses ultrametric features. Because geometry is always connected with a concrete number field [2], in the case of the nonarchimedean geometry it is used to be the field of p-adic numbers Q_p. In the high energy physics, these numbers have been used almost twenty years. The motivation comes from string theory [3]. Generally speaking, p-adic approach should be useful in describing a very early phase of the universe and processes around Planck scale.

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A significant number of papers, motivated by [3], has been published up to now (for a review see [4, 5]). In this short review we are especially interested in application of p-adic numbers and analysis in quantum cosmology [6]. p-Adic quantum mechanics [7] (QM) has been successfully applied in minisuperspace quantum cosmology [8]. We have treated many cosmological models, mainly constructed in four space-time dimensions [6]. Based on that one can ask: is it possible to extend this approach to the multidimensional quantum cosmological models? This article, i.e. its first part is devoted to the formulation of such a model, with two scaling factors and an exotic fluid. We wish to use these models to consider possible p-adic structure of extra dimensions. In addition we explore possibility to formulate a consistent real, p-adic and adelic $(4+D)$ Kaluza-Klein model.

It should be noted that structure of space-time around Planck scale is a very interesting problem. One very attractive approach is based on noncommutative (NC) geometry. Beside noncommutative quantum mechanics [9] (mainly used to formulate some interesting toy models) and a lot of papers related to the NC Quantum Field Theory [10] and NC Standard Model [11], there have been a few attempts to formulate NC Quantum Cosmology [12]. In the second part of this article we formulate and compare $(4+D)$ dimensional model, in particular $(4+1)$, Kaluza-Klein "empty" model on real (and p-adic) commutative space and noncommutative space. First of all we consider and calculate their corresponding quantum propagators, i.e. examine evaluation of the same models constructed on different spaces as possible candidates for true geometrical background at a very early phase of the universe.

After brief mathematical introduction in Section 2, a short review of p-adic and adelic quantum mechanics and cosmology is given. Section 4 is devoted to the classical (4+D)-dimensional cosmological models filled with "exotic" fluid [13]. The corresponding p-adic model is considered in Section 5. In Section 6. we consider zero dimensional version of the field theory of tachyon matter. This paper is ended by short conclusion, including adelic generalization, and suggestion for future research.

2. p -Adic Numbers and Adeles

Perhaps the easier way to understand p-adic numbers is to start with the notion of norm. It is well known that any norm must satisfy three conditions: nonnegativity, homogeneity and triangle inequality. The completion of the field of rational numbers Q with respect to the absolute value, or standard norm $|.|_\infty$, gives the field of real numbers $R \equiv Q_\infty$.

Besides this norm there are another ones which satisfy the first two conditions and the third one in a stronger way

$$\|x+y\| \leq \max(\|x\|, \|y\|), \tag{1}$$

so called strong triangle inequality. The most important of them is p-adic norm $|.|_p$ (p denotes a prime number) [4]. The feature (1), also called ultrametricity, is one of the most important characteristics of the p-adic norm. The number fields obtained by completion of Q with respect to this norm are called p-adic number fields Q_p.

Because Q_p is local compact commutative group the Haar measure can be introduced, which enables integration (for a rather advanced discussion on this and related topics see, for example [14]). In particular, the Gauss integral will be employed

$$\int_{Q_p} \chi_p(\alpha x^2 + \beta x)dx = \lambda_p(\alpha)|2\alpha|_p^{-1/2} \chi_p\left(-\frac{\beta^2}{4\alpha}\right), \quad \alpha \neq 0, \tag{2}$$

where $\chi_p = \exp 2\pi i \{x\}_p$ (a real counterpart $\chi_\infty(x) = \exp(-2\pi i x)$), is an additive character ($\{x\}_p$ is the fractional part of x), and $\lambda_p(\alpha)$ is an arithmetic complex-valued function [4].

Very simple but rather important function in p-adic analysis and p-adic QM is

$$\Omega(|x|_p) = \begin{cases} 1, & |x|_p \leq 1, \\ 0, & |x|_p > 1, \end{cases} \tag{3}$$

which is the characteristic function of Z_p. Note that $Z_p = \{x \in Q_p : |x|_p \leq 1\}$ is the ring of p-adic integers.

