FACTA UNIVERSITATIS
Series: **Physics, Chemistry and Technology** Vol. 4, N^o 2, 2006, pp 261 -272

LORENTZ SYMMETRY ON DEFORMED SPACES

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Abstract. *We consider here noncommutative spaces and their symmetries. For the two special examples of noncommutative spaces the deformed Poincaré symmetry is constructed.*

Key words: *Noncommutative spaces, deformed symmetry, Hopf algebras*

1. INTRODUCTION

The talk given by the author is based on the common work with Paolo Aschieri, Christian Blohmann, Frank Meyer, Peter Schupp and Julius Wess [1]. The concept of symmetry is very important in physics. Classically, symmetries are described by Lie groups or Lie algebras and the physical space is the representation space of the symmetry algebra. Therefore, the question arises if one can introduce the noncommutative (deformed) spaces as representation spaces of some symmetry algebras. It turns out that this is possible in the framework of Hopf algebras and quantum groups [2]. Here we consider two examples of deformed Lorentz symmetry given in terms of Hopf algebras. We start with the short review of noncommutative (deformed) spaces and their representation on the space of commuting coordinates. Then we concentrate on the two special examples, the θ-deformed and the κ-deformed space and construct the deformation of the classical Poncaré algebra using the inversion of the $\star$-product. In the κ-deformed case, the deformation of

Received: 20 September 2005

the Poincaré algebra that is different from the one known in the literature [3] is obtained.

2. NONCOMMUTATIVE SPACES

Definition

Noncommutative (deformed) space is generated by $n+1$ abstract coordinates $\hat{x}^\mu$ which fulfil

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu}(\hat{x}), \qquad \mu = 0, \dots n, \tag{1}$$

where $\Theta^{\mu\nu}(\hat{x})$ is an arbitrary polynomial of coordinates [4], [5]. More precisely, the noncommutative space $\hat{\mathcal{A}}_{\hat{x}}$ is the associative algebra, freely generated by $\hat{x}^\mu$ coordinates and divided by the ideal generated by (1). The elements of this space are all possible polynomials in the coordinates $\hat{x}^\mu$. Before proceeding further, we clarify the notation we use. Coordinates $\hat{x}^\mu$ generate the abstract algebra $\hat{\mathcal{A}}_{\hat{x}}$, while the operators $\hat{\partial}_\rho$, $\hat{L}_{\alpha\beta}$, ... are maps of the abstract algebra $\hat{\mathcal{A}}_{\hat{x}}$ into itself. Variables without the hat symbol, like x^μ, ∂_ρ, ... are usual commutative variables. Sometimes we use $\mathcal{A}_x$ to denote the space of commuting coordinates. The defining relation of the deformed space (1) is very general and one usually consideres some special examples of it. Among them there are three very important ones

$$\begin{aligned} \text{Canonical or } \theta\text{-deformed spaces} \quad [\hat{x}^\mu, \hat{x}^\nu] &= i\theta^{\mu\nu}, & (2)\\ \text{Lie algebra deformed spaces} \quad [\hat{x}^\mu, \hat{x}^\nu] &= iC^{\mu\nu}_\lambda \hat{x}^\lambda, & (3)\\ q\text{-deformed spaces} \quad \hat{x}^\mu \hat{x}^\nu &= \frac{1}{q} R^{\mu\nu}{}_{\rho\sigma} \hat{x}^\rho \hat{x}^\sigma. & (4) \end{aligned}$$

In the case of θ-deformed spaces [6], $\theta^{\mu\nu} = -\theta^{\nu\mu}$ is an antisymmetric constant matrix of mass dimension -2. For Lie algebra deformed spaces [3], [7] $C^{\mu\nu}_\lambda$ are Lie algebra structure constants of mass dimension -1. And finally, $R^{\mu\nu}{}_{\rho\sigma}$ is the dimensionless R-matrix of the quantum space [2]. This three examples fulfil the Poincaré-Birkoff-Witt (PBW) property.

