

ON SPACE AND TIME IN QUANTUM COSMOLOGY

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Abstract. *The paper considers the properties of space and time in quantum cosmology. It presented the basic ideas of the substantial and relational conceptions of space and time, as well the basic ideas of the continuity and discreteness of space and time. The basics of standard quantum cosmology, i.e. quantum cosmology formulated over the field of real numbers R , have also been presented. Quantum cosmology is the application of the quantum theory to the universe as a whole in the early phases of its evolution, when the universe was very small so that all the four interactions were practically unified. In order to obtain the maximum possible information from quantum cosmology it is necessary that it be "complete". The concept "complete" refers here to the formulation of the theory over the field of real numbers and the field of p -adic numbers Q_p . Since p -adic numbers are generally not well-known, the idea of their introduction has carefully been considered. Within the p -adic quantum cosmology representing quantum cosmology over the field of p -adic numbers Q_p , the main results concerning the de Sitter model have been presented. The consequence of this (complete) formulation of the de Sitter model is the radius discreteness of the universe.*

1. INTRODUCTION

Present-day conceptions of space and time are very diverse relying on a synthesis of physical, mathematical and philosophical ideas. At the basis of man's perception of space and time it is possible to differentiate between two sets of questions: 1) do space and time exist as independent entities, and 2) does their existence depend on something more fundamental than themselves (like matter, an atom or a thought)? If the first set of questions has been resolved and if the existence of space-time as a basic or derived reality has been accepted, the next question to ask is what its properties are. In that case, independently of the solution of the first set of questions, it is necessary to establish whether space and time are discrete or continual, and whether time is directed or not [1].

The answer to these questions does not appear to have any practical implications since space, within the great dimensions in which we live, manifests itself as being continual (in fact, we perceive it as such). However, from the point of view of physics as a fundamental science, it is absolutely important to search for the answer. It is also worth mentioning that science has occasionally dealt with an apparently marginal (with little practical value) problem that in the course of time proved to bring about a new industrial revolution and radically change man's conception of life. For centuries, different ideas about whether space-time is continual or discrete have been developed, and different models representing space as flat, curved, finite or infinite have been made. On the other hand, time has been regarded as accelerated, linear or decelerated and its course as cyclic, reversible or irreversible or as represented by a directed "arrow of time". The most acceptable models, however, have been a model of continual three-dimensional space and a model of continual one-dimensional time.

2. ELEMENTS OF THE THEORY OF SPACE AND TIME

2.1 Substantial Conception of Space and Time

The substantial conception of space as a storehouse of all the material objects has been developed by Democritus and Newton. It is rooted in Democritus' concept of emptiness resulting from his division of the universe into a full part and an empty one. Emptiness is devoid of the attribute of "extendedness" although it is a prerequisite, an arena or a framework of the material body dynamics. Newton postulated the existence of space, also independent of the bodies residing in it. According to Newton "absolute space, by its own nature and irrespective of anything external, always remains immovable and similar to itself". Absolute space is not a physical entity comprising material objects and the relationships between them. It is an amorphous frame within which all the physical processes take place but it also serves as a reference for all the physical motions that could be described in terms of the axioms of mechanics. Due to the laws of motion it is imparted a physical reality. Matter is not its intrinsic property and it exists by and in itself which makes it absolute.

The substantial conception of time lying at the basis of classical physics regards time as an immaterial entity of its own kind. This entity exists in itself, is governed by its own laws and is independent of space, substance and fields whose existence and motion are fundamentally affected by it.

Newton's mechanics is based on the idea of the existence of one and the same time in the whole universe. His theory of time presupposes that there is a unique order (arrangement) of instants, that events cannot be identified with them and that they can take place in some of them. Time relations between the events depend on their relations with the instants in which they take place whereas the "before and after" relations are established between different instants. Newton believed that absolute time instants make up a continual successiveness similar to the real number successiveness. He also assumed that the rate at which these instants follow one another is constant and independent of all the actual events and processes. Time relations between events are absolute in nature, i.e. every event has strictly defined and independent time relations with all the other events in the universe regardless of whether they are mutually related by material interactions or not. Time relations do not only involve the "earlier-later" relations but also the relation of

simultaneousness that is regarded as the relation denoting that the event belongs to a certain instant of absolute time.

