

## FUNDAMENTAL AND SECOND HARMONIC AMPLITUDES IN A COLLISIONAL MAGNETOACTIVE PLASMA

UDC 533.9

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**Abstract.** *We present a theoretical investigation of frequency doubling of electromagnetic wave in homogeneous, collisional and magnetized plasma. The coupled nonlinear equations for fundamental ordinary wave and second harmonic extraordinary wave have been solved. The amplitudes of these waves have been calculated for various values of collisional frequency and distance from the plasma boundary.*

### 1. INTRODUCTION

The process of frequency doubling of an electromagnetic wave in plasma has been investigated by numerous authors [1-7]. The reasons for it lie in plasma diagnostics, astrophysical research, problems connected with fusion experiments etc. The influence of electron-ion collisions on the process has been usually neglected.

In this paper, we solve analytically and numerically the problem of the second harmonic generation by the ordinary mode, propagating through a homogeneous magnetized, collisional plasma. The dependence of the amplitudes of the fundamental wave and its second harmonic on collisional frequency and slab thickness is shown when the ooe-phase synchronism conditions are satisfied, i.e. when  $N_0^{(1)} = N_0^{(2)}$ , where  $N_0^{(1)}$  and  $N_0^{(2)}$  are reflection indices for the fundamental ordinary and second harmonic extraordinary waves, respectively.

### 2. BASIC EQUATIONS

We consider plasma in constant external magnetic field  $\mathbf{B}_0 = B_0 \mathbf{e}_z$ . The incoming electromagnetic wave propagates along the  $x$  axis. The standard nonlinear equation describes the varying of the electric field amplitudes [5]:

$$\nabla^{(\eta)} \times [\nabla^{(\eta)} \times \mathbf{E}^{(\eta)}] - \frac{\eta^2 \omega^2 \hat{\varepsilon}^{(\eta)}}{c^2} \mathbf{E}^{(\eta)} = \frac{i\eta\omega\omega\eta}{c^2 \varepsilon_0} \mathbf{j}_{nl}^{(\eta)} \quad (1)$$

where  $\eta = 1, 2$  refers to the pump and frequency-doubled wave, respectively. The dielectric tensor  $\hat{\varepsilon}^{(\eta)}$  has the form:

$$\hat{\varepsilon}^{(\eta)} = \begin{pmatrix} \varepsilon_1^{(\eta)} & i\varepsilon_2^{(\eta)} & 0 \\ -i\varepsilon_2^{(\eta)} & \varepsilon_1^{(\eta)} & 0 \\ 0 & 0 & \varepsilon_3^{(\eta)} \end{pmatrix} \quad (2)$$

where:

$$\varepsilon_1^{(\eta)} = 1 - \frac{v(\eta - is)}{\eta[(\eta - is)^2 - u]}, \quad \varepsilon_2^{(\eta)} = -\frac{v(u)}{\eta[(\eta - is)^2 - u]}, \quad \varepsilon_3^{(\eta)} = 1 - \frac{v}{\eta(\eta - is)}$$

and

$$v = \frac{\omega_p^2}{\omega^2}, \quad u = \frac{\omega_c^2}{\omega^2}, \quad s = \frac{v_{ei}}{\omega}.$$

Here  $\omega_p$  and  $\omega_c$  are the electron plasma and electron cyclotron frequencies, respectively and  $v_{ei}$  is the electron-ion collision frequency.