Representation on the space of commuting coordinates

The deformation quantisation allows us to to describe the properties of a noncommutative space in a perturbative way, order by order in the deformation parameter. In the zeroth order the commutative space-time is obtained. The main idea of the deformation quantisation is to represent a noncommutative space on the space of commuting coordinates. PBW property enables us to map an arbitrary element $\hat{f}(\hat{x})$ of $\hat{\mathcal{A}}_{\hat{x}}$ to the space of commuting coordinates. To extend this vector space isomorphism to an algebra morphism one has to map the multiplication in the abstract algebra $\hat{\mathcal{A}}_{\hat{x}}$ to the space of commuting coordinates

$$\hat{f} \cdot \hat{g}(\hat{x}) \mapsto f \star g(x) \in \mathcal{A}_x. \tag{5}$$

The new product in $\mathcal{A}_x$ is noncommutative and we call it the star product ($\star$-product) of two functions. The algebra of noncommuting coordinates $\hat{\mathcal{A}}_{\hat{x}}$ is then isomorphic to the algebra of commuting variables with the $\star$-product as multiplication. In the following we consider two special examples of noncommutative spaces, the θ-deformed space and the κ-deformed space.

θ-deformed space

In the case of the θ-deformed space (2) the $\star$-product is given by the Moyal-Weyl $\star$-product [9]

$$\begin{aligned} f \star g\,(x) &= \lim_{x \to y} e^{\frac{i}{2}\theta^{\rho\sigma}\frac{\partial}{\partial x^\rho}\frac{\partial}{\partial y^\sigma}} f(x)g(y) \qquad (6) \\ &= \sum_{n=1}^{\infty} \Big(\frac{i}{2}\Big)^n \frac{1}{n!}\theta^{\rho_1\sigma_1}\dots\theta^{\rho_n\sigma_n}\Big(\partial_{\rho_1}\dots\partial_{\rho_n} f(x)\Big)\Big(\partial_{\sigma_1}\dots\partial_{\sigma_n} g(x)\Big). \end{aligned}$$

The abstract derivative $\hat{\partial}_\mu$ that is consistent with (2) fulfils

$$[\hat{\partial}_\rho, \hat{x}^\mu] = \delta^\mu_\rho. \tag{7}$$

Its representation on the space of commuting coordinates is given by the usual partial derivative

$$\hat{\partial}_\rho \mapsto \partial^\star_\rho = \partial_\rho \tag{8}$$

and it has the undeformed Leibniz rule

$$\begin{aligned} (\partial^\star_\rho \star (f \star g)) = \partial_\rho(f \star g) &= (\partial^\star_\rho \star f) \star g + f \star (\partial^\star_\rho \star g) \\ &= (\partial_\rho f) \star g + f \star (\partial_\rho g). \qquad (9) \end{aligned}$$

κ-deformed space

The κ-deformed space is a special example of the Lie-algebra type of deformation (3). The coordinates $\hat{x}^\mu$ fulfil

$$[\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}_\rho \hat{x}^\rho, \tag{10}$$

where

$$C^{\mu\nu}_\rho = a(\delta^\mu_n \delta^\nu_\rho - \delta^\nu_n \delta^\mu_\rho), \quad \mu = 0, \dots, n. \tag{11}$$

Latin indices denote the undeformed dimensions, n denotes the deformed dimension and Greek indices refer to all $n+1$ dimensions. The constant deformation vector a^μ of length a points in the n-th spacelike direction, $a^n = a$. The parameter a is related to the frequently used parameter κ as $a = 1/\kappa$. Written more explicitly (10) reads

$$[\hat{x}^n, \hat{x}^l] = ia\hat{x}^l, \quad [\hat{x}^k, \hat{x}^l] = 0; \quad k, l = 0, 1, \dots, n-1. \tag{12}$$

This space has a quantum group symmetry given in terms of the κ-deformed Poincaré Hopf algebra

Algebra sector

$$\begin{aligned}
[\hat{\partial}_\mu, \hat{\partial}_\nu] &= 0, \\
[M^{ij}, \hat{\partial}_\mu] &= \delta^j_\mu \hat{\partial}^i - \delta^i_\mu \hat{\partial}^j, \qquad [M^{in}, \hat{\partial}_n] = \hat{\partial}^i, \\
[M^{in}, \hat{\partial}_j] &= \delta^i_j \frac{e^{2ia\hat{\partial}_n} - 1}{2ia} - \frac{ia}{2}\delta^i_j \hat{\partial}^l \hat{\partial}_l + ia\hat{\partial}^i \hat{\partial}_j, \\
[M^{\mu\nu}, M^{\rho\sigma}] &= \eta^{\mu\sigma} M^{\nu\rho} + \eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\rho} M^{\nu\sigma} - \eta^{\nu\sigma} M^{\mu\rho}.
\end{aligned} \tag{13}$$