2.2 Relational Conception of Space and Time

The relational conception regards space as a relational structure of material objects. The main proponents of this conception are Aristotle and Leibnitz [2]. While negating the existence of independent space and the separation of space from matter, Leibnitz placed spatiality in the area of the body's properties. Leibnitz believed that if there were no objects, no space or time would exist. His concept of space as a system of relations is inconceivable without the material objects. Somewhat different, but a similar hypothesis was proposed by Descartes who identified "extendedness" or space and considered it identical with matter.

Emphasizing that he regards space and time as purely relative, namely that space is the order of co-existence whereas time is the order of successiveness of phenomena, Leibnitz laid the foundations of the relational conception of time. According to it, events are more fundamental than instants as they do not exist by themselves but are representations of the classes of events. Time cannot exist by itself, i.e. it is not body-independent. The present-day relational conception takes time as a system of relations among physical events. That system of relations does not exist by itself but is determined by the nature of mutual interactions between material systems. Time is a certain relation between material systems that interact or that are capable of interacting. Owing to the ultimate rate of interaction transmission, time relations of an event are not possible with any other events in the universe but only with those events that it is, in principle, capable of interacting with. According to the relational conception of time, the concept of simultaneousness makes interactions between events impossible, thus denoting the absence of time relations. It is assumed in physics that the values of the time variable t form a continual sequence. Each of those values is usually called an instant.

2.3 Structure of Space

Although not historically verified, an idea seems to have been very influential in Newtonian times, namely that geometry is a science of space structure or "pure extendedness" rather than of spatial properties of the material bodies [3]. In the nineteenth century revolutionary steps were made in geometry as a study of space. A decisive step was taken by Lobachevski who discovered dependence of the angle on distance thereby replacing Euclid's axiom of parallels by a new axiom and making a new and consistent system. After Lobachevski, geometry started developing intensively in two directions: 1) Riemann's differential-metric, and 2) Klein's group theory. According to Riemann, it is possible to regard ordinary space as a three-dimensional manifold of points, i.e. as a special case of an n -dimensional manifold. His basic idea was related to the problem of measurement and the basic geometric concept was that of length defined by him in terms of the differential form. In Riemann's geometry, distance between two close points x and $x+dx$ was defined in terms of the metric form $ds^2 = g_{ij} dx^i dx^j$, where ds^2 was a manifold invariant, g_{ij} a metric tensor, and $i, j = 1, \dots, n$.

2.3.1 Continuity of Space and Time

The model that has long been accepted is the model of continual three-dimensional space, sometimes called the continual model or continuum. The proponents of this model were Aristotle, Descartes, Newton, Leibniz, Penrose, etc. According to Penrose, all the events take place within the continual differentiable manifold called space-time continuum. This idea has become so familiar that such a structure of space and time seems obvious to us. The continuum is primarily an ideal, abstract and mathematical construction. Using the continuum to describe the structure of space and time can be justified by the properties of microscopic spaces and time. The space-time continuum is an idealization in those fields where it appears as a physical reality. Conceptions of the continuity of space and time play a significant role in physics where they are widely used in the apparatus of physical theories. Let us just remember Newton's or Einstein's theory of gravitation.

2.3.2 Discreteness of Space

The conception of pure space discreteness is just as old as the conception of continuity. It is rooted in the ideas of the old Greece atomists, particularly Democritus. any of the present-day physicists have also supported the space discreteness approach. In order that the problem of space within the microcosm be resolved, investigations have been carried out in different directions. The most interesting ones are concerned with the generalizations of algebras and groups in the microcosm or with the introduction of the elementary length in the non-linear, non-local and non-Archimedean theories. A discrete space hypothesis has been developed by D. Ivanenko, E. Foradori, V. Ambarcumian, A. March, H. Snider, etc.

