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GENERATION OF THE SHEARED RADIAL ELECTRIC FIELD BY A MAGNETIC ISLAND STRUCTURE

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Abstract. *The effect of the presence of a magnetic island structure on the ambipolar radial electric field is studied in the context of the belt island model. It is shown that the sheared radial electric field region exists on the island position. Depending on the model parameters, the single (ion root) or multiple (one ion and two electron roots) solutions for the radial electric field are obtained at different radial positions. The radially non-local treatment is developed proposing the steady-state plasma conditions. The numerical calculations show that the diffusion of the radial electric field is significant only near the island boundaries. As a result the discontinuities in the ambipolar electric field profile are smoothed.*

1. INTRODUCTION

The investigations of the transport processes in toroidal plasmas (such as in tokamak and stellarators) [1]-[2] have shown that the sheared radial electric field reduces the radial correlation of turbulence and suppresses the anomalous transport in plasma environment. This is of special interest in the context of the ETB (External Transport Barrier or H-mode) and ITB (Internal Transport Barrier) [3]-[6] discharges in confinement devices, which are associated with the large shear of the radial electric field. Thus, the attempt is to investigate the mechanisms for the generation of the sheared radial electric field.

The topology of the magnetic field in torus plasmas changes in the presence of the small error fields, plasma instabilities etc. Depending on the characteristics of the mentioned effects (usually associated with the strength of the magnetic field perturbation) [2], [7] either the magnetic island chains around the rational magnetic surfaces or the magnetic field regions with more or less irregularities appear. These topology changes strongly effect plasma confinement [8]-[11]. For example the plasma density and temperature profiles are changed, and particle fluxes are modified [12]. In order to satisfy the

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ambipolarity constraint [2],[13] (the equality of the ion and electron fluxes), the modified fluxes cause changes in the radial electric field.

Thus, in this paper the effort is done in investigating the generation of the sheared radial electric field by the magnetic island structure. The simple analytical model based on local heat transport process is developed, which is in consistence with the experimental results (temperature profiles and transport scaling [14]). The steady-state spatial profiles of the plasma electron and ion temperature are generated in subsection 2.1 by solving the energy balance equation. In order to simplify the presentation a periodic cylinder approximation of the tokamak geometry and renormalized form of heat equation are used (eq. 4 in [14]). The procedure is based on the two fundamental assumptions: the heat transport is primarily a diffusive process, and the effect of the macroscopic disturbances is predominant in the region where it occurs [14]. The magnetic islands (changes in topology) shorten the effective radial path length for heat diffusion and reduce the temperature gradient inside the reconnected rational surface on which they are formed. This has been observed experimentally as flattening on the temperature profile [12]. Thus, in the present paper the magnetic island is modeled as a belt of uniformly flattened temperature in the vicinity of the corresponding rational surface. The characteristics of the belt island model are presented in subsection 2.2. The behaviour of the radial electric field is discussed in subsection 2.3. At first, the ambipolar radial electric field is numerically calculated following reference [15] in the frame of the nearly steady-state conditions (evolution of the radial electric field with stationary formed island structure). Additionally, the diffusion of the electric field is modeled following procedures developed in papers [2],[16]-[18]. The numerical results confirm that the magnetic islands generate the sheared radial electric field, as illustrated in section 3. It is shown that in general, the ambipolarity relation may have multiple solutions in diverse plasma parameter regions, as noted in literature [13],[19],[20]. However, in the presence of magnetic island the multiplicity is lost inside the island boundaries, i.e. only the ion root is found. Finally, conclusions are given in Section 4.

2. Model

2.1. The plasma temperature profile

The steady-state plasma temperature profile is modeled by the energy balance equation [14]:

$$n\frac{\partial T(r,t)}{\partial t} = -\nabla \cdot \vec{q}(r) + Q(r) \tag{1}$$

where n is the plasma density, r is the radius of the plasma column, t is the time, $\vec{q}(r)$ is the heat flux density and $Q(r)$ is the energy source term (i.e. the sum of all heat sources and sinks in plasma [14]). The ion and electron temperature and the corresponding densities are assumed to be equal due to simplicity. Initial temperature profile is given as:

$$T(0,r) = (T_0 - T_a)\left(1-\frac{r^2}{a^2}\right) + T_a \tag{2}$$

where the parameters T_0 and T_a are the plasma temperature at $r = 0$ and $r = a$, respectively. Parameter a is the minor radius of the plasma column. The balance equation is computed with the boundary conditions:

$$\left.\frac{\partial T(r,t)}{\partial r}\right|_{r=0} = 0, \quad T(a,t) = T_a \tag{3}$$

