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FLUX COMPACTIFICATION OF TYPE IIB SUPERGRAVITY

Klaus Behrndt

Arnold-Sommerfeld-Center for Theoretical Physics
Department für Physik, Ludwig-Maximilians-Universität
Theresienstraße 37, 80333 München, Germany

Abstract. *In order to lift the continuous moduli space of string vacua, non-trivial fluxes may be the essential input. In this talk I summarize aspects of two approaches to compactifications in the presence of fluxes:* (*i*) *generalized Scherk-Schwarz reductions and gauged supergravity and* (*ii*) *the description of flux-deformed geometries in terms of G-structures and intrinsic torsion. Especially the type IIB case is considered.*

Key words: *supergravity, fluxes, compactification*

1. Introduction

A major problem in most string compactifications is the emergence of a moduli space of string vacua. Moduli appear in two guises: (i) the closed string moduli, which are related to deformations of internal cycles and (ii) open string moduli, that parameterize the positions of branes wrapped in the internal space. The standard model of particle physics as well as inflationary cosmology are not compatible with free moduli which is moduli problem. If supersymmetry is broken only at fairly low energies, we need a mechanism that fixes all moduli while still preserving some supersymmetries and so far, only fluxes seem to be able to lift both types of moduli in a supersymmetric way. Fluxes are nothing but RR- and/or NS-forms, which do not vanish in the vacuum and they exert forces on cycles. A non-vanishing flux presents an energy density and hence corresponding cycle tend to expand. The opposite happes with a perpendicular flux, ie. it yields a shrinking cycle. These

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two competing effects yield the stabilization and it is not the quantization condition for fluxes that yield the lifting. The open string moduli are fixed by fluxes due to their coupling to the world volume of the branes provides a potential and since the open string moduli are compact (for compact internal space), the potential will have extrema. In addition to these form-field fluxes exist also metric fluxes, as eg. given by twisted tori.

All fluxes are related to Cherk-Schwarz reductions which are especially interesting because they yield a consistent truncation on the massless KK spectrum. In addition, they are related to gauged supergravity and the potential can be calculated explicitly, which is helpful for many purposes. On the other hand gauged supergravity does not yield the deformed (internal) geometry and thus the lifting to 10 dimensions is in most cases not clear. To get the deformed internal geometry, one has instead to solve directly the 10-dimensional equations, which uncovers not only the embedding of the internal fluxes but also classifies the solution with respect to the torsion classes (or the G-structures as defined by a set of differential forms).

We have organized this paper as follows. In Section 2 we explain the relation between Scherk-Schwarz reductions and gauged supergravity and Section 3 is devoted to the second approach, ie. we solve the 10-dimensional (type IIB) Killing spinor equations and relate fluxes to torsion components and G-structures. For more details we refer to an extended talk on this subject [1].

2. Fluxes, Scherk-Schwarz reduction and gauged supergravity

One may argue that in the vacuum all fields should be trivial and the metric is (Ricci-) flat. This strong restriction is not justified, and non-zero values of RR- and NS-fields can still be considered as a viable vacuum configuration – at least as long as they respect the *4-dimensional* Poincaré symmetry. If so, we are dealing with compactifications in presence of fluxes and one can distinguish between gauge field and metric (or geometric) fluxes, that are related by supersymmetry. These fluxes generate a non-zero energy-momentum tensor and hence the internal metric is in general not Ricci-flat; the resulting geometries can be quite complicated (as we will see later).