The basic assumption of the discrete space hypothesis is the introduction of an elementary length into the geometry of the microcosm. This length is different in content depending on the discrete space hypothesis. Particular significance is attributed to the attempts at establishing a non-Archimedean geometry made by an Austrian physicist A. March, who set out from the measurement problem. In order that space structure could be defined, it is necessary to introduce a metric or a unit of measurement and a test body that could only be represented by the elementary particle. In this way we can be positive that there is no shorter distance in nature. The exactness of measurement is dependent on that distance. The introduced elementary length l_0 is at the same time the unit of measurement. According to A. I Panchenko, the introduction of the hypothesis that assumes the existence of a space quantum of the order of the Planck length

$$l_0 = \sqrt{\frac{Gh}{c^3}} \approx 10^{-35} m \quad (1)$$

(h -Planck constant, c -velocity of light, G -Newtonian gravitational constant) is also determined by the fact that at these distances all the types of physical interactions become equally effective which makes their differentiation impossible. In principle, it is not possible to measure distances smaller than l_0 , which means that at such small distances the Archimedes axiom from the Euclidean geometry is not valid. Space having the properties like these is called non-Archimedean or ultra-metric. According to the non-Archimedean theories, the fundamental length is introduced immediately into space as its intrinsic property. This results in the fundamental question of whether space is continual or discrete?

3. POSSIBLE DIRECTIONS IN SEARCH OF THE ANSWERS?

So far we have considered some basic conceptions about the continual and discrete space-time. Who is right, or to put it in another way, is space-time continual or discrete? If we cannot give a straightforward answer to this question, let us at least try to find out what physical theory can provide the answer. Of the two great theories of the 20th century, quantum mechanics and the theory of relativity, the latter is, in essence, a theory of space-time which is why it must be taken into account when considering the properties of space. Quantum mechanics must also be included in the research since one of its basic assumptions is the existence of quanta, i.e. of something discrete. A question arises immediately of how tenable and how complete these two theories are. The question of tenability is related to the question of why there are two theories if one of the primary goals of physics as a science is the unification of interactions or description of nature in terms of one theory. This was one of the main concerns of physics in the latter part of the 20th century.

The question of completeness is not as present and well-known as the one expressed in the search for one theory that could replace the two existing theories. It is common knowledge that in the process of the physical theories construction the existing mathematical apparatus is both being used and further developed. Let us remember Newton's contribution to mathematics that is reflected in the introduction and further development of infinitesimal calculus; the theory of groups, that was of utmost importance in the unification of strong, weak and electromagnetic interactions, or tensor calculus that turned out to be the basis for the theory of relativity. We, however, do not usually think about one field of mathematics that proved to be rather important - the **number theory**. It is just the application of the number theory in physics, particularly during the last fifteen years of the twentieth century, that led to significant results concerning the space-time structure at small distances. These results seem so important that it is even possible to talk about the relativity of the basic equations in physics related to the number fields [4]. Having that in mind, it is possible to consider the theory that takes account of all the number fields offered by nature as complete.

4. QUANTUM COSMOLOGY

Attempting to unite quantum mechanics and the theory of relativity the first problem to face is the problem of gravity quantization. Namely, it is a theory of space and time, i.e. a geometric theory and all the other interactions are quantized in that space-time. Let us see first whether it is necessary to quantize Einstein's theory of gravitation at all.

It is a closed theory in the sense that it presents both the field equations and the motion equations referring to the bodies in that field. As Hawking and Penrose have demonstrated, global and local singularities are bound to exist in it. That the singularity is global refers to the fact that extrapolating the solutions of the cosmological equations backwards in time we come to the very beginning (Big Bang) when density of the universe was infinite which means that the expansion was initiated from one point. The local singularity is related to the existence of black holes or objects of such a great mass that light (information transmitter) is not capable of leaving them. In spite of being so good, the theory of relativity is faced with the problem of singularities. Although there have been numerous attempts at overcoming it by modifying the very theory of relativity

(e.g. Brans-Dicke gravity), the success of eliminating the singularities was, as a rule, annihilated by the new problems that opened up. In view of the fact that at the beginning of the universe and in the vicinity of the black holes the force of gravitation was so strong that it could be compared with other interactions, there arises a need to define gravitation as a quantum interaction. If a quantum theory like this is applied to the whole universe, the result is a theory called quantum cosmology. It is a product of the end of the twentieth century, being perhaps the boldest theory bequeathed to the new millennium. The results of quantum cosmology sometimes seem so paradoxical that they are easily rejected if not for an unprejudiced interpretation. The basics of quantum cosmology are represented by the work of Wheeler-DeWitt at the end of the 1960s [5]. Still, before the formulation of the inflatory models of the universe, quantum cosmology seemed to be somewhat superfluous being called a "theory for theory's sake" (like the "art for art's sake" theory in the arts).