The operator  $\nabla^{(\eta)}$  is given in the form:

$$\nabla^{(1)} = (ik^{(1)} + \frac{\partial}{\partial x})\mathbf{e}_x, \quad \nabla^{(2)} = (ik^{(2)} + \frac{\partial}{\partial x})\mathbf{e}_x. \quad (3)$$

The nonlinear electric currents on the right-hand side of equation (1) are:

$$\mathbf{j}_{nl}^{(1)} = i\omega\varepsilon_0(1 - \hat{\varepsilon}^{(1)})[\mathbf{v}^{(1)*} \times \mathbf{B}^{(2)} + \mathbf{v}^{(2)} \times \mathbf{B}^{(1)*} - \frac{m}{e}(\mathbf{v}^{(1)*} \cdot \nabla^{(1)})\mathbf{v}^{(1)*}] + en^{(1)}\mathbf{v}^{(2)} + en^{(2)}\mathbf{v}^{(1)*} \quad (4)$$

$$\mathbf{j}_{nl}^{(2)} = 2i\omega\varepsilon_0(1 - \hat{\varepsilon}^{(2)})[\mathbf{v}^{(1)} \times \mathbf{B}^{(1)} - \frac{m}{e}(\mathbf{v}^{(1)} \cdot \nabla^{(1)})\mathbf{v}^{(1)}] + en^{(1)}\mathbf{v}^{(1)} \quad (5)$$

where

$$n^{(\eta)} = i\frac{\varepsilon_0}{e}\mathbf{k}^{(\eta)} \cdot \mathbf{E}^{(\eta)}, \quad \mathbf{v}^{(\eta)} = i\frac{\eta\omega\varepsilon_0}{en_0}(1 - \hat{\varepsilon}^{(\eta)})\mathbf{E}^{(\eta)}, \quad \mathbf{B}^{(\eta)} = \frac{1}{\eta\omega}\mathbf{k}^{(\eta)} \times \mathbf{E}^{(\eta)}.$$

Taking in account relations (2), (3), (4) and (5) one can obtain from (1) the following set of coupled equations:

$$\begin{aligned}\frac{\partial E_z^{(1)}}{\partial x} &= (C_{11} - iC_{12})E_z^{(1)*} E_y^2, \\ \frac{\partial E_y^{(2)}}{\partial x} &= (C_{21} - iC_{22})E_z^{(1)2},\end{aligned}\quad (6)$$

where  $C_{11}$ ,  $C_{12}$ ,  $C_{21}$  and  $C_{22}$  are the coupling constants, depending on  $u$ ,  $v$  and  $s$  in the following way:

$$\begin{aligned}C_{11} &= \{mc^2(N_o^{(1)2} + \kappa_o^{(1)2})(1+s^2)[(8-2s^2-2v-2u)^2 + s^2(v-8)^2]\}^{-1} \times \\ &\times \{s(N_o^{(1)2} + \kappa_o^{(1)2})(6s^2 - vs^2 - 8 - 2u) - (N_o^{(1)}N_e^{(2)} + \kappa_o^{(1)}\kappa_e^{(2)}) \times \\ &\times [8 - 2v - 2u - s^2(v-6)] + s(N_e^{(2)}\kappa_o^{(1)} - N_o^{(1)}\kappa_e^{(2)})(v + 2s^2 + 2u)\} \times \\ &\times ev\sqrt{u} \exp\left[-\frac{2\omega(\kappa_o^{(1)} - \kappa_e^{(2)})x}{c}\right] \\ C_{12} &= \{mc^2(N_o^{(1)2} + \kappa_o^{(1)2})(1+s^2)[(8-2s^2-2v-2u)^2 + s^2(v-8)^2]\}^{-1} \times \\ &\times \{(N_o^{(1)}\kappa_e^{(2)} - N_e^{(2)}\kappa_o^{(1)})[8 - 2v - 2u - s^2(v-6)] + \\ &+ s^2(N_o^{(1)2} + \kappa_o^{(1)2})(16 + 2s^2 + 2u - v) + \\ &+ s(N_o^{(1)}N_e^{(2)} + \kappa_o^{(1)}\kappa_e^{(2)})(v + 2s^2 + 2u)\} \times \\ &\times ev\sqrt{u} \exp\left[-\frac{2\omega(\kappa_o^{(1)} - \kappa_e^{(2)})x}{c}\right] \\ C_{21} &= \{mc^2(N_o^{(1)2} + \kappa_o^{(1)2})(1+s^2)[(8-2s^2-2v-2u)^2 + s^2(v-8)^2]\}^{-1} \times \\ &\times [s(N_o^{(1)}N_e^{(2)} + \kappa_o^{(1)}\kappa_e^{(2)})(16 - 2s^2 - 2u - 3v) + \\ &+ (N_o^{(1)}\kappa_e^{(2)} - N_e^{(2)}\kappa_o^{(1)})(8 + 6s^2 - 2u - 2v - s^2v)] \times \\ &\times ev\sqrt{u} \exp\left[-\frac{2\omega(\kappa_o^{(1)} - \kappa_e^{(2)})x}{c}\right] \\ C_{22} &= \{mc^2(N_o^{(1)2} + \kappa_o^{(1)2})(1+s^2)[(8-2s^2-2v-2u)^2 + s^2(v-8)^2]\}^{-1} \times \\ &\times [s(N_e^{(2)}\kappa_o^{(1)} - N_o^{(1)}\kappa_e^{(2)})(16 - 2s^2 - 2u - 3v) + \\ &+ (N_o^{(1)}N_e^{(2)} + \kappa_o^{(1)}\kappa_e^{(2)})(8 + 6s^2 - 2u - 2v - s^2v)] \times \\ &\times ev\sqrt{u} \exp\left[-\frac{2\omega(\kappa_o^{(1)} - \kappa_e^{(2)})x}{c}\right]\end{aligned}\quad (7)$$