Coalgebra sector

$$\begin{aligned}
\Delta\hat{\partial}_n &= \hat{\partial}_n \otimes 1 + 1 \otimes \hat{\partial}_n, \\
\Delta\hat{\partial}_j &= \hat{\partial}_j \otimes 1 + e^{ia\hat{\partial}_n} \otimes \hat{\partial}_j.
\end{aligned} \tag{14}$$

$$\begin{aligned}
\Delta M^{ij} &= M^{ij} \otimes 1 + 1 \otimes M^{ij}, \\
\Delta M^{in} &= M^{in} \otimes 1 + e^{ia\hat{\partial}_n} \otimes M^{in} + ia\hat{\partial}_k \otimes M^{ik}.
\end{aligned} \tag{15}$$

We do not write here counits and antipodes, but instead refer the reader to [3]. We see that the algebra sector (13) is deformed as well as the coalgebra sector (14), (15) (leading to the deformed Leibniz rules). We mention that one can find the basis in which the algebra sector is undeformed, but the coalgebra sector remains deformed [10]. In the zeroth order in the deformation parameter a this Hopf algebra reduces to the classical Poncaré Hopf algebra. The symmetrically ordered $\star$-product, expanded up to first order in the deformation parameter a reads

$$f \star g\,(x) = f(x)g(x) + \frac{i}{2}C^{\rho\sigma}_{\lambda}x^{\lambda}(\partial_\rho f(x))(\partial_\sigma g(x)) + \mathcal{O}(a^2). \tag{16}$$

3. θ-DEFORMED POINCARÉ ALGEBRA

In this section we construct the symmetry for the θ-deformed space. The method used is general enough so that we can apply it to the κ-deformed space in the next section. Under the classical infinitesimal Lorentz transformations

$$x^\mu \to x'^\mu = x^\mu + x^\nu \omega_\nu{}^\mu, \quad \omega^{\mu\nu} = -\omega^{\nu\mu} = const. \tag{17}$$

the scalar field $\phi(x)$ transforms as

$$\delta^{cl}_\omega \phi(x) \stackrel{\text{def}}{=} \phi'(x) - \phi(x) = -x^\mu \omega_\mu{}^\lambda (\partial_\lambda \phi(x)). \tag{18}$$

We can rewrite (18) in terms of the $\star$-product (6)

$$\delta_\omega \phi = -x^\mu \omega_\mu{}^\lambda (\partial_\lambda \phi) = -(X^\star_\omega \star \phi). \tag{19}$$

Solving (19) perturbatively one finds up to first order in θ

$$X^\star_\omega = x^\mu \omega_\mu{}^\lambda \partial_\lambda - \frac{i}{2}\theta^{\rho\sigma}\omega_\rho{}^\lambda \partial_\lambda \partial_\sigma + \mathcal{O}(\theta^2) \tag{20}$$

and the deformed Lorentz transformation of a scalar field is given by

$$\begin{aligned} \delta_\omega \phi &= -(X^\star_\omega \star \phi) \\ &= -(x^\mu \omega_\mu{}^\lambda) \star (\partial_\lambda \phi) + \frac{i}{2}\theta^{\rho\sigma}\omega_\rho{}^\lambda (\partial_\sigma \partial_\lambda \phi) + \mathcal{O}(\theta^2). \end{aligned} \tag{21}$$

Transformations (21) close in the undeformed algebra

$$[\hat{\delta}_\omega, \hat{\delta}'_\omega] = \hat{\delta}_{[\omega,\omega']}. \tag{22}$$

In the next step we demand (in analogy with the classical Lorentz transformations) that the $\star$-product of two scalar fields transforms like a scalar field

$$\delta_\omega\Big(\phi_1 \star \phi_2\Big) \stackrel{\text{def}}{=} -x^\mu \omega_\mu{}^\lambda \partial_\lambda \Big(\phi_1 \star \phi_2\Big). \tag{23}$$