Simultaneous treatment of real and p-adic numbers can be realized by concept of adeles. An adele $a \in \mathcal{A}$ is an infinite sequence $a = (a_\infty, a_2, \cdots, a_p, \cdots)$, where $a_\infty \in R$ and $a_p \in Q_p$, with the restriction that $a_p \in Z_p$ for all but a finite set S of primes p. The set of all adeles $\mathcal{A}$ can be written in the form

$$\mathcal{A} = \underset{S}{U}\, \mathcal{A}(S), \quad \mathcal{A}(S) = R \times \prod_{p \in S} Q_p \times \prod_{p \notin S} Z_p\ . \tag{4}$$

Also, $\mathcal{A}$ is a topological space. Algebraically, it is a ring with respect to the componentwise addition and multiplication. There is the natural generalization of analysis on R and Q_p to analysis on $\mathcal{A}$ [15].

3. Minisuperspace Quantum Cosmology

Quantum cosmology [16, 17] is the application of quantum theory to the universe as a whole. The investigations in quantum cosmology are often based on the WheelerDeWitt equation. The method is to restrict first the configuration space to a finite number of variables (scale factor, inflation field, . . .) and then to quantize canonically. Since the full configuration space of three-geometries is called "superspace", the ensuing models are called "minisuperspace models". For the minisuperspace models, we use metric in the standard 3+1 decomposition

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + h_{ij}dx^i dx^j, \tag{5}$$

where N is the lapse function. For this models corresponding functional integral is reduced to functional integral over three-metric and configuration of matter fields, and to another usual integral over the lapse function N. For the boundary condition $q_\alpha(t_2) = q''_\alpha$, $q_\alpha(t_1) = q'_\alpha$ in the gauge $\dot{N} = 0$, we have minisuperspace propagator

$$\langle q''_\alpha; q'_\alpha\rangle = \int dN \mathcal{K}(q''_\alpha, N; q'_\alpha, 0), \tag{6}$$

$$\mathcal{K}(q''_\alpha, N; q'_\alpha, 0) = \int \mathcal{D}q_\alpha \chi(-S[q_\alpha]), \tag{7}$$

is an ordinary quantum-mechanical propagator between fixed minisuperspace coordinates (q'_α, q''_α) in a fixed "time" N. S is the action of the minisuperspace model, i.e.

$$S[q_\alpha] = \int_0^1 dtN \left[\frac{1}{2N^2} f_{\alpha\beta}(q)\dot{q}^\alpha \dot{q}^\beta - U(q)\right], \tag{8}$$

where $f_{\alpha\beta}$ is a minisuperspace metric ($ds_m^2 = f_{\alpha\beta}dq^\alpha dq^\beta$) with an indefinite signature $(-, +, +, \ldots)$. This metric includes spatial (gravitational) components as well the matter variables for the given model. It should be noted the necessary condition for the existence of an adelic quantum model is the existence of p-adic ground state $\Omega(|q_\alpha|_p)$ defined by (3), i.e.

$$\int_{|q_\alpha{}'|_p \le 1} \mathcal{K}_p(q_\alpha{}'', N; q_\alpha{}', 0)dq_\alpha{}' = \Omega(|q_\alpha{}''|_p). \tag{9}$$

4. (4+D)-Dimensional Cosmological Models

Over the Field of Real Numbers

The old idea that four dimensional universe, in which we exist, is just our observation of physical multidimensional space-time attracts much attention nowadays. In such models compactification of extra dimensions play the key role and in some of them leads to the period of accelerated expansion of the universe [13, 18, 19]. This approach is supported and encouraged with the results of the astronomical observations. We briefly recapitulate some facts of the real multidimensional cosmological models, necessary for p-adic and adelic generalization. The metric of such Kaluza-Klein model with D-dimensional internal space can be presented in the form [13, 20]

$$ds^2 = -\tilde{N}^2(t)dt^2 + R^2(t)\frac{dr^i dr^i}{(1+\frac{kr^2}{4})^2} + a^2(t)\frac{d\rho^a d\rho^a}{(1+k'\rho^2)^2} \tag{10}$$

