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PHYSICAL PARAMETERS OF THE VENTILATED FILTER CIGARETTE

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Abstract. *The main physical parameter of all ventilated filter cigarettes is the filter ventilation. It has been known for many years that these cigarettes have a possibility to achieve major reductions in all noxious smoke components, including those in the gas phase, and that filter ventilation is a practical tool for controlling smoke deliveries. The term filter ventilation in this case describes the supply of diluting air to the main-stream smoke via the ventilated cigarette filter. Smoking of a lit cigarette is a nonlinear dynamic process, and filter ventilation depends on the interrelationship between a number of factors. Fortunately, the total ventilation measured on an unlit cigarette during constant and standard air flow at mouth end is somewhat lower than for a lit cigarette.*
It was shown that linear models used here make it possible to estimate filter ventilation degree for various commercial unlit cigarettes from nondestructive pressure measurements and geometrical data.

Key words: *ventilated filter cigarette, filter ventilation, air flow, pressure*

1. INTRODUCTION

A filter cigarette of the diameter D represents a junction of the tobacco and filter parts connected by a tipping paper (TP) which wraps all of the filter part and a small portion of the tobacco part. The lengths of the tobacco part, the filter part and the TP are $l_T = ET$, $l_F = MT$, and $l_{TP} = MS = l_F + l_C$, respectively, where $l_C = ST$ is the length of the tobacco part wrapped with TP, Fig.1. The filter part wrapped with TP is well known as a cigarette filter or, simply, filter.

A nonventilated filter cigarette (NVFC) has no lateral (ventilating or diluting) air flow entering the filter, whereas a ventilated filter cigarette (VFC) has this flow. Thus, nonventilated and ventilated filters correspond to the NVFC and VFC, respectively. The tobacco part contains a tobacco rod (TR) wrapped with cigarette paper (CP) which is always permeable for air flow. The filter part contains a filter rod (FR) of various constructions wrapped with air impermeable or permeable plug wrap paper (PWP) in the case of the nonventilated or ventilated filter, respectively.

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The NVFC has air impermeable TP. This paper, used for VFC, has at least one permeable (ventilated) zone of the length $l'_V = AB = X'Y'$ placed between two impermeable (nonventilated) zones of the lengths $l'_1 = MA = MX' = l_M \geq 1$ cm and $SB = SY' = l_C + l'_0$. Thus, l_C and $l_S = l_T - l_C$ are the lengths of the nonventilated and ventilated segments of the TR, respectively. The lengths of the nonventilated filter segments are l'_1 and $l'_0 = TB = TY'$, where $l_F = l'_0 + l'_V + l'_1$, Fig.1.a.

The length of the ventilated filter which has TP with K ventilated zones of the lengths $l'_{Vk} = X'_k Y'_k$, $k = 1,2,...,K$ and $K+1$ nonventilated zones, can be expressed as

$$l_F = l'_0 + \sum_{k=1}^{K} (l'_{Vk} + l'_k), \tag{1}$$

where l'_k are the lengths of the nonventilated filter segments. First and last nonventilated filter segments have lengths $l'_0 = TB = TY'_1$ and $l'_K = MA = MX'_K = l_M$, Fig.1.b. Distance of the k-th ventilated filter segment from the mouth end M is $l'_{Mk} = MX'_k = l'_k + \sum_{k+1}^{K} (l'_{Vk} + l'_k)$.

The behaviour of each of the ventilated zones depends on the perforation manner of the TP. The TP whose ventilated zones have chaotic and small perforation holes is known as porous TP. If a TP has ventilated zones with equidistant rows of small perforation holes, it is called perforated TP.

A commercial VFC has FR made from cellulose acetate and wrapped with porous PWP and perforated or porous TP. Very often, the perforated TP has one ventilated zone with a single row of holes or multiple rows of holes. In the first case, $l'_V = 0$ and $A \equiv B$; in the other case, $l'_V = (N-1)\Delta x$ and $l'_0 \geq \Delta x/2$, where N is the number of rows and Δx is the distance between two rows. Rarely, the perforated TP has K ventilated zones of the lengths $l'_{Vk} = X'_k Y'_k = (N_k - 1)\cdot\Delta x_k$, $k = 1,2,..., K$. In this case, N_k is the number of equidistant rows of holes of the k-th ventilated zone and Δx_k is the distance between two rows in this zone. For nonventilated filter segments are $l'_0 \geq \Delta x_1/2$ and $l'_k > \Delta x_k + \Delta x_{k+1}$, $k = 1,2,..., K-1$. If the k-th ventilated zone has a single row of holes, then $l'_{Vk} = 0$. In this case, if $1 < k < K$, then $l'_k > \Delta x_{k+1}$ and $l'_{k-1} > \Delta x_{k-1}$, if $k = 1$, then $l'_1 > \Delta x_2$ and if $k = K$, then $l'_{K-1} > \Delta x_{K-1}$.