This request leads to the deformed Leibniz rule

$$\delta_\omega\Big(\phi_1\star\phi_2\Big) = (\delta_\omega\phi_1)\star\phi_2+\phi_1\star(\delta_\omega\phi_2) - \frac{i}{2}\theta^{\rho\sigma}\Big((\delta_{\partial_\rho\omega}\phi_1)\star(\partial_\sigma\phi_2)+(\partial_\rho\phi_1)\star(\delta_{\partial_\sigma\omega}\phi_2)\Big)+\mathcal{O}(\theta^2), \tag{24}$$

where $\delta_{\partial_\rho\omega}\phi_1 = -\omega^\lambda{}_\rho(\partial_\lambda\phi_1)$. We rewrite (21) in a more familiar way

$$\delta_\omega\phi = -\frac{1}{2}\omega^{\alpha\beta}L_{\alpha\beta}\phi, \tag{25}$$

where $L_{\alpha\beta}$ is the orbital part of the Lorentz generator $M_{\alpha\beta}$ given by

$$\begin{aligned} L_{\alpha\beta} &= x_\alpha\partial_\beta - x_\beta\partial_\alpha, \\ &= x_\alpha\star\partial_\beta - x_\beta\star\partial_\alpha + \frac{i}{2}\theta^{\rho\sigma}(\eta_{\rho\alpha}\partial_\beta-\eta_{\rho\beta}\partial_\alpha)\partial_\sigma+\mathcal{O}(\theta^2). \end{aligned} \tag{26}$$

In the second line this result is rewritten in terms of the $\star$-product such that it also has a meaning in the abstract algebra[2]. Equation (22) rewritten in terms of the generators $M_{\alpha\beta}$ reads

$$[M_{\rho\sigma}, M_{\alpha\beta}] = \eta_{\rho\beta}M_{\sigma\alpha}+\eta_{\sigma\alpha}M_{\rho\beta}-\eta_{\rho\alpha}M_{\sigma\beta}-\eta_{\sigma\beta}M_{\rho\alpha}. \tag{27}$$

If derivatives are included as well,

$$[M_{\alpha\beta},\partial_\mu] = \eta_{\mu\alpha}\partial_\beta - \eta_{\mu\beta}\partial_\alpha \quad \text{and} \quad [\partial_\mu,\partial_\nu]=0 \tag{28}$$

we see that the algebra sector of the θ-deformed Poincaré transformations is undeformed. Let us now look at the coproduct for this transformations. From (24) it follows

$$\begin{aligned} \Delta M_{\alpha\beta} &= M_{\alpha\beta}\otimes 1 + 1\otimes M_{\alpha\beta} \\ &\quad +\frac{i}{2}\theta^{\rho\sigma}\Big((\eta_{\rho\alpha}\partial_\beta-\eta_{\rho\beta}\partial_\alpha)\otimes\partial_\sigma+\partial_\rho\otimes(\eta_{\sigma\alpha}\partial_\beta-\eta_{\sigma\beta}\partial_\alpha)\Big)+\mathcal{O}(\theta^2). \end{aligned} \tag{29}$$

Splitting $M_{\alpha\beta}$ into orbital and spin parts gives

$$\begin{aligned} \Delta L_{\alpha\beta} &= L_{\alpha\beta}\otimes 1 + 1\otimes L_{\alpha\beta} \\ &\quad +\frac{i}{2}\theta^{\rho\sigma}\Big((\eta_{\rho\alpha}\partial_\beta-\eta_{\rho\beta}\partial_\alpha)\otimes\partial_\sigma+\partial_\rho\otimes(\eta_{\sigma\alpha}\partial_\beta-\eta_{\sigma\beta}\partial_\alpha)\Big)+\mathcal{O}(\theta^2), \end{aligned} \tag{30}$$

$$\Delta\Sigma_{\alpha\beta} = \Sigma_{\alpha\beta}\otimes 1 + 1\otimes\Sigma_{\alpha\beta}. \tag{31}$$

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[2] In the abstract algebra $L_{\alpha\beta}$ reads

$$L_{\alpha\beta} = \hat{x}_\alpha\hat{\partial}_\beta - \hat{x}_\beta\hat{\partial}_\alpha + \frac{i}{2}\theta^{\rho\sigma}\big(\eta_{\rho\alpha}\hat{\partial}_\beta - \eta_{\rho\beta}\hat{\partial}_\alpha\big)\hat{\partial}_\sigma.$$

We see that the coproduct for the orbital part of the generator $M_{\alpha\beta}$ is deformed, while for the spin part we obtain the undeformed coproduct. For the completeness we rewrite the Leibniz rule (9) in terms of the coproduct

$$\Delta\partial_\mu = \partial_\mu \otimes 1 + 1 \otimes \partial_\mu. \tag{32}$$

One can check that these coproducts are coassociative and consistent with the algebra (27), (28). Adding counits and antipods defines the θ-deformed Poncaré Hopf algebra[3]. One should notice that the generators $M_{\alpha\beta}$ do not close the Hopf algebra themselves since in (29) derivatives appear. Using different approaches, this result was obtained in [11] also.

