

## ON THE HARMONY OF THE UNIVERSE: FROM ANTIQUITY TO KEPLER

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**Abstract.** *"If I have seen further, it is by standing on the shoulders of giant", is what Isaac Newton said. One of them was Johannes Kepler (1571 - 1630), whose research substantially contributed to the heliocentric figure of the universe we now adhere to. We have in mind the planets moving around the Sun along elliptical paths, expressed by his laws, and the hypothesis that they are influenced by it. It was the birthplace of Newton's universal law of gravitation. This confirmed the ancient idea of a well-built harmonic universe, supported by the Pythagorean idea of a number that regulates all things in the universe, and Plato's five regular solids and so on, as well as Aristotle's doctrine of heavenly spheres, up until Copernicus heliocentric system.*

**Key words:** *harmony, elliptic path, law, Pythagoras, Plato, Copernicus, Newton.*

Fundamental questions as to the origin and nature of man, of the world, but especially how the universe came into being and our place within it, have forever exerted a profound influence on the mind of man. These concerns find expression in the earliest literature of the Greeks (Homer and Hesiod) among others, and can be traced to prehistoric times. Some of the basic concepts necessary for later astronomy were enunciated in the fifth century BC: Parmenides hypothesized the sphericity of the Earth and stated that the moon receives its light from the sun while Empedocles developed this further by inferring the cause of solar eclipses. These ideas have progressed down to contemporary scientific astronomy. To use Steven Hawking's phrase "science, indeed the whole of civilization, is a series of incremental advances each building on what went before."

The fourth century BC saw the introduction of the most characteristic Greek contribution to astronomical theory, the idea that the apparently irregular motion of the heavenly bodies should be explained by geometrical models based on uniform circular motion, that is, harmonic laws of the universe. Later sources attribute this to Plato. It is perhaps more certain that the first system of homocentric spheres centered on the fixed, spherical Earth, rotating with uniform motion around different poles was constructed by Plato's contem-

porary Eudoxus. Significantly, it was Eudoxus who first established axiomatic rigor in geometry. We may surmise, as Kepler did much later, that such success led to the notion that the explanatory power of geometry could be extended to other fields, including the heavens. These ideas were transmitted down to contemporary scientific astronomy, and continue to evolve. Newton remarked: "If I have seen further, it is by standing on the shoulders of giants."

Without Copernicus, and Kepler, there might have been no General Relativity, no "Big Bang" theory on the origin, evolution and ultimate fate of the universe.

My paper will explore the contribution to astronomy of one such "giant" upon whose shoulders Newton stood, with particular reference to his greatest work *Harmonices Mund*<sup>1</sup>, Kepler's vision of cosmic perfection.

Few scientists have repeatedly captured the imagination of novelists (Max Brod and John Banville among others). Fewer still have had a five-act opera (Paul Hindemith's *Die Harmonie der Welt*) based on their work, but then Kepler was no ordinary scientific thinker, devoting his restless life to the scholarly study of astronomy, epistemology, mathematics, optics, philology, philosophy and theology.

At the dawn of the scientific revolution the German astronomer Johannes Kepler (1571-1630) refashioned the ancient world picture of the universe. A contemporary of Galileo, near-contemporary of Copernicus, Descartes and Newton, Kepler lived in interesting though turbulent times of the Renaissance, the Reformation and Counter-Reformation, when seventeenth century Europe was racked by political and religious conflict known as the Thirty Years' War.

Like Copernicus whose work inspired him, Kepler was a deeply religious man. He viewed his continual study of universalities as a fulfilment of his Christian duty to comprehend the universe God had created. His life was anything but tranquil or lacking in contrast. Chaos and the quest for harmony were the driving forces of his life.

At Tübingen University Kepler was privately taught about the then new, controversial Copernican heliocentric theory soon becoming its forceful advocate.

No homocentric system can account for the obvious variation in size and brightness of, for example, the moon and Venus. To address this, the epicyclic and eccentric hypotheses for planetary motion were proposed. The Ptolemaic theory, a derivative of Aristotle's *De Caelo*, was based on a complex combination of epicycles and eccentrics employing uniform circular motions. Attempting to fit a kinematical model into a unified physical system based on Aristotelian notions, especially that Nature is not wasteful, Ptolemy described the universe so that each planetary "shell" is contiguous with that of the bodies immediately above and below. This system allowed the computation of all celestial positions and phenomena pertinent to the ancients with reasonable accuracy. In order to do so Ptolemy had to assume that the moon's orbit was along a path that brought it twice as close to the Earth as at other times, a flaw he was quick to recognize. The Ptolemaic vision of a small, completely determined universe, became the canonical view in the Middle Ages, albeit not universally accepted even in late antiquity, and for 1300 years dominated astronomy.