It is more convenient to analyze the flow processes in the ventilated filter, whose TP has K perforated zones, by using perforated filter segments of the lengths

$$l_{Vk} = X_k Y_k = l'_{Vk} + \Delta x_k = N_k \Delta x_k, \quad k = 1,2,...,K \tag{2}$$

instead of ventilated filter segments of the lengths l'_{Vk} [1]. In this case $X_k X'_K = Y_k Y'_K = \Delta x_k/2$. A perforated filter segment of the length l_{Vk} represents the ventilated filter segments of the length l'_{Vk} enlarged by $\Delta x_k/2$ at both side. Lengths of the nonperforated filter segments are now

$$l_0 = TY = TY_1 = l'_0 - \Delta x_1 / 2, \quad l_K = MX = MX_K = l_M - \Delta x_K / 2, \tag{3.a}$$

$$l_k = X_k Y_{k+1} = l'_k - (\Delta x_k + \Delta x_{k+1}) / 2, \quad k = 1,2,...,K-1. \tag{3.b}$$

The distance of the k-th perforated filter segment from the mouth end M is

$$l_{Mk} = MY_k = l_k + \sum_{k+1}^{K} (l_{Vk} + l_k). \tag{4}$$

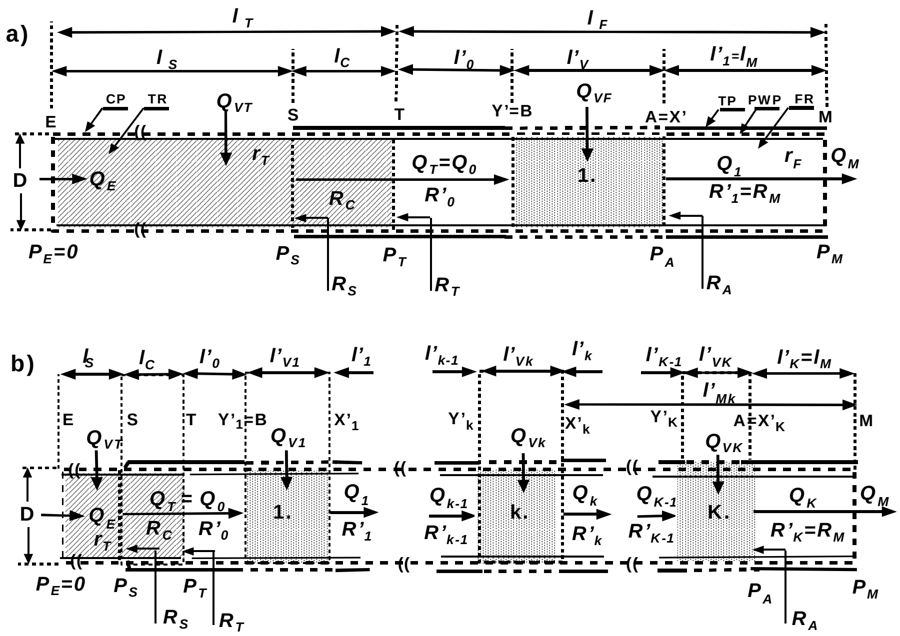

Fig.1. Structure and flows in the VFC with:
a) one ventilated filter segment, b) K ventilated filter segments

In the case of the filter with one perforated zone on the TP, lengths of the perforated and nonperforated filter segments are

$$l_V = XY = N\Delta x, \quad l_0 = TY = l_0 - \Delta x/2 \text{ and } l_1 = MX = l_M - \Delta x/2,$$

where $XX' = YY' = \Delta x/2$.

The pressure P_M [Pa] measured at mouth end M of the unlit VFC with respect to the atmosphere pressure during constant and standard air flow $Q_M = 17{,}5\ 10^{-6}\mathrm{m}^3\mathrm{s}^{-1}$ at this end is the source of all axial and lateral flows. Therefore, the pressure at end E is $P_E = 0$, Fig. 1. The flow Q_M is the sum of the flows $Q_T = Q_0$ and Q_{VF} entering the filter from the TR and from the filter vents, respectively. Therefore, Q_{VF} is the total lateral flow through the filter. The flow Q_T is the sum of the flows Q_E and Q_{VT} entering the TR from the end E and from the vents on the CP, respectively, where Q_{VT} is the total lateral flow through the TR. The entrance of the flows Q_E, Q_{VT} and Q_{VF} being at atmosphere pressure.

The definitions of the ventilations, for both lit and unlit VFC are

$$V_F = \frac{Q_{VF}}{Q_M} = \frac{1}{Q_M}\sum_{k=1}^{K} Q_{Vk} = \frac{Q_M - Q_0}{Q_M} = 1 - \frac{Q_0}{Q_M} = = 1 - \prod_{k=1}^{K}\frac{Q_{k-1}}{Q_k} = 1 - \prod_{k=1}^{K} A_k = 1 - A, \tag{5}$$

$$V_T = \frac{Q_{VT}}{Q_M} = \frac{Q_0 - Q_E}{Q_M} = \frac{Q_0}{Q_M}(1 - \frac{Q_E}{Q_0}) = (1 - V_F)(1 - \frac{Q_E}{Q_0}), \tag{6}$$

$$V = \frac{Q_{VF} + Q_{VT}}{Q_M} = V_F + V_T, \tag{7}$$

where, V_F, V_T and V are the filter, tobacco and total ventilations, respectively. The definition of V_F includes a case of the filter with K ventilated segments. Thus, Q_{Vk} is the total lateral flow entering the filter from the k-th ventilated segment, Fig.1.b. It is known that total ventilation increases once the cigarette is lit [2]. The order of the V_T degree is 0.05 (or 5%) which is low compared to the V_F degree and V_F degree is usually higher than 0.2 (or 20%).