To make this more explicit let us note, that in the simplest case gauge field fluxes can be generated by a linear dependence (on the internal coordinates) of the KK scalars coming from gauge fields. The gauge symmetry implies that these scalars appear only via derivatives in the Lagrangian/equations of motion and hence, a linear dependence on the internal coordinates leaves the Lagrangian still independent of the internal coordinates and one can integrate over them. This is known as the (generalized) Scherk-Schwarz reductions, which can be applied to any global symmetry and especially to those KK scalars, that parameterize an isometry of the moduli space. In general, this procedure does not commute all supersymmetry transformations and hence supersymmetry is at least partially broken. In the original Scherk-Schwarz reduction this was done with respect to a fermionic phase transformation, which did not commute with the supersymmetry transformation and hence supersymmetry was broken completely [2]. In the case at hand, we apply it to isometries of the moduli spaces and some supersymmetries remain unbroken, or in other words, some supersymmetry transformations commute with the (generalized) Scherk-Schwarz reduction. More details on these reductions are given in the literature, see [3] – [7].

From the lower dimensional point of view Scherk-Schwarz reductions correspond to a gauging of the corresponding global symmetry and following [8] let us discuss a simple example. If we just keep the axion-dilaton coupling, the type IIB supergravity action reads

$$S \sim \int \sqrt{g}\,\Big[R - \frac{g^{MN}\partial_M\tau\partial_N\bar{\tau}}{|\tau-\bar{\tau}|^2}\Big] \tag{1}$$

which exhibits, as part of the SL(2,R) symmetry, the axionic shift symmetry $\tau \to \tau + c$ for any $c = const.$ In the Scherk-Schwarz reduction over one coordinate (say y) one assumes $c = my$ and hence one writes

$$\tau(x,y) = \tau(x) + my\ . \tag{2}$$

For the metric, one makes the usual KK-Ansatz

$$ds^2 = e^{2\sigma}(dy + A_\mu dx^\mu)^2 + g_{\mu\nu}dx^\mu dx^\nu$$

where ∂_y is a Killing vector. Thus, the kinetic term yields

$$\frac{g^{MN}\partial_M\tau\partial_N\bar{\tau}}{|\tau-\bar{\tau}|^2} \;=\; \frac{g^{\mu\nu}D_\mu\tau(x)D_\nu\bar{\tau}(x)}{|\tau(x)-\bar{\tau}(x)|^2} + \frac{m^2}{|\tau(x)-\bar{\tau}(x)|^2}\,e^{-2\sigma}$$

where the second term is a (run-away) potential and the covariant derivative in the kinetic term is $D_\mu\tau = \partial_\mu\tau(x) - mA_\mu$. Therefore, the scalar field $\mathrm{Re}(\tau)$ is now charged with respect to the *local* shift transformation

$$\tau \to \tau + c(x) \quad , \qquad A \to A + \frac{1}{m}dc$$

and in the original metric the gauge transformation in A can be absorbed by a coordinate transformation $y \to y + c(x)$. Obviously, the same result can be obtained by a gauging of the global shift symmetry in the reduced theory. The charged scalar, $\mathrm{Re}(\tau)$, does not enter the potential and represents a flat direction, which is required by gauge invariance and which in turn can be used to gauge away the scalar giving a mass to the gauge boson (the kinetic term for $\mathrm{Re}(\tau)$ becomes a mass term for A_μ).

This Scherk-Schwarz reduction was related to the isometry ∂_y and was generated an internal flux given by the 1-form: $d_y \tau = m dy$ and as result the scalar field $\mathrm{Re}(\tau)$ became charged under the corresponding KK vector field. A general Calabi-Yau space has no isometries and the internal metric does not give rise to 4-dimensional vector fields, but nevertheless there is an analogous mechanism which relates flux compactification to gauged supergravity. To be concrete we follow now [9], consider the type IIB case with fluxes for the NS-2-form B and write

$$B + iJ = u^a(x,y)\omega_a = [u^a(x) \, + c^a(y)] \, \omega_a \ .$$

The coefficients $c^a(y)$ are fixed by the requirement that the corresponding field strength yields a real *internal* 3-form (= flux), which can be expanded in the basis $\{\chi^k, \chi_k\}$ with the *constant* coefficients m_k, ie. $dc^a(y) \wedge \omega^a = m^k \chi_k = H^{flux}$ giving