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<sup>1</sup> *The Harmony of the World*, 1619.

In 1473, Copernicus had noticed that the Ptolemaic theory involved a great number of identical epicycles and found he could eliminate them by assuming the Earth rotated around the sun. Kepler's conception of the universe, was that it was heliocentric, with planets orbiting the slightly off center sun along perfectly circular paths. His initial ideas were thus little more than a Copernican gloss on the generally accepted Ptolemaic system. However, the Copernican theory predicted a relative spacing of the planets such that the heavenly spheres could no longer be contiguous as postulated. Without such spheres to hold them in place, how could the planets remain in their regular orbits? Further, why were there only six then known planets?

Kepler was struck by a revelation that set him on a passionate journey changing the course of his life. It was, he felt, the secret key to comprehending the universe. He drew an equilateral triangle within a circle and another circle within the triangle. The ratio of the circles was, it occurred to him, indicative of the ratio of the orbits of Saturn and Jupiter. Encouraged by this revelation he assumed that all six then known planets were so arranged around the sun, that geometric figures would fit perfectly between them. He thought, that is, the planets were maintained in their orbits at varying distances from the sun determined by suitable nesting of celestial spheres.

Initial testing of his hypothesis with two-dimensional plane figures was unsuccessful. Kepler then tried the so-called Pythagorean or Platonic solids, namely the cube, the tetrahedron, the octahedron, the dodecahedron and the icosahedron. By fitting them into the spaces separating the planets, he demonstrated it was possible to produce the inter-planetary spacing predicted by Copernicus, without disturbing the classical view of the heavenly spheres. In addition, he had resolved why in his time there were only six known planets: there being only five perfectly regular or "harmonious" three-dimensional forms, this geometric framework could support only six planets. Indeed, he considered the Creator had planned these five regular polyhedra as beautifully elegant spacers between the six known planets.

Kepler spent the rest of his life in the quest for mathematical proof and observational data to validate his theory, albeit this turned out to be a mirage.

Subsequently inheriting a wealth of highly accurate astronomical data from the legendary Tycho Brahe, Kepler hoped to develop a proper cosmological model but to little avail. He examined the data for underlying mathematical patterns but became dissatisfied with Copernicus's revision of Ptolemy, and abandoned his own polyhedral theory ideas for lack of observational support. It seemed he could only impose accepted theory on the data if the observations were flawed by some large errors. Kepler's faith in Tycho Brahe's observational skills militated against so assuming. Tycho Brahe having paid special attention to measuring the movements of Mars, Kepler concentrated his efforts on elucidating the Martian data.

In October 1604, Kepler witnessed the extraordinary sighting of a new star which became one of the brightest in the sky before slowly fading into invisibility (a "supernova"). The appearance of such a star clearly indicated the heavens were not constant or unchangeable, and, in contradiction of Ptolemy's celestial spheres, that the universe had no natural boundary.

Continuing his work on the Martian observations, Kepler assumed the orbits in the new heliocentric model would be perfect circles centred on the sun. Notwithstanding innumerable, exhausting computations he was unable to match circular, or even elaborate

pseudo-circular orbits to the data. Recalling the old eccentric circles in Ptolemaic astronomy, Kepler experimented with fitting elliptical orbits to the data from Mars. Elliptical orbits fit almost exactly. Planetary orbits are elliptical, he determined, not ideally circular.

Once the concept of elliptical orbits had become established in his mind Kepler announced his first two laws of planetary motion, known even today as Kepler's laws. These assert that a planet moves in an elliptical orbit with the sun as one focus; and an imaginary line joining the center of a planet to the center of the sun ("radius vector") sweeps out equal areas in equal time intervals, allowing us to determine the position of a planet at a given time, or the time when a planet will be in a given position.

Meanwhile, despite religious upheaval in his new homeland and personal adversities, Kepler managed to publish two more important books: *Epitome Astronomica Copernicae*, and *Harmonices Mundi*<sup>2</sup>, a series of five books. Kepler's passion to seek and understand harmonic proportions in the solar system animated his writing *Harmonices Mundi*. Max Caspar, author of a standard biography of Kepler, describes *The Harmony of the World* as a passionate work, "a great cosmic vision woven out of science, poetry, philosophy, theology and mysticism".