The approximate values of V_F, V_T, and V will be calculated using linear mechanical flow models based on the laminar and viscous flow regime for which there is a linear relationship between flow and the corresponding pressure. It will be shown that the necessary data can be obtained from the next nondestructive pressure measurements:

P_{FC} - on the encapsulated filter,

P_{ME} - on the encapsulated VFC,

P_{MC} - on the VFC with encapsulated filter and

P_{MBk} - on the VFC with the unencapsulated only one of the K ventilated filter segments and closed end E. This means that K measurements are necessary in the case of a filter with K-ventilated segments. All mentioned measurements are illustrated in Fig. 2. As can be seen, the illustrated measurements of pressures P_{MBk} refer to the VFC with several perforated filter segments. The measured pressure P_{ME} is unnecessary if V_F is of interest only. Evidently, the situation during the measurement of P_{MC}, Fig.2, corresponds to the NVFC. In this case, $V_F = 0$ and the definition of the tobacco ventilation V'_T is

$$V'_T = \frac{Q_{VTC}}{Q_M} = \frac{Q_M - Q_{EC}}{Q_M} = 1 - \frac{Q_{EC}}{Q_M}. \tag{8}$$

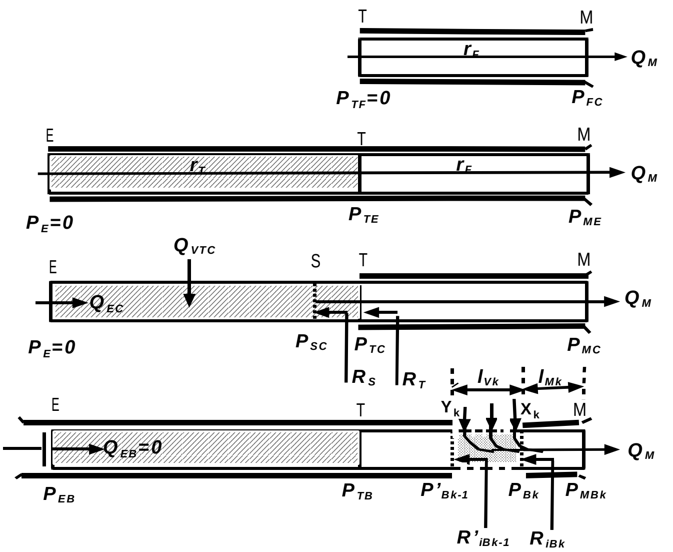

Fig. 2. Pressure measurements

Thus, the V_T of the VFC depends on the V'_T of the NVFC, or

$$V_T = (1 - V_F)V'_T \tag{9.a}$$

where

$$Q_{EC} / Q_M = Q_E / Q_0. \tag{9.b}$$

The pressures P_{FC}, P_{ME}, P_{MC}, and P_{MBk}, k=1,2,…,K can be measured using an accurate pressure drop machine calibrated to the standard air flow Q_M which has a rubber sleeve encapsulator with adjustable length [2]. The measurements of P_{MBk} still require an apparatus which must also has a rubber sleeve encapsulator with adjustable length, from l_T to l_T+l_F–l_M–l'_{VK}, and a rubber stopper for closing the entrance of the flow Q_E.

Till now, the recommended pressure measurements were P_{FC}, P_{ME}, P_{MC}, and P_M - pressure on the VFC [1,2,3] and P_{FP} – the pressure on the filter prototype (filter detached from the tobacco part which has one ventilated segment of the length $l'_V = AT$ placed at end T), [4]. The distribution of the lateral flows used here and in the references [1,3,4] represents a better approximation than those reported by Keith [2]. However, using this better distribution and pressures P_{FC}, P_{ME}, P_{MC}, and P_M, it is impossible to estimate V_F, V_T and V without the knowledge of the air permeabilities of the PWP and TP, except in the case when TP has only one row of holes [1]. On the other hand, the estimated values of V_F, V_T and V of the VFC which has K ventilated filter zones can be found from pressures P_{FC}, P_{ME}, P_{MC}, and P_{FPk} k = 1,2,…,K, but this is not practical in consideration of the necessity to make K experimental filter prototypes each time [4].

2. FLOW REGIMES IN THE COMPONENTS OF THE VFC

The calculation of the V_F, V_T, and V requires the knowledge of the flow regimes in the components of the VFC. In other words, it is necessary to know the relationship between pressure and air flow in the nonventilated or encapsulated segments of the filter and tobacco rod, and the relationship between the air flow and pressure differential through various types of papers used in the construction of the VFC.

The linear relationship between pressure drop $\Delta p = p$ and laminar and viscous air flow q of the form

$$\Delta p = p_2 - p_1 = p = r \cdot \Delta x \cdot q = R_x \cdot q \tag{10}$$

is nearly exact only for the encapsulated or nonventilated filter rod segment of the length Δx if this rod is made from cellulose acetate. This relationship is usable as a good approximation for encapsulated tobacco rods, too.

The coefficient r[Pa s m^{-4}] represents an air flow resistivity (air flow resistance per unit length) of filter or tobacco rod and R_x[Pa s m^{-3}] is the air flow resistance of the encapsulated or nonventilated filter or tobacco rod segment of the length Δx. This air flow resistivity depends on the air viscosity and on the diameter and density of the rod.

The CP and PWP use the ulteriorly perforated naturally porous papers, whereas the TP uses the air impermeable paper which is ulteriorly perforated. The air flow through paper wrappers of interest is through natural pores and through holes which are ulteriorly perforated using any of several perforation techniques. The thickness of these wrappers is usually from 20 μm to 40 μm.

The naturally porous paper usually has an interlocking network of cellulose fibers interspersed with chalk particles and spaces (pores) in this matrix. The order of these spaces is 1 μm wide, which is small compared to the paper thickness. For typical values of the cigarette flow, the Reynolds number in this case is Re<<1, and the air flow through natural pares is governed by viscous forces. Thus, if pressure difference P across this paper exists, the air capillary flow through the pores may be expressed as

$$Q_{cap} = \omega \cdot A \cdot P = G \cdot P, \tag{11}$$

where ω [$mPa^{-1}s^{-1}$] and G[$m^3Pa^{-1}s^{-1}$] are the air permeability and air flow conductance of the paper due to viscous flow, respectively, and A[m^2] is the area of the paper exposed to the flow. The paper in consideration can be treated as a bundle of parallel capillary tubes (channels) with circular cross section. Using Poiseulle's Law of viscous flow through each tube, Baker [5] shows that ω depends on the air viscosity, paper thickness, mean diameter of pores, and number of pores per unit area. The Coresta unit for air permeability ω is 1Cu =1cm/(min kPa).