Application

Having the θ-deformed Poincaré symmetry at hand, one can construct theories that are invariant under this symmetry and analyse their properties. We give one very simple example. Let us consider ϕ^3 theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi) \star (\partial^\mu\phi) - \frac{m^2}{2}\phi \star \phi - \lambda\phi \star \phi \star \phi. \tag{33}$$

One checks that under the deformed Poincaré transformations this Lagrangian density transforms as

$$\delta_\omega \mathcal{L} = -(X^\star_\omega \star \mathcal{L}) = -x^\alpha \omega_\alpha{}^\lambda (\partial_\lambda \mathcal{L}). \tag{34}$$

To construct the action we use the usual integral and obtain

$$S = \int \mathrm{d}^4x \,\Big(\frac{1}{2}(\partial_\mu\phi) \star (\partial^\mu\phi) - \frac{m^2}{2}\phi \star \phi - \lambda\phi \star \phi \star \phi\Big). \tag{35}$$

From (34) it follows that this action is invariant. Using the variational principle

$$\begin{aligned} \delta S &= \delta\Bigg(\int \mathrm{d}^4x \,\Big(-\frac{1}{2}\phi \star (\partial^\mu\partial_\mu\phi) - \frac{m^2}{2}\phi \star \phi - \lambda\phi \star \phi \star \phi\Big)\Bigg) \\ &= \int \mathrm{d}^4x \,\delta\phi(x) \star \Big(-2\frac{1}{2}(\partial^\mu\partial_\mu\phi) - 2\frac{m^2}{2}\phi - 3\lambda\phi \star \phi\Big) \end{aligned} \tag{36}$$

gives the following equation of motion

$$(\partial^\mu\partial_\mu\phi) + m^2\phi + 3\lambda\phi \star \phi = 0. \tag{37}$$

[3] All the results presented in this section are known to all orders in the deformation parameter [1]. Because of the simplicity only the first order expansions are presented here.

The other way to obtain (37) (expanded in the deformation parameter) is to first expand the $\star$-products in the action (35) and then vary it with respect to the field ϕ.

5. κ-DEFORMED POINCARÉ SYMMETRY

Now we apply the technique from the previous section to construct a deformed symmetry for the κ-deformed space. The underlying idea is to compare the symmetry obtained in this way with the already known κ-Poincaré symmetry described shortly in the first section. The transformation law of a scalar field under the infinitesimal diffeomorphisms is

$$\begin{aligned}\delta_\xi\phi &= -\xi^\mu\partial_\mu\phi = -(X^\star_\xi \star \phi)\\ &= -\xi^\mu \star (\partial_\mu\phi) + \frac{i}{2}C^{\rho\sigma}_\lambda x^\lambda(\partial_\rho\xi^\mu)\star(\partial_\sigma\partial_\mu\phi) + \mathcal{O}(a^2),\end{aligned} \tag{38}$$

where $\xi^\mu(x)$ is an arbitrary function. For the special case of translations, $\xi^\mu = b^\mu = const.$ (38) gives

$$\delta^t_\xi\phi = -b^\mu \star (\partial_\mu\phi) = -b^\mu(\partial_\mu\phi). \tag{39}$$

For the Lorentz rotations, $\xi^\mu = x^\nu\omega_\nu{}^\mu$ we have

$$\begin{aligned}\delta^l_\xi\phi &= -x^\lambda\omega_\lambda{}^\mu \star (\partial_\mu\phi) + \frac{i}{2}C^{\rho\sigma}_\lambda x^\lambda\omega_\rho{}^\mu \star (\partial_\sigma\partial_\mu\phi)\\ &= -\frac{1}{2}\omega^{\alpha\beta}(L_{\alpha\beta}\phi),\end{aligned} \tag{40}$$

where $L_{\alpha\beta} = x_\alpha\partial_\beta - x_\beta\partial_\alpha$[4]. Transformations (39) and (40) close in the undeformed algebra

$$\begin{aligned}[][L_{\mu\nu}, L_{\rho\sigma}] &= \eta_{\mu\sigma}L_{\nu\rho} + \eta_{\nu\rho}L_{\mu\sigma} - \eta_{\mu\rho}L_{\nu\sigma} - \eta_{\nu\sigma}L_{\mu\rho},\\ [\partial_\rho, \partial_\sigma] &= 0,\\ [L_{\mu\nu}, \partial_\rho] &= \eta_{\nu\rho}\partial_\mu - \eta_{\mu\rho}\partial_\nu.\end{aligned} \tag{41}$$