The inflatory theory states that the observable part of the universe (probably even the whole of it) came to be through the fast expansion of an area as long as the Planck length which perhaps did not contain a single particle. Quantum cosmology, as any quantum theory, involves the calculation of the wave function as a solution of the Wheeler-DeWitt equation or the corresponding path integral [6,7]. Unlike the standard quantum mechanics wave function, the wave function of the universe $\psi(a)$ is not dependent on time but only on the "radius" of the universe a .

The fact that the wave function of the universe is not time-dependent is one of the basic paradoxes in quantum cosmology according to which the universe does not change with time. The term "change in time" presupposes the existence of something unchangeable, something external to the universe, and something that serves as a reference for its evolution. If the universe includes **everything**, then there could be no outside observer in reference to whom the universe could change. In reality, we never ask why the universe changes but why we perceive it as changing. Thus we automatically divide it into two parts, the macroscopic observer (we) and everything else. The "everything else" can change with time in spite of the fact that the wave function of the universe does not depend on time.

One of the most interesting questions related to quantum cosmology is that of the boundary condition. Namely, adequate boundary conditions should be imposed on the solutions of the Wheeler-DeWitt equation (or to the corresponding path integral). What boundary conditions, however, are we to take if the system whose wave function we are determining contains **everything** and if there is nothing outside it which means that it has no boundaries. In 1983, Hartle and Hawking seem to have resolved this problem in an acceptable way by introducing the condition that exactly reflects this fact, i.e. the no-boundary condition [6].

An additional problem is also the problem of the physical laws at the "time of the birth" of the universe. Namely, if we presuppose something rather obvious, like the fact that at one time our universe did not exist, the existence of the "then" laws defining its emergence and further evolution becomes meaningless because time did not exist either. The laws defining our biological evolution are embedded in our genetic code, but where were the laws of physics embedded if there was no universe? In other words, what existed before the Big Bang (if it happened at all or at least in the way we believe it happened)?

Quantum cosmology aims at solving some theological questions as well. Let us remember that there are two dominant religious conceptions about the creation of the world. In keeping with the Judeo-Christian teachings, the universe had a certain beginning. That is stated in the well-known hypothesis about the Creation, according to which the universe emerged from the Cosmic Egg. On the other hand, the basis of the Hindu-Buddhist teachings is Nirvana according to which the universe is timeless and has neither the beginning nor the end. Quantum cosmology proposes an excellent synthesis of these two different viewpoints. In the beginning there was Nothing. No space, no matter and no energy. According to quantum mechanics, however, the Nothing, or quantum vacuum, is unstable. The Nothing could start boiling as well as producing a great many "bubbles" each expanding at a very fast rate and representing one universe. If that is so, our universe, or rather that part of the multiverse of parallel and timeless universes, resembles Nirvana.

5. NUMBER FIELDS AND PLANCK SCALE PHYSICS

In standard quantum cosmology, the argument of the wave function of the universe is a real number. However, in addition to real numbers, there are also p -adic numbers in mathematics (Figure 1.). In order to explain this figure, let us make a short excursion into the number theory. The first set of numbers that we become familiar with in our young age is a set of natural numbers N containing numbers like 1,2,3... Since solving equations of the type $a + x = b$, $b \leq a$, becomes impossible within the set of natural numbers, it is necessary to introduce negative numbers and zero which, together with natural numbers, give a set of integers $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