The naturally porous paper and air impermeable paper which are ulteriorly perforated may have perforation holes with various shapes, area, and arrangement. These holes usually have the mean diameter of the same order as paper thickness. In this case, the Reynolds number for typical cigarette flows, is from 20 to 200. Therefore, the air flow through perforation holes is governed by inertial forces. This flow is still laminar, but the relationship between the flow and pressure is not linear.

The perforation holes of the diameter less than 8 μm act like capillary tubes and viscous flow regime through them is dominant. Both the paper with chaotic, dense, and small perforation holes and the paper with single or multiple equidistant rows of small perforation holes, may be approximately treated as naturally porous paper, eq. (11). The slang name of these kinds of papers is often "porous" and "perforated" paper, respectively. The air permeability measurements for perforated tipping papers were carried out with 1 cm^2 of paper exposed to the air flow at the pressure of 1 kPa. As recommended by Coresta methods, a 1 cm length of the perforated row must be included in the exposed area.

3. Linear Flow Models

From eq.(10), linear flow model of the nonventilated segment of the filter or tobacco rod of the length l is illustrated in Fig.3.a. The corresponding equations are

$$P = P_a - P_b = r \cdot l \cdot Q = R \cdot Q \tag{12.a}$$

$$R_{ib} = P_b / Q; \quad R_{ia} = P_a / Q = R + R_{ib}, \tag{12.b}$$

where R_{ia} and R_{ib} are corresponding input flow resistances.

A linear flow model of the rod segment wrapped with ulteriorly perforated paper which has either multiple rows of holes or porous structure (both papers have very small perforation holes) can be formed using eqs. (10,11). The models of the short rod segment of the length Δx, which contain only one row of holes (perforated subsegment), are presented in Fig.3.b and c. The difference between these subsegments is in the location of the row of holes. Because of that, the length of a rod segment which has N rows of holes at a small distance Δx, composed from the model in Fig. 3.b (Γ - perforated subsegment) or from those in Fig. 3.c (T - perforated subsegment) is l'_V or $l_V = N\Delta x = l'_V + \Delta x$, respectively. The Γ- perforated subsegment is suitable for deriving flow models of the porous rod segment (rod segment of the length l'_V wrapped with porous paper), but the T - perforated subsegment is more convenient for deriving a flow model of the perforated rod segment (rod segment of the length l_V wrapped with perforated paper), [1].

The equations for Γ- perforated subsegment are

$$p(x+\Delta x) - p(x) = \Delta p(x) = r \cdot \Delta x \cdot q(x+\Delta x) \tag{13}$$

$$q(x+\Delta x) - q(x) = \Delta q(x) = g_V \cdot \Delta x \cdot p(x) = G_V \cdot p(x), \tag{14}$$

where g_V [$m^2Pa^{-1}s^{-1}$] is the air flow conductivity which depends on the air permeability of the paper, G_V is the air flow conductance of each row of holes into perforated rod segment and $r \cdot \Delta x$ represents the air flow resistance of the rod between two rows. For FR, parameter g_V depends on the effective air permeability of the combination of the TP and PWP.

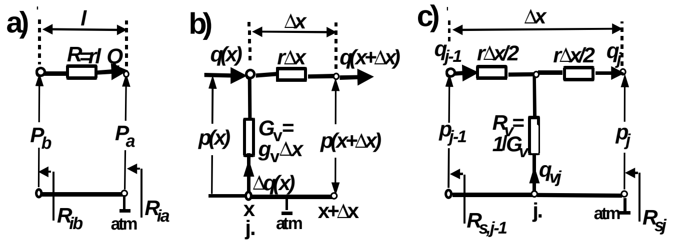

Fig. 3. Linear flow model of the: a) nonventilated rod segment, b) Γ - perforated subsegment, c) T - perforated subsegment

The T – perforated subsegment has general lumped flow parameters $R_V = G_V^{-1}$ and $R_R = r \cdot \Delta x/2$, and represents a j-th subsegment of the perforated rod segment of the length l_V. The corresponding equations of this segment are

$$-p_{j-1} = (R_R + R_V)q_{j-1} - R_V q_j = -R_{s,j-1} q_{j-1}, \tag{15}$$

$$p_j = -R_V q_{j-1} + (R_R + R_V) q_j = R_{sj} q_j = R_{sj}(q_{Vj} + q_{j-1}), \tag{16}$$

$$R_{s,j-1} = p_{j-1} / q_{j-1}; \quad R_{sj} = R_R + R_V \parallel (R_R + R_{s,j-1}), \quad j = 1,2,\dots,N, \tag{17}$$

where $R_{s,j-1}$ and R_{sj} are the corresponding input resistances. In this case, $p_N = P_1$ and $q_N = Q_1$ are pressure and flow at the input end **X** of the perforated rod segment, and $p_0 = P_0$ and $q_0 = Q_0$ are the pressure and flow at the output end **Y** of this segment. Thus, $R_{sN} = R_{i1}$ and $R_{s0} = R'_{i0}$ represent input resistances of the perforated rod segment and of the rod segment which precede the considered rod segment at end **Y**.