where $\tilde{N}(t)$ is a lapse function, $R(t)$ and $a(t)$ are the scaling factors of 4-dimensional universe and internal space, respectively; $r^2 \equiv r^i r^i$ $(i = 1, 2, 3)$, $\rho^2 \equiv \rho^a \rho^a$ $(a = 1, ...D)$, and $k, k' = 0, \pm 1$. The form of the energy-momentum tensor is

$$T_{AB} = diag(-\rho, p, p, p, p_D, p_D, ..., p_D), \tag{11}$$

where indices A and B run over both spacetime coordinates and the internal space dimensions. If we want the matter to be confined to the four-dimensional universe, we set all $p_D = 0$.

Now, we examine the case for which the pressure along all the extra dimensions vanishes $p_D = 0$ (in braneworld scenarios the matter is to be confined to the four-dimensional universe), so that all components of T_{AB} are set to zero except the spacetime components [13]. We assume the energy-momentum tensor of spacetime to be an exotic fluid χ with the equation of state

$$p_\chi = \left(\frac{m}{3} - 1\right)\rho_\chi, \tag{12}$$

(p_χ and ρ_χ are the pressure and the energy density of the fluid, parameter m has value between 0 and 2).

4.1 Classical Model

Dimensionally extended Einstein-Hilbert action (without a higher-dimensional cosmological term) is

$$S = \int \sqrt{-g}\tilde{R} dt d^3 R d^D \rho + S_m = \kappa \int dt L \tag{13}$$

where κ is an irrelevant constant, $\tilde{R}$ is the scalar curvature of the metric, so we can read off the Lagrangian of the model (for flat internal space)

$$L = \frac{1}{2\tilde{N}} R a^D \dot{R}^2 + \frac{D(D-1)}{12\tilde{N}} R^3 a^{D-2} \dot{a}^2 + \frac{D}{2\tilde{N}} R^2 a^{D-1} \dot{R}\dot{a} - \frac{1}{2} k\tilde{N} R a^D + \frac{1}{6}\tilde{N}\rho_\chi R^3 a^D. \tag{14}$$

For closed universe ($k = 1$), substitution of the equation of state in the continuity equation

$$\dot{\rho}_\chi R + 3(p_\chi + \rho_\chi)\dot{R} = 0 \tag{15}$$

leads to the energy density in the form

$$\rho_\chi(R) = \rho_\chi(R_0)\left(\frac{R_0}{R}\right)^m, \tag{16}$$

where R_0 is the value of scaling factor in arbitrary reference time. If we define cosmological constant as $\Lambda = \rho_\chi(R)$, Lagrangian becomes

$$L = \frac{1}{2\tilde{N}} R a^D \dot{R}^2 + \frac{D(D-1)}{12\tilde{N}} R^3 a^{D-2} \dot{a}^2 + \frac{D}{2\tilde{N}} R^2 a^{D-1} \dot{R}\dot{a} - \frac{1}{2}\tilde{N} R a^D + \frac{1}{6}\tilde{N}\Lambda R^3 a^D. \tag{17}$$

Growth of the scaling factor R, according to (16) leads to the decrease of the cosmological constant by the relation

$$\Lambda(R) = \Lambda(R_0)\left(\frac{R_0}{R}\right)^m. \tag{18}$$

This decaying Λ term may also explain the smallness of the present value of the cosmological constant since, as the universe evolves form its small to large size, the large initial value of Λ decays to small values. If we take $m = 2$, initial condition for cosmological constant and scaling factor $\Lambda(R_0)R_0^2 = 3$, for laps function $\tilde{N}(t) = R^3(t)a^D(t)N(t)$, the Lagrangian (17) becomes

$$L = \frac{1}{2N}\frac{\dot{R}^2}{R^2} + \frac{D(D-1)}{12N}\frac{\dot{a}^2}{a^2} + \frac{D}{2N}\frac{\dot{R}\dot{a}}{Ra}. \tag{19}$$