The heat flux density is given as:

$$\vec{q}(r) = -n\chi(r)\nabla T(r) \tag{4}$$

with the thermal diffusivity modeled as:

$$\chi(r) = \chi_0 \exp(\alpha(r/a)^2) \tag{5}$$

where the parameter α is determined from the values of the thermal diffusivity at $r = 0$ and $r = a$:

$$\alpha = \ln\left(\frac{\chi_a}{\chi_0}\right) \tag{6}$$

The heat source term is derived specifying the total input power P_{in} as the volume integral $P_{in} = \int_V Q(r)dV$:

$$Q(r) = \frac{P_{in}}{2\pi^2 R_0 a^2 \Delta x(1 - \exp(-1/\Delta x^2))} \exp\left(-\frac{r^2}{a^2 \Delta x^2}\right) \tag{7}$$

where the parameter Δx is the measure for the radial extent of the heat deposition [14] and R_0 is the major radius of the plasma column.

2.2. Magnetic field configuration

The equilibrium, regular magnetic field structure is chosen in the form

$$B = B_0(1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(2\theta - 10\varphi)) \tag{8}$$

where ε_t and ε_h denote the toroidal and helical inhomogenity of the magnetic field strength B. Parameter B_0 is the magnetic field strength at the magnetic axis and θ/φ are the poloidal/toroidal angles, respectively. The poloidal polarity and the toroidal field period numbers are assumed as 2 and 10 as in the Large Helical Device (LHD) [4] with $\varepsilon_t = 0.10$ and $\varepsilon_h = 0.32$.

2.3. Belt island model

Small error fields and small magnetic field perturbations generically connected with some plasma instabilities could result in reconnection of the magnetic field lines on the rational surfaces, according to the KAM theorem [7]. When the perturbation is small magnetic field structure still provides strong constraints on the particle motion. Changes in topology shorten effective radial path length for heat diffusion. It can cause reduction of temperature gradient inside the reconnected rational surface which has been observed experimentally as flattening on the temperature profile [12]: the magnetic island structure is formed. The belt model ignores the helical structure of the magnetic island and replaces it by a belt of uniformly flattened temperature in the vicinity of the rational surface.

To simulate the temperature flattening the thermal diffusivity $\chi(r)$ is assumed to be expressed by

$$\chi(r) = \chi_0 \exp\left(\alpha\left(\frac{r}{a}\right)^2\right) + \chi_{is} \exp\left(-\beta\frac{(r-r_0)^2}{a^2}\right) \tag{9}$$

where the thermal diffusivity in the interior of the magnetic island, χ_{is}, is chosen as a large value in comparison with $\chi(a)$, $\chi(0)$ to simulate temperature flattening. The parameter β indicates the radial extent of the magnetic island which is determined specifying the central position x_0 and the width h of the belt magnetic island.

2.3. The model equations for the radial electric field

The radial electric field is calculated from the ambipolarity equation [13] :

$$\Gamma_e = \Gamma_i \,. \tag{10}$$

In the present paper the electron and ion fluxes $\Gamma_{e,i}$, respectively, are calculated adopting the Kovrizhnykh's formulae [15]. Accordingly, the E_r value depends on the local value of the plasma parameters.

At the next step, the evolution of the E_r is modelled as:

$$\frac{\partial E}{\partial t} = -\frac{1}{\varepsilon_0\varepsilon_r}\sum_j e_j\Gamma_j + \frac{1}{r}\frac{\partial}{\partial r}\left(rD_E\frac{\partial E}{\partial r}\right) \tag{11}$$

combining the Ampere's law, continuity equations for density and temperature in the context of the drift-kinetic theory [2]. In equation (11) the parameter D_E is the diffusion coefficient, ε_r is the low frequency perpendicular dielectric function, ε_0 is the dielectric constant of the vacuum and j describes particle species. The approximative value of D_E is estimated properly adopting Braginskii's results [21] to the present model. Initial and boundary conditions are chosen as:

$$E_r(r,t=0) = E_{ramb}, \quad \left.\frac{dE_r(r,t)}{dr}\right|_{r=0} = 0 \tag{12}$$

Finally, it is worth to stress explicitly that the adopted model is based on the two fundamental assumptions: the heat transport is primarily a diffusive process, and the effect of the macroscopic disturbances is predominant in the region where it occurs.

3. NUMERICAL RESULTS

Effect of the stationary magnetic island on the generation of the sheared radial electric field and the evolution of formed radial electric field are considered following the three staged numerical procedure.

Firstly, the steady-state plasma temperature profile is obtained as a long time solution of the equation (1) proposing initial and boundary conditions (2,3). As examples, the resulting temperature profiles without and with the belt magnetic island are shown in figure 1 for the set of parameters $P_{in} = 5$ MW, $P_{in} = 20$ MW, $T_0 = 100$ eV, $T_a = 1000$ eV, $x_0 = 0.55$ and $h/a = 0.1$. In the presence of the belt island the monotonic temperature profile in Figure 1 is changed by the profile with flat region on the island position. Corresponding the heat diffusivity profiles are plotted in Figure 2.