The complex amplitudes  $E_z^{(1)}$  and  $E_y^{(2)}$  can be expressed through the real amplitudes  $A_1$  and  $A_{2y}$ :

$$E_z^{(1)} = A_1 e^{i\varphi_1}, E_y^{(2)} = A_{2y} e^{i\varphi_2}. \quad (8)$$

Now, the generalized phase is introduced:

$$\Psi = 2\varphi_1 - \varphi_2 + \frac{2\omega\Delta N x}{c}. \quad (9)$$

$$A_{2y} = \frac{A_2}{\sqrt{C_{31}^2 + 1}}, \quad (10)$$

where  $C_{31}$  is the function of  $u$ ,  $v$  and  $s$ .

On the basis of equations (6), (8), (9) and (10) the following system of equations is obtained:

$$\begin{aligned} \frac{dA_1}{dx} &= \frac{C_{11}\cos\Psi - C_{12}\sin\Psi}{\sqrt{C_{31}^2 + 1}} A_1 A_2 \\ \frac{dA_2}{dx} &= \sqrt{C_{31}^2 + 1} (C_{21}\cos\Psi + C_{22}\sin\Psi) A_1^2 \\ \frac{d\Psi}{dx} &= \left( C_{22}\sqrt{C_{31}^2 + 1} \cdot \frac{A_1^2}{A_2} - \frac{2C_{12}}{\sqrt{C_{31}^2 + 1}} A_2 \right) \cos\Psi \\ &\quad - \left( C_{21}\sqrt{C_{31}^2 + 1} \frac{A_1^2}{A_2} + \frac{2C_{11}}{\sqrt{C_{31}^2 + 1}} A_2 \right) \sin\Psi \end{aligned} \quad (11)$$

If we compare this system with that which corresponds to collisionless plasma [8] we can establish the differences: 1) first two equations in (11) have additional terms with  $\cos\Psi$ , 2) the third equation in (11) includes amplitudes  $A_1$  and  $A_2$ .

Using the Runge-Kutt [9] method the solutions ( $A_1$ ,  $A_2$  and  $\Psi$ ) of the system (11) are obtained. For  $x = 0$  the phase and the amplitudes are given by:

$$A_1(0) = 2 \sin \Phi \sqrt{\frac{2S^i}{c\epsilon_0 \left[ (1 + N_0^{(1)})^2 + \kappa_0^{(1)^2} \right]}}, \quad A_2(0) = 0, \quad \Psi_0 = \frac{\pi}{2} \quad (12)$$

where  $S^i$  is the incident energy flux and  $\Phi$  is the angle between  $E^{(1)}(0)$  and  $y$  axis.