$$d(B + iJ) = du^a(x) \, \wedge \omega_a + (m^k \, \chi_k + cc) \ .$$

To keep the notation simple, we drop all indices ($m^k \to m$) and collecting the terms containing this mass deformation yields for the 5-form

$$F_5 \ = \ dC_4 - \frac{1}{4} C_2 \wedge H \ = \ [dA - \frac{1}{4} m \, C_2^{(ext)}] \wedge \chi + cc + \cdots$$

where C_2^{ext} is the external component of the 2-form [note, a Calabi-Yau has no non-trivial 5-forms and therefore $\omega \wedge \chi = 0$]. Now, the kinetic term for this 5-form yields exactly a massive 2-form coupling in 4 dimensions

$$\Big[(dA)_{\mu\nu} - \frac{1}{8} \, m \, C^{(ext)}_{\mu\nu}\Big]^2$$

and this expression has to be dressed up with the metric of the complex structure moduli space (we suppressed the indices). This massive 2-form can be dualized to a massive vector, where the charged scalar is given by the dual of $C_2^{(ext)}$.

The realization of flux compactifications within gauged supergravity opens the possibility to understand the moduli stabilization within gauged supergravity and we shall summarize in the following some essentials. The starting point is the Lagrangian of ungauged supergravity with N=2 supersymmetry in 4 dimensions, which is obtained from standard Kaluza-Klein reduction giving rise to a continuous moduli space: $\mathcal{M} = \mathcal{M}_V \times \mathcal{M}_H$, where $\mathcal{M}_V$ and $\mathcal{M}_H$ are parameterized by the scalars belonging to the vector multiplets and to the hyper multiplets, respectively. Potentials that are consistent with $\mathcal{N} = 2$ supersymmetry are obtained by performing a gauging of various global symmetries. There are two different types of gaugings, namely (i) one can either gauge some of isometries of the moduli space of ungauged $\mathcal{N} = 2$ supergravity or (ii) one can gauge (part of) the $SU(2)$ R-symmetry, which only acts on the fermions. We are interested in a gauging that generate a potential for both types of scalars and we discuss gaugings of isometries of $\mathcal{M}_H$.

Scalar fields in hyper multiplets parameterize a quaternionic Kahler manifold $\mathcal{M}_H$ and these spaces possess three complex structures J^x as well as a triplet of Kahler two-forms K^x ($x = 1, 2, 3$ denotes the $SU(2)$ index). The holonomy group of these spaces is $SU(2) \times Sp(n_H)$ and the Kahler forms are covariantly constant with respect to the $SU(2)$ connection. The isometries of $\mathcal{M}_H$ are generated by a set of Killing vectors $k_I = k_I^u \partial_u$

$$q^u \to q^u + k_I^u \epsilon^I \tag{3}$$

where "I" counts the different isometries and q^u are the scalar fields of hyper multiplets. The gauging of (some of) the Abelian isometries gives gauge covariant derivatives $dq^u \to dq^u + k_I^u A^I$ so that the vector field become massive. In order to maintain supersymmetry, the gauging has to preserve the quaternionic structure, which implies that the Killing vectors have to be tri-holomorphic, which is the case whenever it is possible to express the Killing vectors in terms of a triplet of real Killing prepotentials $\mathcal{P}_I^x$ as follows:

$$K_{uv}^x k_I^v = -\nabla_u \mathcal{P}_I^x \equiv -\partial_u \mathcal{P}_I^x - \epsilon^{xyz} \omega_u^y \mathcal{P}_I^z \tag{4}$$

where ω_u^y is the $SU(2)$ connection giving the Kahler forms by $K_{uv}^x = -\nabla_{[u}\omega_{v]}^x$. By using the Pauli matrices σ^x one can also use a matrix notation: $\mathcal{P}_I = \sum_{x=1}^3 \mathcal{P}_I^x \, \sigma^x$. With these Killing prepotentials one can define an SU(2)-valued superpotential by [10, 11]