Their coproducts are

$$\begin{aligned}\Delta\partial_n &= \partial_n\otimes 1 + 1\otimes\partial_n,\\ \Delta\partial_j &= \partial_j\otimes(1-\frac{ia}{2}\partial_n) + (1+\frac{ia}{2}\partial_n)\otimes\partial_j + \mathcal{O}(a^2).\end{aligned} \tag{42}$$

$$\begin{aligned}\Delta L_{\alpha\beta} &= L_{\alpha\beta}\otimes 1 + 1\otimes L_{\alpha\beta}\\ &\quad -\frac{ia}{2}\Big(\delta^n_\alpha(\partial_\beta\otimes x^\lambda\partial_\lambda - x^\lambda\partial_\lambda\otimes\partial_\beta) - \alpha\longleftrightarrow\beta\Big) + \mathcal{O}(a^2).\end{aligned} \tag{43}$$

[4]Note that $L_{\alpha\beta}$ can also be written in terms of the $\star$-product in analogy with (26).

From (43) it is obvious that $\Delta L_{\alpha\beta}$ does not close in the algebra of derivatives and Lorentz generators (Poincaré algebra). Therefore, we have to enlarge the algebra and include coordinates as well. The way that coordinates appear in (43) suggests introducing dilatation operator. Inserting $\xi^\mu = \epsilon x^\mu$ with ϵ real constant in (38) gives for infinitesimal dilatations

$$\delta^d_\xi \phi = -\epsilon x^\mu \star (\partial_\mu \phi) = -\epsilon x^\mu (\partial_\mu \phi) = -\epsilon D\phi. \tag{44}$$

As the next step we check that generators ∂_μ, $L_{\alpha\beta}$ and D close in the undeformed algebra[5]. In addition to (41) we obtain

$$\begin{aligned} [D, D] &= 0, \\ [D, \partial_\mu] &= \partial_\mu, \\ [D, L_{\mu\nu}] &= 0. \end{aligned} \tag{45}$$

Coproduct of the generator of dilatations is

$$\Delta D = D \otimes 1 + 1 \otimes D - \frac{ia}{2}\Big(\partial_n \otimes D - D \otimes \partial_n\Big) + \mathcal{O}(a^2). \tag{46}$$

Coproduct of the Lorentz generators (43) can now be rewritten as

$$\begin{aligned} \Delta L_{\alpha\beta} = {} & L_{\alpha\beta} \otimes 1 + 1 \otimes L_{\alpha\beta} \\ & - \frac{ia}{2}\Big(\delta^n_\alpha(\partial_\beta \otimes D - D \otimes \partial_\beta) - \alpha \longleftrightarrow \beta\Big) + \mathcal{O}(a^2). \end{aligned} \tag{47}$$

From (47) we see that $\Delta L_{\alpha\beta}$ closes in the algebra of ∂_μ, $L_{\alpha\beta}$ and D generators. Adding counits and antipodes we obtain the κ-deformed Weil Hopf algebra. In this way we have constructed another deformed symmetry for the κ-deformed space. Comparing this result with the κ-Poincaré Hopf algebra, we see that this two quantum symmetries are not equal. The problem is that in the "$\star$-product inversion" approach coordinates naturally appear and one is forced to exchange Poincaré algebra for a larger one (in this case Weil algebra). The connection between these two symmetries is still not understood properly and this is left for the further research.

[5] This step is obvious. The transformations (39), (40) and (44) are classical transformations and therefore the algebra is undeformed.

6. CONCLUSIONS

We presented here two special examples of noncommutative spaces. Based on the "inversion of the $\star$-product method" we constructed the deformed Poincaré symmetry that acts on these spaces. Using these symmetries invariant actions and equations of motion can be obtained. However, the question of conserved quantities is still not clear. For the example in the 4th sectionone can construct the energy-momentum tensor, but it seems to be either conserved or symmetric and not both. This remains as an open question and will be considered in the future.

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