### 3. NUMERICAL RESULTS

The amplitudes  $A_1$  and  $A_2$  are investigated numerically for parameters  $x$  and  $s$  and the results are plotted in figures 1-4.

Figures 1 and 2 show the amplitudes of the ordinary fundamental wave  $A_1$  and the extraordinary second harmonic wave  $A_2$ , respectively, against the slab thickness  $x$  for given (resonant) values of plasma density and external magnetic field intensity. These figures demonstrate that collisionless plasma (series 1) is the most suitable for the second

harmonic generation, because the second harmonic amplitude increases rapidly while the fundamental wave amplitude decreases. The similar spatial dependence of amplitudes  $A_1$  and  $A_2$  in laser plasma ( $s = 10^{-4}$ ) and collisionless plasma is evident.

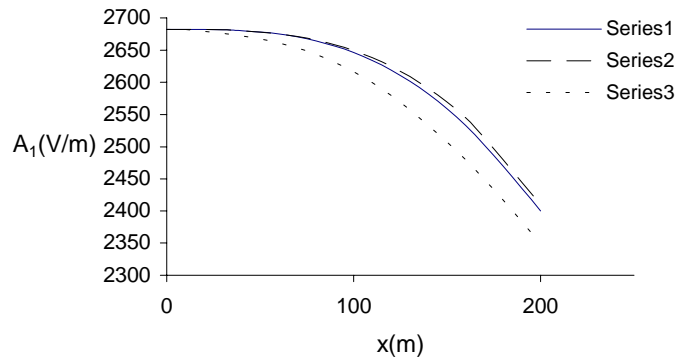


Fig. 1. Fundamental wave amplitude  $A_1$  as a function of slab thickness  $x$  at various collisional parameter  $s$ . Series 1,2 and 3 correspond to  $s = 10^{-20}$ ,  $10^{-4}$  and  $10^{-2}$  respectively. Other parameters:  $\omega = 2.092\text{GHz}$ ,  $\omega_p^2/\omega^2 = 0.8$ , resonant value of  $\omega_c^2/\omega^2$ ,  $S^i = 10^4 \text{ W/m}^2$ ,  $\Phi = 45^\circ$ .

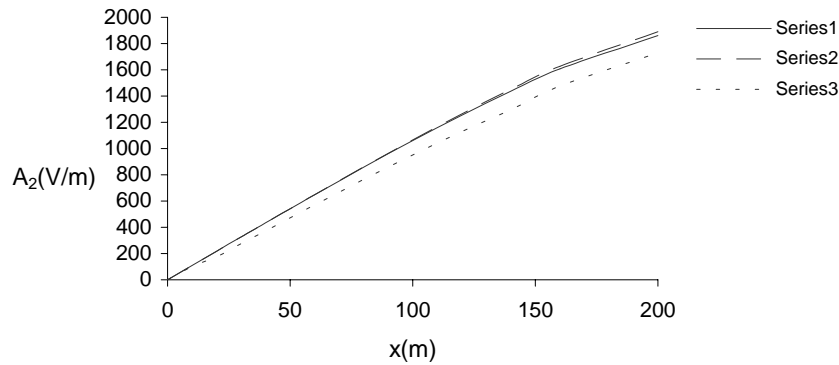


Fig. 2. The second harmonic wave amplitude  $A_2$  as a function of slab thickness  $x$  at various collisional parameter  $s$ . Series 1,2 and 3 correspond to  $s = 10^{-20}$ ,  $10^{-4}$ ,  $10^{-2}$  respectively. Other parameters:  $\omega = 2.092\text{GHz}$ ,  $\omega_p^2/\omega^2 = 0.8$ , resonant value of  $\omega_c^2/\omega^2$ ,  $S^i = 10^4 \text{ W/m}^2$ ,  $\Phi = 45^\circ$ .