The configuration of the T – perforated subsegment is the same as that of the linear electrical and symmetric T – section with lumped resistive parameters R_R [Ω] and R_V [Ω]. The characteristic parameters of this section are transmission constant b and characteristic impedance Z_b [Ω]. Thus, the configuration of the perforated rod segment of the length l_V corresponds to the linear passive electrical twoport which represents a cascade connection of N identical T – sections. Using electromechanical analogies between flow and electrical current and between pressure and electrical voltage [1], the eqs. (15-17) describe this electrical twoport whose characteristic parameters are $\beta = Nb$ and Z_b [Ω]. Returning to the mechanical flow models, the relations between flow parameters R_R, R_V, b, β and Z_b [m^{-3}Pa s] are

$$chb = (R_R / R_V) + 1 \approx 1 + b^2 / 2! \quad \Rightarrow \quad R_V \approx 2R_R / b^2 = r \cdot \Delta x / b^2 \tag{18}$$

$$Z_b = R_V shb \cong R_V (b + b^3 / 3!) \approx (r \cdot \Delta x / b)(1 + b^2 / 6). \tag{19}$$

$$\beta = Nb \tag{20}$$

The approximation used here is valid for a greater number of the perforated tobacco or filter segment of the commercial VFC. In this chase, equations for perforated rod segment are:

$$P'_0 = p_0 = R'_{i0} Q_0 \tag{21}$$

$$P_1 = p_N = P'_0 \, ch\beta + Q_0 Z_b sh\beta \tag{22}$$

$$Q_1 = q_N = (P'_0 / Z_b) sh\beta + Q_0 ch\beta \tag{23}$$

$$A'_1 = Q_1 / Q_0 = (R'_{i0} / Z_b) sh\beta + ch\beta \tag{23}$$

$$R_{i1} = P_1 / Q_1 = (R'_{i0} \, ch\beta + Z_b \cdot sh\beta) / A'_1. \tag{25}$$

The symbol of the flow model of the perforated rod segment of the length l_V = **XY** is presented in Fig. 4.a.

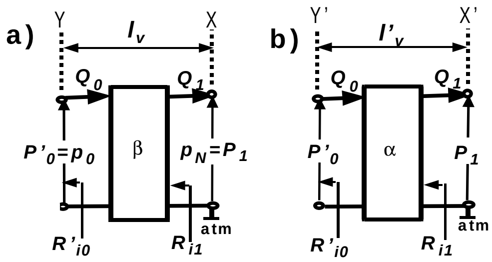

Fig. 4. Symbol of the: a) perforated rod segment, b) porous rod segment

A flow model of the porous rod segment of the length l'_V = **X'Y'** can be derived using Γ-perforated subsegment and conditions $N \to \infty$ and $\Delta x \to dx$ on the segment $x \in [0, l'_V]$. In this case, eqs. (13,14) become

$$dp(x) / dx = r \cdot q(x) \quad \text{and} \quad dq(x) / dx = g_v p(x) \tag{26.a}$$

Or

$$d^2 p(x)/dx^2 = a^2 \cdot p(x) \quad \text{and} \quad d^2 q(x)/dx^2 = a^2 q(x) \tag{26.b}$$

where r and g_v are general distributed flow parameters of the incremental transmission flow line which represents an incremental porous subsegment. The combination constant a[m^{-1}] is

$$a = \sqrt{r \cdot g_V} \tag{27}$$

It is not difficult to show that particular solutions of the linear differential eqs. (26) for a transmission flow line of the length l'_V, which represents a porous rod segment of the same length, give

$$P'_0 = p(0) = R'_{i0} Q_0 \tag{28}$$

$$P_1 = p(l'_V) = P'_0 \, ch\alpha + Q_0 Z_a sh\alpha \tag{29}$$

$$Q_1 = q(l'_V) = (P'_0 / Z_a) sh\alpha + Q_0 ch\alpha \tag{30}$$

$$\alpha = a \cdot l'_V \tag{31}$$

$$Z_a = a / g_V = r / a = \sqrt{r / g_V} = r \cdot l'_V / \alpha \tag{32}$$

$$A'_1 = Q_1 / Q_0 = (R'_{i0} / Z_a) sh\alpha + ch\alpha \tag{33}$$

$$R_{i1} = P_1 / Q_1 = (R'_{i0} \, ch\alpha + Z_a \cdot sh\alpha) / A'_1 , \tag{34}$$

where $\alpha = a \cdot l'_V$ and Z_a [Pa s m^{-3}] are characteristic parameters of this segment. The symbol of the flow model of the porous rod segment which has the length l'_V = **X'Y'** is presented in Fig.4.b. Using electromehanical analogies, eqs. (27-34) describe a linear passive electrical twoport with uniformly distributed resistive parameters or a linear electrical transmission line of the length l'_V, whose characteristic parameters are the propagation constant a [m^{-1}] and the characteristic impedance Z_b [Ω].

4. Calculation and Estimation of the Physical Parameters from the Measured Pressures

Linear flow models, Fig. 5, of the encapsulated filter and VFC during pressure measurements illustrated in Fig. 2, represent corresponding combinations of the models from Figs. 3 and 4. The models obtained in this way make it possible to calculate some of the physical parameters of the unlit VFC and to estimate the remainders.

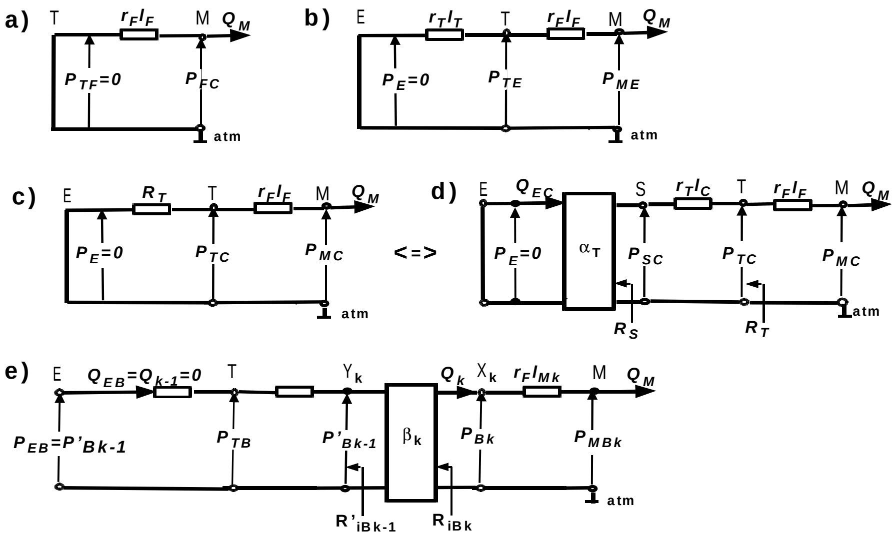

Fig.5. Linear flow models of the: a) encapsulated filter b) encapsulated VFC c, d)VFC with encapsulated filter, e) VFC with unencapsulated k-th perforated filter segment and closed end E