$$W^x = X^I \mathcal{P}_I^x \equiv X^I(z) \mathcal{P}_I^x(q) \ , \tag{5}$$

where $\{z, q\}$ denote collectively the scalars from vector and hyper multiplets. A real valued superpotential can be defined as $W^2 = e^K \det(W^x \sigma^x)$, where K is the Kahler potential of the special Kahler manifold $\mathcal{M}_V$. Supersymmetric vacua are extrema of the real superpotential, which are equivalent to a covariantly constant superpotential W^x. This gives as constraints for supersymmetric vacua

$$(i) \quad (\nabla_A X^I)\mathcal{P}^x_I = 0 \quad , \qquad (ii) \quad X^I(\nabla_u \mathcal{P}^x_I) = K^x_{uv}(X^I k^v_I) = 0 \tag{6}$$

where $\{\nabla_A, \nabla_u\}$ denote the Kahler/SU(2)-covariant derivatives with respect to the scalars $\{z^A, q^u\}$ in vector/hyper multiplets and $X^I = X^I(z)$ is part of the symplectic section (X^I, F_I) [F_I is the derivative of the prepotential $F(X)$ with respect to X^I].

In order to fix the moduli from the vector as well as hyper multiplet it was important that we gauged an isometry of the quaternionic space $\mathcal{M}_H$ or equivalently to add fluxes (3-form flux on the IIB e.g.) which make a scalar of a hyper multiplet massive. This is only the minimal requirement, on top of this gauging one may also consider to gauge isometries of vector multiplet moduli space $\mathcal{M}_V$. The resulting superpotential obtained from gauged quaternionic isometries was given in (5) with $X^I = X^I(z)$ as the "electric" part of the symplectic section $V = (X^I, F_I)$. It is a known problem, that gauged supergravity prefers the electric part and does not produces the magnetic part of the superpotential. But by taking into account also (massive) tensor multiplets, one can promote it to a manifestly symplectic expression [12]. An important property of this setup is however, that by a symplectic transformation one call always go into a strictly perturbative regime where all magnetic charges vanish so that the potential in (5) can always be considered. This property that the electric and magnetic charges are mutually local is a consequence of supersymmetric Ward identities.

We can now discuss the conditions of getting a complete lifting of the moduli space. A necessary condition for this is that the variations of the hyperino and gaugino vanish for constant scalars which yielded eqs. (6). The condition (ii) is equivalent to the existence of a fixed point for the Killing vector

$$k = X^I k_I$$

and the *complete* hyper multiplet moduli space is lifted if k has a NUT fixed point, ie. if it represents a *point* on $\mathcal{M}_H$. This excludes by the way, axionic shift symmetries and requires a *compact* isometry [11]. The fixed point set of a Killing vector field is always of even co-dimension, which is related to the rank of the 2-form dk calculated on the fixed point set. In fact, if the rank is maximal, ie. $\det(dk) \neq 0$, the fixed point set is in fact a point on the manifold and dk parameterizes a rotation around the fixed point. Otherwise, any zero mode of dk would parameterize a shift symmetry of the fixed point set and hence if $\det(dk) = 0$, the potential will have some flat directions. Therefore, we get the following two conditions for lifting the hyper multiplet moduli space