The solutions of corresponding equations of motion are

$$R(t) = C_1 e^{\alpha t}, \tag{20}$$

$$a(t) = C_2 e^{\beta t}, \tag{21}$$

where the constants C_1, C_2, α and β depend on initial conditions. It is a reasonable assumption that the size of all spatial dimensions is the same at $t = 0$. It may be assumed that this size would be the Planck size, i.e. $R(0) = a(0) = l_P$. Above solutions can be read in terms of Huble parameter $H = \dot{R}/R$ [13]

$$R(t) = l_P e^{Ht}, \tag{22}$$

$$a(t) = l_P e^{-Ht}. \tag{23}$$

Depending of the dimensionality of the internal space, we have

$$R(t) = l_P e^{Ht}, \tag{24}$$

$$a_\pm(t) = l_P e^{\frac{2Ht}{D}[-1\pm\sqrt{1-\frac{2}{3}(1-\frac{1}{D})}]^{-1}}, \tag{25}$$

for $D = 1$ and

$$R_\pm(t) = l_P e^{\frac{D\beta t}{2}[-1\pm\sqrt{1-\frac{2}{3}(1-\frac{1}{D})}]}, \tag{26}$$

$$a(t) = l_P e^{\beta t}, \tag{27}$$

for $D > 1$. The solution corresponding to $D = 1$ predicts an accelerating (de Sitter) universe and a contracting internal space, with exactly the same rates. In the case $D > 1$ analysis is complicated, but results are similar.

4.2 Quantum Model

Quantum solutions are obtained from the Wheeler-DeWitt equation

$$H\Psi(R, a) = 0, \tag{28}$$

where H is the Hamiltonian and Ψ is the wave function of the universe. For this model above equation is read in the new variables ($X = \ln R$ and $Y = \ln a$)

$$\left[(D-1)\frac{\partial^2}{\partial X^2} + \frac{6}{D}\frac{\partial^2}{\partial Y^2} - 6\frac{\partial}{\partial X}\frac{\partial}{\partial Y}\right]\Psi(X, Y) = 0. \tag{29}$$

By the new change $x = X\frac{3}{D+3} + Y\frac{D}{D+3}$, $y = \frac{X-Y}{D+3}$, Wheeler-DeWitt equation takes a simple form

$$\left[-3\frac{\partial^2}{\partial x^2} + \frac{D+2}{D}\frac{\partial^2}{\partial y^2}\right]\Psi(x, y) = 0. \tag{30}$$

Equation (30) has the four possible solutions [13]

$$\Psi_D^\pm(x, y) = A^\pm e^{\pm\sqrt{\frac{\gamma}{3}}x \pm\sqrt{\frac{\gamma D}{D+2}}y}, \tag{31}$$

$$\Psi_D^{\pm}(x,y) = B^{\pm} e^{\pm\sqrt{\frac{\gamma}{3}}x \mp \sqrt{\frac{\gamma D}{D+2}}y}, \tag{32}$$

where $A^{\pm}$ and $B^{\pm}$ are the normalization constants. It is possible to impose the boundary conditions to get a $\Psi_D(R,a) = 0$. For the further details see [13].

5. (4+D)-Dimensional Model Over the Fields of p-Adic Numbers

Consideration of (4+D)-dimensional Kaluza-Klein model over the field Q_p will be started from the Lagrangian in the form (19). All quantities in this Lagrangian will be treated as the p-adic ones. Taking again replacement $X = \ln R$ and $Y = \ln a$, it becomes

$$L = \frac{1}{2N}\dot{X}^2 + \frac{D(D-1)}{12N}\dot{Y}^2 + \frac{D}{2N}\dot{X}\dot{Y}. \tag{33}$$

The corresponding p-adic equations of motion are

$$\ddot{X} + \frac{D}{2}\ddot{Y} = 0, \quad \ddot{X} + \frac{D-1}{3}\ddot{Y} = 0. \tag{34}$$