The equations (12) and the model from Fig. 3.a are used to form linear flow models of the encapsulated filter, Fig. 5.a, encapsulated VFC, Fig. 5.b, and of the VFC with the encapsulated filter, Fig. 5.c. From the measured pressure P_{FC}, the flow resistivity r_F of the FR is obtained by

$$r_F = P_{FC} /(Q_M \cdot l_F). \tag{35}$$

Having the measured pressures P_{FC} and P_{ME}, the flow resistivity r_T of the TR is obtained by

$$r_T = P_{TE} /(Q_M \cdot l_T) = (P_{ME} - P_{FC}) /(Q_M \cdot l_T). \tag{36}$$

From the measured pressures P_{FC}, P_{ME}, and P_{MC}, the input flow resistance R_T of the tobacco part, the flow resistance R_C of the nonventilated TR segment, and the input flow resistance R_S of the ventilated TR segment are obtained by

$$R_T = P_{TC} / Q_M = r_T l_C + R_S = (P_{MC} - P_{FC}) / Q_M \tag{37.a}$$

$$R_C = r_T l_C = [(P_{ME} - P_{FC}) / Q_M] \cdot (l_C / l_T) \tag{37.b}$$

$$R_S = P_{SC} / Q_M = R_T - R_C. \tag{37.c}$$

The pressure P_{TC} represents at the same time the pressure at end T of the unlit tobacco part detached from the filter. Evidently, the resistance R_S depends on the properties of the TR and the CP.

The equations (27-34) describe a ventilated TR segment of the length $l_S = l'_V$ if this TR is wrapped with a porous CP. These equations and symbol presented in Fig. 4.b are used to form the flow model in Fig. 5.d, which is equivalent to the model in Fig. 5.c. The corresponding equations are now:

$$a_T = a = \sqrt{r_T g_{VT}}; \quad P_E = P'_0 = 0; \quad R_E = R'_{i0} = 0; \quad Q_{EC} = Q_0$$
$$P_{SC} = P_1 = Q_{EC} \cdot Z_T \cdot sh\alpha \;; \quad Q_M = Q_1 = Q_{EC} \cdot ch\alpha_T; \tag{38}$$

$$\alpha_T = a_T l_S, \quad Z_T = Z_a = a_T / g_{VT} = r_T / a_T = r_T \cdot l_S / \alpha_T \tag{39}$$

$$A'_T = A'_1 = Q_M / Q_{EC} = ch\alpha_T \tag{40}$$

$$R_S = R_{i1} = P_{SC} / Q_M = Z_T th\alpha_T \approx \frac{r_T l_S}{\alpha_T} \frac{\alpha_T + \alpha_T^3 / 3!}{1 + \alpha_T^2 / 2!} = r_T l_S \frac{1 + \alpha_T^2 / 6}{1 + \alpha_T^2 / 2}, \tag{41}$$

where g_{VT} is the air flow conductivity of the porous CP. The approximation used in eq. (41) is always valid for a porous TR segment [1]. The only unknown value in this equation is α_T. The solution of the equation

$$F_T = \frac{R_S}{r_T l_S} = \frac{P_{MC} - P_{FC}}{P_{ME} - P_{FC}} \frac{l_T}{l_S} - \frac{l_C}{l_S} = \frac{1 + \alpha_T^2 / 6}{1 + \alpha_T^2 / 2}, \tag{42}$$

obtained from eqs.(35-41), is

$$\alpha_T = [6(1 - F_T)/(3F_T - 1)]^{1/2}, \quad \frac{1}{3} << F_T < 1. \tag{42.b}$$

Using the approximate value α_T and eqs.(39), the approximate values of the remaining parameters are:

$$a_T = \alpha_T / l_S, \quad Z_T = r_T l_S / \alpha_T \text{ and } g_{VT} = a_T / Z_T. \tag{43}$$

From eqs. (8,40), the tobacco ventilation V'_T is

$$V'_T = \frac{Q_{VTC}}{Q_M} = 1 - \frac{1}{ch\alpha_T}. \tag{44}$$

Therefore, V'_T represents the ventilation of the unlit tobacco part detached from the filter, too.

Evidently, eqs. (27-34) also describe either a porous filter segment of the length l'_V or the k-th porous filter segment of the length l'_{Vk} if this filter has a TP with one or several porous zones, respectively.