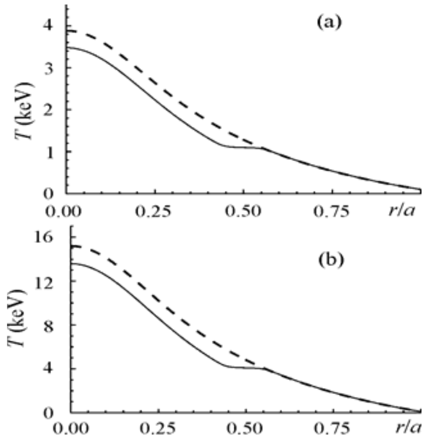

Fig. 1. The steady-state temperature profiles without (dashed curves) and with the magnetic island (straight curves) for plasma parameters: $T_0(0) = 100$ eV, $T_a(0) = 1000$ eV, $x_0 = 0.55$, $h = 0.1$, $n = 3 \cdot 10^{19}$ m^{-3}, $a = 0.5$ m, $P_{in} = 5$ MW (a), $P_{in} = 20$ MW (b).

At the second stage the radial electric field is numerically calculated from the ambipolarity constraint (Eq. 10) assuming the steady-state plasma with temperature profile generated previously (Fig. 1). The profiles of the ambipolar radial electric field are plotted in Fig. 3. The monotonous radial electric field profiles, which are obtained for the temperature profile without flattening (dashed curves in Fig. 3), are changed by the profiles with sheared region (solid curves in Fig. 3).

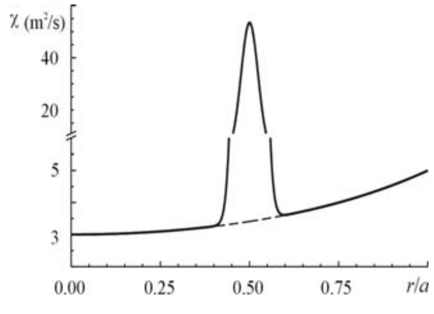

Fig. 2. The thermal diffusivity profile without magnetic island and in the presence of the belt island are plotted by straight and dashed line, respectively. Parameters are $\chi_0 = 0.3$ m^2/s, $\chi_a = 5$ m^2/s, island position $x_0 = 0.55$ and island width $h = 0.1$.

Depending of the radial position E_r profile consists of one branch ($E_r < 0$) or of the three branches (two with $E_r > 0$ and one with $E_r < 0$), i.e. the multiple solutions for the ambiplar radial electric field are found. In other words, the radial electric field solution bifurcates with r [17]. As stated in literature [18]-[20], the negative and positive branches of the E_r are called the ion and the electron root, respectively. They appear as a consequence of the highly non-trivial and nonlinear flux dependence on the E_r in the configurations of interest [17]- [20]. When the island structure is created inside the radial domain previously characterized by the multiple solution, the multiple solution is changed by the single solution. In other words, instead of the three roots only one -negative root survives and it corresponds to the sheared radial electric field. As stated in literature [17]-[20], discontinuities in the E_r profile are attached to the proposed steady-state of the plasma variables and the locality of the ambipolar equation. Thus, it is proved that the presence of the magnetic island induces generation of the sheared radial electric field and at the positions of the island boundaries the E_r bifurcates (one to three branches of the E_r and so on).

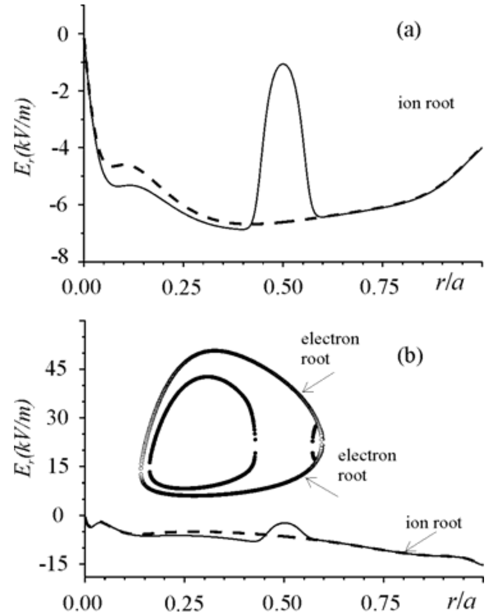

Fig. 3. The ambipolar E_r profiles for the same set of parameters as in Figure 1. In case (a) one ion root ($E_r < 0$) is obtained, and in case (b) one ion or one ion and two electron roots ($E_r > 0$) are observed at different radial positions. The magnetic island generates the sheared electric field at its position (solid lines in Figure 3). Only the ion root exists in the domain of sheared E_r.