$$|k| = 0 \quad , \qquad \text{with :} \quad \det(dk) \neq 0 \ . \tag{7}$$

If we can find a Killing vector that satisfies both conditions, the hyper multiplet moduli space will be lifted in the vacuum. We should place a warning here. Although, the isometries on the classical level are well understood it is unclear whether the full quantum corrected moduli space has isometries at all, which makes the moduli fixing issue obscure – at least from the supergravity point of view. But we do not want to speculate here about the quantum moduli space for hyper multiplets and shall instead continue with the discussion of the second condition in (6). If the hyper scalars are fixed, the Killing prepotentials are some fixed functions of the scalars of the vector multiplets, ie. $\mathcal{P}_I = \mathcal{P}_I(q(z))$ and hence they vary over $\mathcal{M}_V$. If $\mathcal{P}_I$ would be constant, only one vacuum can occur, namely at the point where this constant symplectic vector is a *normal vector* on $\mathcal{M}_V$ [13]. But since $\mathcal{P}_I$ varies now, it might become normal at different points, related to the appearance of multiple critical points. If we calculate the second covariant derivatives on $\mathcal{M}_V$ at this fixed point, ie. $\nabla_{\bar{A}}\nabla_B X^I \mathcal{P}_I^x$ and use relations from special geometry[2] we find that all these critical points are isolated – at least as long as the metric does not degenerate. Therefore, there are no further constraints from the vector multiplet moduli space and the crucial relations that have to be realized are the ones in (7).

In the discussion so far we did not mention the fact that the potential obtained in gauged supergravity is always independent of the charged scalar field(s) and we have to ask whether this indicates some flat directions of the potential. There are two reasons why this does not spoil our discussion so far. On one hand, if the fixed point set is zero-dimensional (ie. a point) this flat direction is only an artificial angular coordinate on the moduli space. On the other hand, as we mentioned already before, the charged scalars "can be eaten" by the vector fields giving them a mass that corresponds to the eigenvalues of $(h_{uv} k_I^u k_J^v)|_{|k|=0}$. So, there are no moduli related to these scalars anymore.

[2]Because: $\nabla_{\bar{A}}\nabla_B X^I \sim g_{\bar{A}B} X^I$.

3. Deformed geometry and G-structures

Supersymmetry exchanges fermionic with bosonic degrees of freedom and in a supersymmetric vacua with trivial fermions, the fermionic variations have to vanish [the variations of the bosonic field vanish identical for trivial fermionic fields]. For type II supergravity, these are the gravitino $\delta\Psi_\mu$ (spin 3/2) and dilatino $\delta\lambda$ (spin 1/2) variation. On the IIB side, both Majorana-Weyl spinors have the same chirality and can be combined into a single (complex) Weyl spinor so that the variations can be written in the Einstein frame as [14, 15]

$$\begin{array}{llcl} \text{IIB}: & \delta\Psi_M & = & \Big[D_M - \frac{i}{2}Q_M + \frac{i}{480}F^{(5)}\Gamma_M\Big]\epsilon - \frac{1}{96}\Big[G^{(3)}\,\Gamma_M + 6G^{(3)}_M\Big]\epsilon^\star\ , \\ & \delta\lambda & = & i\,P\epsilon^\star - \frac{i}{24}G^{(3)}\epsilon \end{array} \tag{8}$$

with: $P = \frac{1}{1-|T|^2}dT$, $G^{(3)} = \frac{1}{\sqrt{1-|T|^2}}(F_3 - TF_3^\star)$, $F^{(5)} = dC_4 - \frac{1}{4}\,(C_2 \wedge dB)$ and $Q = \frac{1}{1-|T|^2}\mathrm{Im}(TdT^\star)$, $T = \frac{1+i\tau}{1-i\tau}$. In these variations all indices are contracted with Γ-matrices. The number of unbroken supersymmetries is given by the zero modes of these equations, i.e. the number of Killing spinors for which these variations vanish. The 10-dimensional spinor can be expanded in all independent internal and external spinors.

Having only one internal spinor η, which is SU(3) singlet, SU(3)-structures are given by the 2-form and a 3-form

$$\eta^\dagger\,\gamma_{mn}\eta = i\,J_{mn} \quad , \qquad \eta^T\gamma_{mnp}\eta = i\,\Omega_{mnp} \tag{9}$$