The equations (18-25) describe either a perforated filter segment of the length $l_V = N \cdot \Delta x$ or the k-th perforated filter segment of the length $l_{Vk} = N_k \cdot \Delta x_k$if this filter has a perforated TP with one or several perforated zones, respectively. These equations, the symbol presented in Fig. 4.a, the eqs. (12,4) and the model from Fig. 3.a are used to form a linear flow model of the VFC with unencapsulated only k-th perforated filter segment and with closed end **E**, Fig. 5.e. In this case,

$$Q_{EB} = Q_{k-1} = 0, \; Q_M = Q_k, \; P_{EB} = P_{TB} = P'_{Bk-1} \text{ and } R'_{iBk-1} = P_{EB} / Q_{EB} = \infty .$$

From the measured pressures P_{MBk} and P_{FC}, the input flow resistance R_{iBk} of the k-th perforated filter segment at distance l_{Mk} from the mouth end M is given by

$$R_{iBk} = \frac{P_{Bk}}{Q_M} = \frac{P_{MBk} - r_F l_{Mk} Q_M}{Q_M} = \frac{P_{MBk}}{Q_M} - \frac{P_{FC}}{Q_M}\frac{l_{Mk}}{l_F}. \tag{45}$$

Thus, eqs.(18-25) for $Q_{k-1} = Q_0 = 0$, $P'_{Bk-1} = P'_0$, $R'_{iBk-1} = R'_{i0} = \infty$, $b_k = b$, $R_{Vk} = R_V$, $N_k = N$ and

$$R_{Fk} = R_R = r_F \Delta x_k / 2, \quad \beta_k = \beta = N_k b_k, \tag{46}$$

become

$$chb_k = (R_{Fk} / R_{Vk}) + 1 \approx 1 + b_k^2/2 \;\Rightarrow\; R_{Vk} = r_F \Delta x_k / b_k^2, \tag{47}$$

$$Z_{Fk} = Z_b = R_{Vk} shb_k \approx \frac{r_F \Delta x_k}{b_k}(1 + \frac{b_k^2}{6}) = \frac{r_F l_{Vk}}{\beta_k}(1 + \frac{\beta_k^2}{6N_k^2}), \tag{48}$$

$$P_{Bk} = P_1 = P'_{Bk-1}\, ch\beta_k, \quad Q_M = Q_1 = (P'_{Bk-1} / Z_{Fk}) sh\beta_k, \quad A'_k = \infty,$$

$$R_{iBk} = R_{i1} = P_{Bk} / Q_M = Z_{Fk} cth\beta_k \approx r_F \cdot l_{Vk} \cdot (1 + \frac{\beta_k^2}{6N_k^2}) \cdot \frac{1 + \beta_k^2/2}{\beta_k^2 + \beta_k^4/6} \tag{49}$$

where R_{Vk} is the effective air resistance of each row of holes and depends on the air permeabilities of the TP and PWP. The only unknown in the eq. (49) is β_k. The equation

$$F_k = \frac{R_{iBk}}{r_F l_{Vk}} = \frac{P_{MBk}}{P_{FC}}\frac{l_F}{l_{Vk}} - \frac{l_{Mk}}{l_{Vk}} = (1 + \frac{\beta_k^2}{6N_k^2})\frac{1 + \beta_k^2/2}{\beta_k^2 + \beta_k^4/6}, \tag{50.a}$$

obtained from eqs.(35, 45, 49), has solution

$$\beta_k^2 = \frac{-c_1 + \sqrt{c_1^2 - 4c_2 c_0}}{2c_2}, \tag{50.b}$$

$$c_2 = 2N_k^2 F_k - 1 > 0, \quad c_1 = 12N_k^2 F_k - 6N_k^2 - 2 > 0, \quad c_0 = -12N_k^2,$$

$$c_1^2 - 4c_2 c_0 > 0.$$

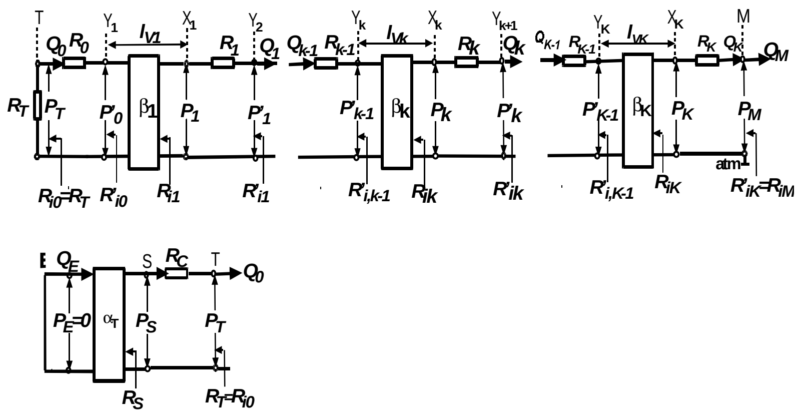

Fig.6. Linear flow model of the VFC with K-perforated filter segment and porous CP.

The obtained approximate value of β_k make it possible to calculate approximate values of b_k, R_{Vk} and Z_{Fk} from eqs. (46-48).

For example, the linear flow model of the VFC with K perforated filter segments is presented in Fig.6. Each of these segments is modeled using the symbol from Fig.3.a. The calculation procedure of approximate values of the V_F, V_T and V of this cigarette is the following:

(1) Having the geometrical data l_T, l_C, l_F, l'_{Vk}, l'_k, N_k and Δx_k and measured pressures P_{FC}, P_{ME}, P_{MC}, and P_{MBk}, $k = 1,2,...,K$ the obtained results are l'_0 from eq.(1), l_{Vk} from eq.(2), l_0, l_K, l_k, $k = 1,2,...,K-1$, from eq. (3), l_{Mk} from eq.(4), r_F, r_T, R_T, R_C, , and R_S from eqs. (35-37), α_T, a_T, Z_T and g_{VT} from eqs. (42-43), β_k from eq. (50), b_k, R_{Vk} and Z_{Fk} from eqs. (46-48) and R_0 and R_k from equations

$$R_0 = r_F l_0, \quad R_k = r_F l_k, \quad k = 1,2,...,K.$$

(2) from Fig.6: $R_{i0} = R_T = R_C + R_S$

$$R'_{i0} = R_0 + R_{i0}$$

$$A = 1$$

(3) $k = 1,2,...,K$

from eq. (24): $A'_k = (R'_{ik-1} / Z_{Fk}) sh\beta_k + ch\beta_k = Q_k / Q_{k-1} = (A_k)^{-1}$