At the third stage of the numerical calculations the non-local electric field equation (11) is solved with the ambipolar E_r as the initial condition. The radial electric field diffusion coefficient is estimated to be of order 0.1 – 1 m^2/s for the actual set of parameters. Calculations show that the radial electric field diffusion is effective only in the neighborhood of the magnetic island boundaries, as illustrated in Figures 4 and 5. Diffusion delocalizes dynamics connecting the close radial positions characterized with the large E_r

shear. Consequently, the discontinuities in the E_r profile are lost (figures 4 and 5). It is a proof of the theoretical conclusions that the diffusion takes part only in the presence of the large E_r gradients [19], [20].

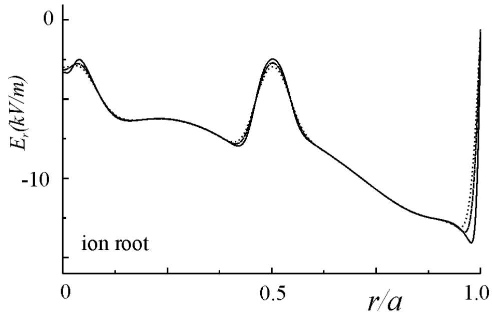

Fig. 4. The ion root evolution for parameter set (b) in Figures 1 and 2, and the diffusion coefficient D_E = 0.1 m^2/s. Solid, dashed and dotted curves are obtained for time t = 0.2, 0.6 and 1.0 ms respectively. The figure is obtained proposing the steady-state plasma conditions.

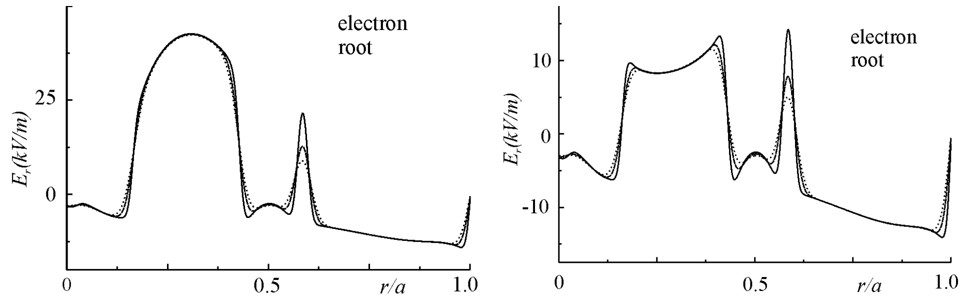

Fig. 5. The evolution of the E_r profiles with ion branches for parameter set (b) in Figure 2 and the diffusion coefficient D_E = 0.1 m^2/s. Solid, dashed and dotted curves are obtained for time t = 0.2, 0.6 and 1.0 ms, respectively.

4. CONCLUSIONS

The main results of the present paper can be summarized as:

– The sheared ambipolar radial electric field is created due to the presence of the magnetic island in the context of the belt island model.
– The single (ion) or the multiple (one ion and two electron) the radial electric field roots are obtained by changing the model parameters. The discontinuities in the radial electric field profile are associated with the locality of the adopted model and steady-state conditions on plasma parameters.
– In the region of the sheared electric field, i.e. the magnetic island, the single ion root is unique solution for the ambipolar electric field.
– The diffusion of the radial electric field has produced the observable delocalization effect only at the position with the high electric field shear, i.e. at the island boundaries. Thus, the discontinuities in the electric field profile are smoothed.

The investigations in this paper are only the first step of trying to understand the generic connection between the sheared radial electric field and the magnetic islands. As the next step the feedback effect of the formed sheared radial electric field to the particle dynamics will be considered. Also, the intentions are to change the simple belt model with more realistic thin island model and to obtain more correct diffusion term in the radial electric field equation.

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STVARANJE RADIJALNOG ELEKTRIČNOG POLJA SA "SHEAR"-OM MAGNETNIM OSTRVIMA

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Proučavan je uticaj prisustva magnetnih ostrva na ambipolarno radijalno električno polje u kontekstu belt modela. Pokazano je da region "shear" radijalnog električnog polja postoji na poziciji ostrva. U zavisnosti od parametara, jednostruka ili višestruka rešenja su dobijena za radijalno električno polje na različitim radijalnim pozicijama. Numerički proračuni pokazuju da je difuzija radijalnog električnog polja značajna samo u blizini granica ostrva. Kao rezultat, dolazi do poravnanja profila ambipolarnog električnog polja.