[with $1 = \eta^\dagger\eta$] where J is a symplectic form with $J^2 = -1$ and can be used to define (anti) holomorphic coordinates and Ω is then the holomorphic 3-form. All other fermionic bi-linear vanish as result of identities for 6-d γ-matrices and hence no further (regular) singlet forms can be build. If the spinor is covariantly constant, these forms are closed and the structure group is identical to the holonomy – if not, the holonomy is not inside SU(3) and the space cannot be Calabi-Yau (not even complex in general). The failure of the structure group to be the holonomy is measured by torsion classes. Following the literature [16, 17], one introduces five classes $\mathcal{W}_i$ by

$$\begin{array}{lcl} dJ & = & \frac{3i}{4}\,(\mathcal{W}_1\bar\Omega - \bar{\mathcal{W}}_1\Omega) + \mathcal{W}_3 + J\wedge\mathcal{W}_4\ , \\ d\Omega & = & \mathcal{W}_1 J\wedge J + J\wedge\mathcal{W}_2 + \Omega\wedge\mathcal{W}_5 \end{array} \tag{10}$$

with the constraints: $J\wedge J\wedge\mathcal{W}_2 = J\wedge\mathcal{W}_3 = \Omega\wedge\mathcal{W}_3 = 0$. Depending on which torsion components $\mathcal{W}_i$ are non-zero, one can classify the geometry of the internal space. E.g., if only $\mathcal{W}_1 \neq 0$ the space is called nearly Kahler, for $\mathcal{W}_2 \neq 0$ almost Kahler, the space is complex if $\mathcal{W}_1 = \mathcal{W}_2 = 0$ and it is Kahler if only $\mathcal{W}_5 \neq 0$.

For the 4-dimensional external space, supersymmetry requires that it has to be, up to warping, flat or anti-deSitter and hence we make the Ansatz for the metric

$$ds^2 = e^{2A(y)} \Big[g_{\mu\nu} dx^\mu dx^\nu + h_{mn}(y)\, dy^m dy^n \Big] \tag{11}$$

where $g_{\mu\nu}$ is either flat or AdS_4, h_{mn} is the metric of the internal space and the warp factor depend only on the coordinates of the internal space. In the vacuum all off-diagonal terms should vanish and the fluxes should have only internal components or are proportional to the 4-dimensional volume form. These constraints are required by 4-dimensional Poincaré invariance.

The type IIB side has been discussed in the literature already extensively, see [18, 19, 20] and we want to summarize here only some aspects. Again we admit only fluxes that are consistent with 4-dimensional Poincaré symmetry, ie. they have components along the internal space with the only exception of the 5-form, that has to have components along the external space; required by the self-duality.

An important property on the IIB side is, that as long as one keeps SU(3) structures, the vacuum has to be flat, ie. a cosmological constant can be generated by fluxes [21]. This may indicate, that SU(3) structures always yield potentials of the no-scale form which are positive definite, but this needs to be verified for the most general fluxes consistent with SU(3) structures. Recall, the no-scale structure is only an approximation and corrections (quantum corrections, D3-instanton corrections etc.) do not respect this property and yield anti deSitter vacua and therefore these corrections have to break the SU(3) down to SU(2) structures. Let us stress that we are using here only supersymmetry and therefore our approach is valid for classical and quantum geometry as long as at least four supercharges remain unbroken!

Depending on the concrete form of the spinor one finds again different solutions and the most general spinor, consistent with SU(3) structures can be written as

$$\epsilon = a\ [\theta \times \eta] + b^\star\ [\theta^\star \times \eta^\star] \tag{12}$$

where both spinors are chiral and a and b are complex coefficients. There are two special cases: (i) for $a = b$, the 10-dimensional spinor ϵ is Majorana-Weyl which gives the NS-sector solution and (ii) if $ab = 0$ which was explored to a large extend in [18] and this case still allows the internal space to be Calabi-Yau.