If we also want to solve equations of the type $ax = b$, $a, b \in Z$, $a \neq 0$, it is necessary to introduce rational numbers as well (numbers that could be represented as fractions a/b , the numerators and denominators being integers). This set of numbers is usually denoted by Q . It is well-known, however, that this set does not contain the so-called irrational numbers. If we add them too, a set or a field of real numbers R is obtained. In a strictly mathematical sense, this completion of the set of rational numbers is performed by including the limits of the Cauchy's sequences which are calculated with respect to the usual norm, i.e. the absolute value $|\cdot|_R$. This, however, is not the only norm to be used. According to the Ostrowski theorem [4], the so-called p -adic norm $|\cdot|_p$ could also be used. Index p appearing here is used to denote prime numbers 2,3,5,7,..., or those natural numbers that are larger than one and that are only divisible by themselves and by one. If the p -adic norm is used for calculating the limits of the Cauchy's sequences, completion of the field of rational numbers with respect to that norm yields the fields of p -adic numbers Q_p . Let us note that the p -adic numbers were introduced by a German mathematician K. Hensel in 1899 and that they were first successfully applied in high-energy physics in 1987 [4].

A logical question arises here: Is there any sense in also using the p -adic numbers when constructing physical models? The answer is affirmative since there are some indications that space-time at small distances has some unusual properties that are not explicable in terms of the geometries constructed on the basis of the field of real numbers. Namely, at extremely small distances (Planckian), the Archimedean axiom of the standard Euclidean geometry becomes violated. According to that axiom $\forall x, y \in N$ for

which it holds that $|x|_R < |y|_R$, there exists a natural number n , such that $|nx|_R > |y|_R$. It is necessary to remember that this axiom actually enables measurement of distances in our great-dimension world.

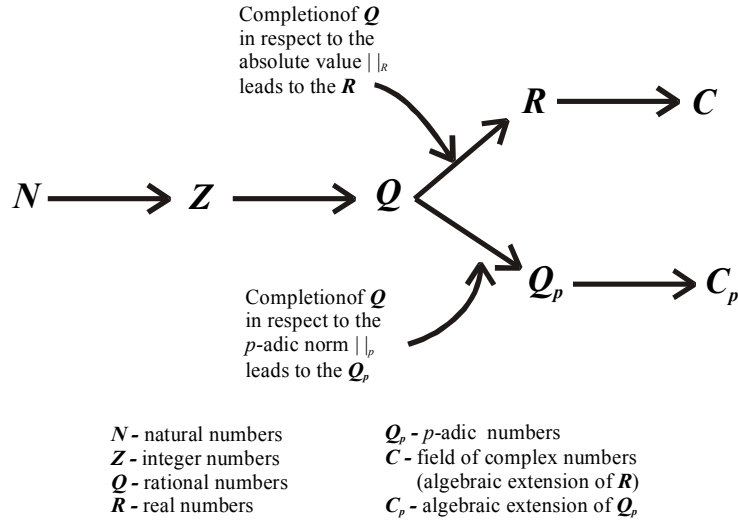


Fig. 1. Logical and historical sequence of acquiring the sets of the numbers used in mathematics and physics today.

Let us see now what is happening at supersmall distances of the order of the Planck length. One of the consequences of quantum gravity implies that there is a general limitation concerning the exactness of measurement [8] and that, according to it, it is not possible to measure distances shorter than the Planck length l_0 . In fact, when these distances are concerned the measurement error is greater than the distance that is being measured.

This indicates that the Archimedean axiom is not valid within these distances, which are fantastically small for us. Since the Archimedean axiom does not hold, it follows that geometry cannot be Euclidean (the one that we know from everyday life) either. Geometry is related to the number field that we use, and the number fields that produce geometry like that (non-Archimedean) represent the p -adic number fields. For these number fields it holds that $\forall x, y \in N$ where $|x|_p < |y|_p$, so that any natural number n yields $|nx|_p \leq |x|_p$ and, of course, $|nx|_p \leq |y|_p$. Let us see now what are the effects of using these number fields in cosmology.