Book V (subtitled *On the most perfect harmony of the heavenly motions*) of *Harmonices Mundi* contains Kepler's third law of planetary motion stating that the cubes of mean distances of the planets from the sun are proportional to the squares of their periods of revolution. This is the so-called harmonic law relating the sizes of the planetary orbits to the time to revolve around the sun. What Kepler surely hoped to find as he began his research for *Harmony* was some harmonic fine-tuning to explain the discrepancies between the polyhedral predictions of his first two laws and the actual planetary spacing. If this was ultimately a saga of disappointment, the climax of *Harmonices Mundi* is Kepler's extending his theory of harmony to music, geometry, epistemology and astronomy, giving each planet's orbital motion a musical note (defined as the difference between its maximum and minimum angular velocities between perihelion and aphelion) and attempting to write music based on the harmonies he thought might exist.

Kepler was evidently no mere curve-fitter, painstakingly trying to fit one trajectory after another onto Tycho Brahe's rich observational legacy until finally discovering, fortuitously, that an ellipse matched the data perfectly. He was a man in search of aesthetic harmony and order, his groundbreaking discoveries informed by profound religious faith and ardent Pythagorean beliefs.

Deeply moved by the vision of a world created by the Almighty with geometrical harmony according to an architectural plan mathematically conceived, which man could investigate and understand, Kepler transformed the Copernican heliocentric hypothesis of heliocentricity into an indispensable tool for practical astronomy, at once simplifying it, and enunciating the three laws of planetary motion that bear his name. While in Copernicus the sun was at the center around which the planets revolved, it played no role in causing their motion. In contrast, *Kepler's* sun emanated a force impelling the planets in their orbits around it. He initially interpreted this as a manifestation of the Neoplatonic *soul* of the sun before relegating it to the domain of physical forces. It was Kepler's Third Law, which inspired Newton's universal law of gravitation.

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<sup>2</sup> *Epitome of Copernican Astronomy*, 1617-21, *The Harmony of the World*, 1619.

His philosophical perspective is clearly the progeny of Pythagoras and his school, of Plato and the Neoplatonists. It is generally recognised that the conception of Pythagorean philosophy in later antiquity may be attributed to Plato (*Timaeus* becoming the classic formulation for the cosmological aspects of the Pythagorean world view) and his school. Aristotle reports that for Pythagoreans all things are numbers or imitate numbers. Philolaus writes that it is by number and proportion that the world becomes organized and knowable. The generation of numbers with the One in the center appears to coincide with the structure of the universe. There must be enough cosmic bodies to correspond to the perfect number ten (graphically represented by the ten-dot figure *tetractys*) with the Earth being one celestial body rotating around a central fire.

Kepler believed that the Sun, as the source of both Life and Light, represents God in the universe; that the world was spherical as this was the perfect, ideal form; that perfection in the universe may be attributed to geometrical principles governing its structure and in particular that the heavenly bodies may be represented by the only five, perfectly regular three-dimensional forms existing, the so-called Platonic solids, namely the cube, the tetrahedron, the octahedron, the dodecahedron and the icosahedron. Indeed he considered that God had planned these five regular polyhedra as beautifully elegant spacers between the then six known planets and that the planets maintained their orbits by suitable nesting of celestial spheres. This was of course consonant with the Ptolemaic description of the universe, in which each planetary "shell" is contiguous with that of the bodies immediately above and below it.

The Pythagorean perception of the universe was that the Ordering One had created it with regularity, correspondence, beauty, proportion and harmony in consonance with the symmetries and correlations intrinsic to numbers and their geometrical representation. As Kepler observed "matter follows God: where there is matter so there is geometry." This is arguably in the tradition of Plato's *Timaeus*, presenting a comprehensive image of a sufficient universe with a unique Creator, elaborating how the nature of the universe is derived from this source, detailing the cosmos, musical harmony, geometric figures, human nature and the elements.

Pythagoras was reputedly struck by the correspondence between relations in the musical and mathematical domains. He observed that when a taut string of a musical instrument was divided by a bridge so that its two segments yielded notes in the fundamental musical relations of octave, fifth, and fourth, the ratios of lengths formed the orderly series 1:2, 2:3, and 3:4. Pythagoreans inferred that all musical relations correspond to mathematical formulae and are governed by intelligible mathematical principles; and since music is a paradigm of harmonious order, they sought to describe the ordering of the whole cosmos and of the soul in terms of similar numerical relations and principles.