Interestingly it is possible to solve the Killing spinor equations without making any assumptions [21]. The type IIB supergravity in our notation has a local U(1) symmetry which becomes manifest if we define the phase $e^{2i\theta} = \frac{1+i\bar{\tau}}{1-i\tau}$ and write the fields in as [15]

$$Q_M = \partial_M \theta - \tfrac{\partial_M \tau_1}{2\tau_2} \quad , \quad P_M = ie^{2i\theta} \tfrac{\partial_M \tau}{2\tau_2} \quad , \quad G_{(3)} = i\tfrac{e^{i\theta}}{\sqrt{\tau_2}}(dA_{(2)} - \tau dB_{(2)}) \ . \tag{13}$$

The phase θ drops out from the equations of motion as well as from the Bianchi identities and the underlying symmetry is the local U(1) gauge transformation

$$\epsilon \to e^{ig}\epsilon \quad , \qquad \theta \to \theta + g \tag{14}$$

for some function g. This local symmetry is due to the coset $SL(2,R)/U(1)$ which is parameterized by the scalar fields of type IIB supergravity and implies that the phase θ can be chosen freely, one can take $\theta = 0$ (string theory convention) or $e^{2i\theta} = \frac{1+i\bar{\tau}}{1-i\tau}$ (supergravity convention) or any other value. Recall, we are working in the Einstein frame which explains the prefactor $\tau_2^{-1/2} = e^{\phi/2}$ in the 3-form G_3.

We can write the spinor (12) as

$$\epsilon = e^{\frac{A-i\omega}{2}} \Big(\sin\alpha \, [\zeta \otimes \chi] + \cos\alpha \, [\zeta^\star \otimes \chi^\star] \Big) \tag{15}$$

where the appearance of the warp factor is a consequence of the gravitino variation [22, 21]. We absorbed the common phase of a and b into the spinor ($\chi = e^{i\beta}\chi_0$) and this phase drops out in most of the calculations.

The 5-form flux is again parameterized by the function Z and for the 3-form flux one find the form

$$G = \frac{1}{4} e^{-2A-i\omega} J \wedge \Big(\cot\alpha \ P_i dz^i + \tan\alpha \ P_{\bar{i}} d\bar{z}^i \Big) + G^{(prim)} \tag{16}$$

with the primitive part obeying: $J \wedge G^{(prim)} = \Omega \wedge G^{(prim)} = \bar{\Omega} \wedge G^{(prim)} = 0$; P_i is the holomorphic part of the vector introduced in (13) and z^i denote the three coordinates parameterizing the internal space. Now, the solution of the Killing spinor equation is given in terms of one holomorphic function

$$f = f(z^i) \tag{17}$$

and can be written as [21]

$$\begin{aligned}
\tau &= c_0 + i\,e^{-\phi_0}\,\frac{|f|^2\,\cos 2\alpha}{f\,\sin^2\alpha + f^\star\,\cos^2\alpha}\;, \\
e^{-4A} &= \frac{\mathrm{Re}f}{4|f|^2}\,\frac{\sin^2 2\alpha}{\cos 2\alpha}\;, \\
Z &= \frac{|f|^2}{\mathrm{Re}f}\,\frac{\cos^2 2\alpha}{\sin^2 2\alpha} \\
\tan(\theta+\omega) &= -\frac{\mathrm{Im}f}{\mathrm{Re}f}\cos 2\alpha\;.
\end{aligned} \tag{18}$$

Using the local symmetry (14) we can set ω or θ to any fixed value, but the combination $\theta + \omega$ is gauge invariant. Note, supersymmetry leaves one function (in addition to the holomorphic function f) free which has to be fixed by the Bianchi identities or equations of motion; this is the master function in [23]. We chose here α, which is the mixing angle between the two spinors, but one may also take Z which can be fixed by the Bianchi identity $dF_5 \sim G \wedge \bar{G}$. The Calabi-Yau case is of course a special (where $\alpha \simeq 0$), where the axion-dilaton and the vector P are holomorphic. For the general case, the internal geometry is a complex manifold and becomes (conformal) Kahler if: (i) if the primitive part of G vanishes and (ii) $dZ \wedge dA = 0$, which can be seen as a constraint on the function f. Another special case are the solutions describing supergravity flows, that correspond to the case where the holomorphic function is constant $f = constant$. For more discussion we refer to [21].

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