6. COMPLETE QUANTUM COSMOLOGY

If we formulate quantum cosmology with $a \in \mathcal{Q}_p$ then we obtain p -adic quantum cosmology. If the p -adic ground state wave function is calculated for the de Sitter model of the universe (the universe with no matter but with a cosmological constant), it follows that they have the form of the so-called characteristic function over the integer p -adic numbers

$$\psi(a) = \Omega(|a|_p) \quad (2)$$

(in the units $\hbar=c=G=l_0=1$) where this function is expressed as

$$\Omega(|a|_p) = \begin{cases} 1, & |a|_p \leq 1 \\ 0, & |a|_p > 1 \end{cases} \quad (3)$$

The so-called adelic wave function of the universe (the wave function which contains the standard wave function ψ_R together with all the p -adic wave functions ψ_p , i.e. the "complete" wave function) is thus a product of the wave function over the real and p -adic numbers

$$\psi(a) = \psi_R(a) \cdot \psi_2(a) \cdot \psi_3(a) \cdots = \psi_R(a) \cdot \prod_{p=2,3,\dots} \psi_p(a) \quad (4)$$

with some conditions (3).

How is it possible to interpret this wave function? Let us note that the field of rational numbers $\mathcal{Q}$ is a point of departure for both real and p -adic numbers (Figure 1.). It is also important to note that the results of all the measurements belong to $\mathcal{Q}$. Consequently, the adelic wave function interpretation becomes meaningful only if its argument is a rational number. In view of the equation (4) and property (3) of the p -adic wave function, calculating the square of the adelic wave function module providing the probability of "finding" the universe with radius a , yields

$$|\psi(a)|^2 = \begin{cases} |\psi_R(a)|_R^2, & |a|_p \leq 1 \\ 0, & |a|_p > 1 \end{cases} \quad (5)$$

If the radius of the universe is expressed in units l_0 , it follows from (5) that the probability of the universe having the radius of the Planck length multiplied by an integer, is equal to the square of the wave function module $\psi_R(a)$; if, on the other hand, the radius of the universe is a fraction (non-integer) number of the Planck length, the probability that there could exist a universe with such a radius is equal to zero. This results in the conclusion that space is discrete or that there is "nothing" within the Planck distance.

This is not the only result of the adelic quantum mechanics and quantum cosmology formulated by B. Dragovich in 1995 [10]. Some other adelic (complete) models have been constructed recently. The free adelic relativistic particle is one of them [11]. By applying the analysis of the results of the adelic theory used in the analysis of the de Sitter model and now applied to the free relativistic particle, one comes to the conclusion that time is discrete too and that its time interval equals t_0 , where

$$t_0 = \sqrt{\frac{G\hbar}{c^5}} \approx 10^{-45} s \quad (6)$$

which is the Planck time.

In conclusion it is possible to say that the aim of combining the theory of relativity and the quantum theory resulted in the formulation of quantum cosmology. On the other hand, the need for a complete theory in our sense, i.e. the theory which exhausts all the number fields, resulted in the formulation of the adelic quantum mechanics and quantum cosmology leading to the discreteness of space and time on the Planck scale.

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O PROSTORU I VREMENU U KVANTNOJ KOSMOLOGIJI

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U radu su razmatrane karakteristike prostora i vremena u kvantnoj kosmologiji. Prezentovane su osnove ideja o supstancijalnoj i relacionoj koncepciji kao i o neprekidnosti i diskretnosti prostor i vremena. Dati su takodje elementi standardne kvantne kosmologije, odnosno kvantne kosmologije formulisane nad poljem realnih brojeva R . Kvantna kosmologija predstavlja primenu kvantne teorije na vasionu kao celinu u ranim fazama njene evolucije kada je univerzum bio veoma mali tako da su sve interakcije praktično bile ujedinjene. U postupku dobijanja maksimalno moguće informacije na osnovu kvane kosmologije neophodno je da ona bude "kompletna". Pojam "kompletna" se odnosi na formulisanje teorije nad poljem realnih brojeva kao i poljem p -adičnih brojeva Q_p . Obzirom da polja p -adičnih brojeva nisu dobro poznata, u radu je prikazana ideja o njihovom uvođenju u fiziku. U okviru p -adične kvantne kosmologije, koja predstavlja kvantnu kosmologiju nad poljem p -adičnih brojeva Q_p , prezentovani su glavni rezultati koji se odnose na De Sitterov model. Posledica ovakve (kompletne) formulacije De Sitterovog modela je diskretnost radijusa univerzuma koji on opisuje.