Figures 3 and 4 show the amplitudes of the ordinary fundamental wave  $A_1$  and the extraordinary second harmonic wave  $A_2$  against the collisional parameter  $s$  when ooe-phase synchronism conditions are satisfied. For small values of parameter  $s$  ( $s < 10^{-3}$ ) the amplitudes do not depend on the collisional frequency. In plasmas with very frequent collisions ( $s > 10^{-3}$ ) the amplitudes attenuate with the increase of parameter  $s$ .

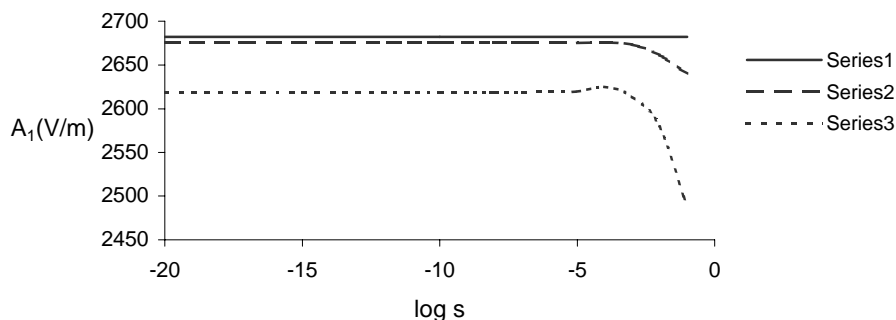


Fig. 3. The fundamental wave amplitude  $A_1$  as a function of collisional frequency. Series 1,2 and 3 correspond to  $x = 10$  m, 60 m and 120 m, respectively. Other parameters are the same as in figure 1.

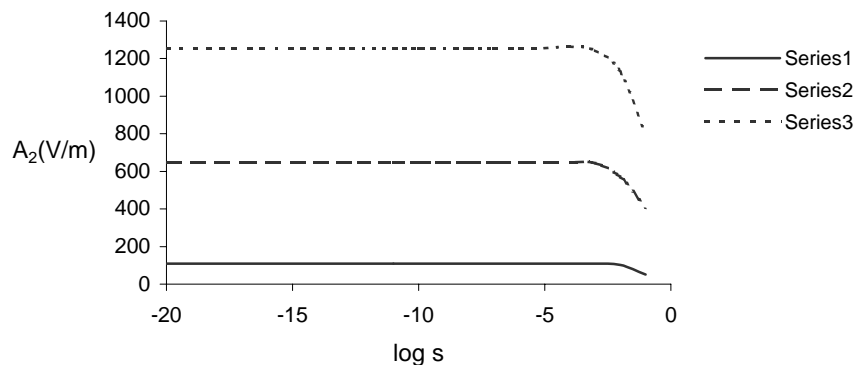


Fig. 4. The second harmonic wave amplitude  $A_2$  as a function of collisional frequency. Series 1,2 and 3 correspond to  $x = 10$  m, 60 m and 120 m, respectively. Other parameters are the same as in figure 3.

#### 4. CONCLUSION

At the end of our paper we can conclude that collisions in the magnetoactive plasma exert influence on the resonant generation of the second harmonic amplitude, especially in plasmas with very frequent collisions ( $s > 10^{-3}$ ). The influence of collisions on this process is negative, because the value of the fundamental wave amplitude decreases with the increasing of collisional frequency.

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## **AMPLITUDE OSNOVNOG I DRUGOG HARMONIKA U KOLIZIONOJ MAGNETOAKTIVNOJ PLAZMI**

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*Prikazuje se teorijsko istraživanje dupliranja frekvencije elektromagnetnog talasa u homogenoj, kolizionoj i magnetizovanoj plazmi. Rešene su spregnute nelinearne jednačine za osnovni redovni i neredovni talas drugog harmonika. Amplitude ovih talasa su računane za različite vrednosti koliziona frekvencije i rastojanja od granice plazme.*