from eq. (25): $R_{ik} = (R'_{ik-1} \cdot ch\beta_k + Z_{Fk} \cdot sh\beta_k) / A'_k$

from Fig.6: $R'_{ik} = R_k + R_{ik}$

$$A''_k = A \cdot (A'_k)^{-1}$$

$$A = A''_k$$

Therefore, for $k = K$

the obtained results are $A = \prod_1^K A_k, \quad R_{iK}$, and

$$R'_{iK} = R_{iM} = R_K + R_{iK}$$

(4) from eq.(5): $V_F = 1 - A, \quad Q_0 = (1 - V_F) Q_M, \quad Q_{VF} = V_F Q_M$

(5) from eqs. (8,44): $V'_T = 1 - \dfrac{1}{ch\alpha_T}$

(6) from eqs. (6,9): $V_T = (1 - V_F) V'_T, \quad Q_E = Q_0 / ch\alpha_T, \quad Q_{VT} = V_T Q_M$

(7) from eq. (7): $V = V_F + V_T,$

(8) from Fig. 6: $P_M = R_{iM} Q_M = R'_{iK} Q_M = (R_K + R_{iK}) Q_M$

The steps from (1) to (8) of the presented algorithm give possibilities to calculate the approximate values of the pressure P_M of the unlit VFC which has K perforated filter segments and values of all its physical parameters. It is also possible to calculate the approximate values of pressures P_{TD} and P_{FD} at end T of the unlit tobacco part detached from the filter and at mouth end M of the filter detached from the tobacco part, respectively, during standard air flow at these ends. The corresponding ventilation degrees are V_{TD} and V_{FD}, respectively.

The pressures P_{TD} and P_{TC} are the same and ventilations V_{TD} and V'_T are the same too, see Fig.2 and eqs. (8,44). Therefore, $P_{TD} = P_{TC} = P_{MC} - P_{FC}$ (step (1), eq.(37a)) and $V_{TD} = V'_T$ (step(5)), where parameter α_T is obtained from eq.(42) in step (1).

The linear flow model of the VFC, Fig.6, for $P_T = 0$ and $R_{i0} = R_T = 0$ corresponds to the filter with K perforated segments detached from the tobacco part. Therefore, from steps (1-4) and (8), the ventilation V_{FD} and pressure P_{FD} are obtained, respectively, by setting $R_{i0} = 0$ in step (2). The values of A, R_{iK}, and R'_{iK} obtained from the K-th iteration in step (3) correspond to this filter, while the values of V_F, step (4), and P_M, step (8), correspond to the values of V_{FD} and P_{FD}, respectively.

The pressures P_M, P_{TD} and P_{FD} can be measured, too. The difference between the calculated and measured pressures would exist, among the rest, because of the adopted linearity between the flow and pressure and because of series expansion of hyperbolic functions by the first two terms.

Moreover, the algorithm presented here can be modified to give the approximate value of the pressure P_M of unlit VFC which has K porous filter segments and values of all its physical parameters. In this case, eqs. (27-34) describe the porous filter segment of the length l'_V or the k-th porous filter segment of the length l'_{Vk}. The symbol from Fig.3.b corresponds to each of these segments.

Evidently, the calculation of the air permeabilities of the PWP and of each of the ventilation zones on the TP is not possible because these parameters are inseparable by nondestructive pressure measurements proposed here.

5. Conclusion

The set of nondestructive pressure measurements proposed here and the adopted linear relationship between pressure and flow make it possible to obtain practically exact values of the some physical parameters of the commercial unlit VFC and approximate values of the remaining parameters. Among them, filter ventilation is of greatest interest with respect to the health of the smoker. The complete calculation procedure was shown in the case of the VFC which has several perforated filter segments including a possibility for obtaining the approximate values of the pressures at mouth end of this cigarette, at end T of the tobacco part detached from the filter, and at mouth end of the filter detached from the tobacco part. It was indicated that this algorithm is flexible enough to include the case of the VFC with several porous filter segments.

A special advantage of the described procedure is its ability to verify the filter, tobacco, and total ventilation degrees of the manufactured cigarettes having tipping paper with one or more ventilated zones from the nondestructive pressure measurement instead from the properties of the materials. Therefore, this procedure represents a useful tool for quality control of cigarette production.

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FIZIČKI PARAMETRI VENTILIRANE FILTER CIGARETE

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Glavni fizički parametar svih ventiliranih filter cigareta je ventilacija preko filtera. Odavno je već poznato da se ove cigarete odlikuju najvećim sniženjem svih štetnih komponenata dima, uključujući i one u gasnoj fazi, kao i da je filter ventilacija praktičan 'alat' za kontrolu oslobodjenih komponenata dima. Izraz 'filter ventilacija' opisuje dovod razredjujućeg vazduha u glavnu struju dima preko ventiliranog cigaretnog filetra. Pušenje upaljene cigarete je nelinearan dinamički proces u kome filter ventilacija zavisi od brojnih faktora i njihovih medjuzavisnosti. Srećom, ukupna ventilacija merena na neupaljenoj cigareti tokom konstantnog standardnog protoka vazduha na njenom pušačkom kraju je nešto niža od ventilacije upaljene cigarete.

Pokazano je da linearni modeli, korišćeni ovom prilikom, omugućavaju procenu stepena filter ventilacije za različite komercijalne neupaljene cigarete samo na osnovu nedestruktivnih merenja pritisaka i geometrijskih podataka.

Ključne reči: *ventilirana filter cigareta, ventilacija filtra, vazdušni protok, pritisak.*