Let use again the change $x = X\frac{3}{D+3} + Y\frac{D}{D+3}$, $y = \frac{X-Y}{D+3}$ to separate variables and make further analyzes of this model rather simple. We use the fact that this action is quadratic with respect to variables (x,y) [21, 22].In these variables the classical action and the kernel of evolution operator read

$$\begin{aligned} &\bar{S}_p(x'',y'',N;x',y',0) \\ &= \frac{1}{2N}(1+\frac{D(D+5)}{6})(x''-x')^2 - \frac{1}{2N}D(D+3)(y''-y')^2, \end{aligned} \tag{35}$$

$$\begin{aligned} \mathcal{K}_p(x'',y'',N;x',y',0) &= \lambda_p\left(\frac{6+D(D+5)}{6}\right)\lambda_p\left(-\frac{D(D+3)}{12N}\right) \\ \times \left|\frac{D(D+3)}{2N^2}\left(1+\frac{D(D+5)}{6}\right)\right|_p^{1/2} &\chi_p(-\bar{S}_p(x'',y'',N;x',y',0)). \end{aligned} \tag{36}$$

Now, we can examine when p-adic wave function has the form corresponding to the simplest ground state [24] (here in two dimensions (3))

$$\Psi_p(x,y) = \Omega(|x|_p)\Omega(|y|_p). \tag{37}$$

Putting kernel of the operator of evolution (36) in equation (9) we get that the required state exists if both conditions

$$|N|_p \leq \left|1+\frac{D(D+5)}{6}\right|_p, \quad |N|_p \leq |D(D+3)|_p,\ p \neq 2, \tag{38}$$

are fulfilled. To answer the question: "can this conditions be useful in determination of dimensionality of the internal space?" one needs further careful analysis.

Going back to the "old variables", p-adic ground state wave function for this model is

$$\Psi_p(x,y) = \Omega\left(\left|\left(1-\frac{D}{D+3}\right)X + \frac{D}{D+3}Y\right|_p\right)\Omega\left(\left|\frac{X-Y}{D-3}\right|_p\right). \tag{39}$$

We can also write down the solutions in the variables R and a [23].

6. Tachyons: Classical and quantum

In string theory, when physicists calculate mass of the particles, in some cases, their squared mass turned out to be negative. Such particles are called tachyons. For such a theory vacuum state is unstable, A. Sen proposed a field theory of tachyon matter few years ago [25, 26]. The action is given as:

$$S = -\int d^{n+1}x V(T)\sqrt{1+\eta^{ij}\partial_i T\partial_j T} \tag{40}$$

where $\eta_{00} = -1$, $\eta_{\mu\nu} = \delta_{\mu\nu}$, $\mu,\nu = 1,...,n$, $T(x)$ is the scalar tachyon field and $V(T)$ is the tachyon potential, which unusually appears in the action as a multiplicative factor and has (from string field theory arguments) exponential dependence with respect to the tachyon field:

$$V(T) = e^{-\alpha T(x)}. \tag{41}$$

It is very important to understand and investigate lower dimensional analogs of this tachyon field theory, i.e. a quantum mechanical model [27] in context of quantum cosmology (first of all rolling tachyons and inflation). The corresponding zero dimensional analogue of a tachyon field can be obtained by the correspondence: $x^i \to t$, $T \to x$, $V(T) \to V(x)$. The action reads:

$$S = -\int dt V(x)\sqrt{1-\dot{x}^2}. \tag{42}$$

Corresponding equation of motion, including $V(x) = e^{-\alpha x}$, is:

$$\ddot{x} + \alpha\dot{x}^2 = \alpha, \tag{43}$$

and coincide with the equation of motion for the system under gravity in the presence of quadratic damping:

$$m\ddot{y} + \beta\dot{y}^2 = mg. \tag{44}$$

This equation can be derived from the action:

$$S = -\int dte^{-\beta y/m}\sqrt{1-\frac{\beta}{mg}\dot{y}^2}. \tag{45}$$

The solution can be found perturbatively:

$$y(t) = y_0(t) + y_1(t), \tag{46}$$

where $y_0(t)$ is solution of Eq. (44) for $\beta = 0$, and $y_1(t)$ is obtained from the same equation after inserting $y_0(t)$ and neglecting all non linear terms:

$$\ddot{y}_1 + at\dot{y}_1 = -bt^2, \tag{47}$$

where $a = \frac{2\beta g}{m}$, $b = \frac{\beta g^2}{m}$. For $y_0(0) = 0$ and $\dot{y}_0(0) = 0$ the final solution is given by:

$$y(t) = \frac{g}{2}t^2 + \frac{b}{2a}t^2({}_2F_2[1,1;3/2,2;-\frac{at^2}{2}] - 1), \tag{48}$$

where ${}_2F_2[1,1;3/2,2;-\frac{at^2}{2}]$ is hypergeometric function. For small t it gets quite simple form:

$$y(t) = \frac{g}{2}t^2 - \frac{\beta g^2}{12m}t^4. \tag{49}$$

We do not discuss here details, which will be presented elsewhere, just to mention that here we consider very simple forms choosing the corresponding initial and final space-time points.

As we note, according to Feynman's idea, dynamical evolution of the system is completely described by the kernel $K(y'',T;y',0)$ of the unitary evolution operator $U(0,T)$, where y'', y' are initial and final positions and T is "total" time:

$$K(y'',T;y',0) = \int Dye^{2\pi i/h\int_0^T Ldt}. \tag{50}$$

There is very useful semi-classical expression for the kernel if the classical action $\bar{S}(y'',T;y',0)$ is polynomial quadratic in y' and y'' (which holds for both real and p-adic number fields):

$$K(y'',T;y',0) = \left(\frac{i}{h}\frac{\partial^2\bar{S}}{\partial y'\partial y''}\right)^{1/2} e^{\frac{2\pi i}{h}\bar{S}(y'',T;y',0)}. \tag{51}$$

One can go back to Eq. (45) and for very small β, it leads to the new form of action:

$$S \to_{(\beta\to 0)} S' = -\int dt\left(\frac{\beta}{2mg}\dot{y}^2 + \frac{\beta}{m}y - 1\right). \tag{52}$$

This action is quadratic with respect to velocity, and standard procedure can be engaged for the path integral and tachyon quantization.

Conclusion

In this paper, we demonstrate firstly how a p-adic version of the quantum (4+D)-Kaluza-Klein model with exotic fluid can be constructed. It is an exactly soluble model. From equations (31), (32) and (37), i.e. (39), it is possible to construct an adelic model too, i.e. model which unifies standard and all p-adic models [12]. The investigation of its possible physical implication and discreteness of space-time deserves much more attention and room.

Let us note that adelic states for the (4+D)-dimensional Kaluza-Klein cosmological model (for any D which satisfies (38)) exist in the form

$$\Psi_S(x,y) = \Psi^{\pm}_{D,\infty}(x_\infty, y_\infty) \prod_{p\in S} \Psi_p(x_p, y_p) \prod_{p\notin S} \Omega(|x_p|_p)\Omega(|y_p|_p), \tag{53}$$

As it is well known in p-adic quantum theory this result leads to some discretization of minisuperspace coordinates x, y. Namely, probability to observe the universe corresponding to our minisuperspace model is nonzero only in the integer points of x and y. Keeping in mind that Ω-function is invariant with respect to the Fourier transform, this conclusion is also valid for the momentum space. Note that this kind of discreteness depends on adelic quantum state of the universe. When some p-adic states are different from $\Omega(|x|_p)\Omega(|y|_p)$ $(S \neq \emptyset)$, then the above adelic discreteness becomes less transparent.

Further investigations could include determination of conditions for existence of the ground states in the form $\Omega(p^\nu|x|_p)\Omega(p^\mu|y|_p)$ and p-adic delta function. We should emphasize that investigation of dimensionality D of internal space from the conditions (38) could additionally contributes to better understanding of the model, especially from its p-adic sector.

In final, we would stress that Sen‘s interesting proposals and conjectures have increased our interests to understand tachyon matter, especially its quantum aspects. Perturbative solutions for classical particles analogous to the tachyons offer many possibilities for further investigations and toy models in quantum mechanics, quantum and string field theory and cosmology on archimedean and nonarchimedean spaces. Our hope is that in can be useful in understanding of nature of dark matter and dark energy effects, as well as for better understanding of the inflation scenario, in particular, on nonarchimedean spaces.

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