Philolaus gave a complete mathematical analysis of the diatonic octave, allied to speculations about cosmic *harmonia*. Plato provides the first account of the "harmony of the spheres" which music Pythagoras alone was said to hear. Pythagoras's dictum: "There is geometry in the humming of the strings, there is music in the spacing of the spheres" clearly underpins Kepler's cosmic structure involving the Platonic solids. Whereas Pythagoras's harmony of the spheres was metaphysical, Kepler's is *real*, albeit possessing frequencies dependent on orbital periods rendering it well beyond the human audible range.

Everything comprised in the cosmic order is structured as a harmonious whole. The analogy between the concept of harmony and that of the soul is striking. In *Timaeus*, Plato

uses the ratios of a musical attunement to describe the structure of the soul of the universe. The attunement is identical with that of Philolaus but Plato situates it in a sophisticated theory of means and proportions. Pythagoreans also believed that harmony acts positively on the human soul, as does music.<sup>3</sup> Music engages with metaphysics, a view espoused by Pythagoras, Plato and Aristotle.

Deeply moved by the vision of a world created by the Almighty with geometrical harmony according to an architectural plan mathematically conceived, which man could investigate and understand, Kepler transformed the Copernican heliocentric hypothesis of heliocentricity into an indispensable tool for the practice of astronomy.

Since the world was for Kepler a reflection of God it must necessarily be a perfect world and thus the manifestation of geometric principles. All is comprised in the cosmic order structured as a harmonious whole. The analogy between the concept of harmony and that of the soul is striking.

It was Kepler's passion to seek and understand harmonic proportions in the solar system that animated his writing *Harmonices Mundi* or "Harmony of the World". In the work Kepler investigates harmony and congruence in geometrical forms and physical phenomena.

His legacy to astronomy is the three laws of planetary motion which bear his name. The first of these states that planetary orbits are elliptical rather than cyclical with the Sun as one focus. The second asserts that the time taken by a planet to reach a particular position is represented by the area swept out by the radius vector drawn from the fixed Sun: the closer a planet is to the Sun, the faster it spins. The third law states that the cubes of the mean distances of the planets from the Sun are proportional to the squares of their periods of revolution. The third law makes it possible to construct a complete scale model of the solar system.

As a theologian Kepler recognized man's limitations and acknowledged that his thoughts were not of his own making but attributable to God's will. As he expressed it: "whatever I have been able to comprehend is attributable to You. Thank you, God, that I am able to appreciate all that You have created".

#### REFERENCES

1. *Cosmic mysteries*, Vol. 5, Kapopoulos Publishing (TimeLife Books), Athens, p.22.
2. Ferguson, Kitty, *Measuring the Universe*, Walker Publishing, New York, 1999.
3. Hawking, Stephen, *A brief history of time*, Bantam books, London 1995.
4. Hawking, Stephen, *On the shoulders of giants: The great works of physics and astronomy*, Penguin Books, 2002.
5. Homblower, Simon and Spawforth, Anthony, *The Oxford classical dictionary* (third edition), Oxford 1996.
6. Kahn, Charles, *Pythagoras and the Pythagorians*, Enalios Publishing, Athens 2005.
7. Logothetis, K., *The Philosophy of the Renaissance*, Athens 1955, pp. 560 - 587.
8. Moutsopoulos, Evangelos, *Motions of sounds, bodies and souls* (Prolegomena 1), Athens 2002, pp. 113 - 118.
9. Moutsopoulos, Evangelos, *Metaphysics and music*, Academy of Athens, 1998, pp. 274 - 275.
10. Sakellarios, G., *Pythagoras*, Kaktos Publishing, Athens 2006.
11. Wright, F., A., *Lempriere's classical dictionary of proper names mentioned in ancient authors*, Routledge, London 1978.
12. Xanthakis, I., Georgalas, G., Adamopoulos, G., Karapiperis, L., Makris, K., *The mysteries of the Universe*, Faros Publishing, Athens 1957, p. 11.

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<sup>3</sup> See Evangelos Moutsopoulos, *Metaphysics and Music*, Athens, 1998 and *Motions of sounds, bodies and souls*, Prolegomena, 2002.

## O HARMONIJI SVEMIRA: OD ANTIKE DO KEPLERA

**Norma Pastelidou**

*"To što sam video dalje, stoga je što sam gledao s ramena giganata", rekao je Isak Njutn. Jedan od njih bio je Johan Kepler (1571-1630), čije je istraživanje suštinski doprinelo heliocentričnoj slici svemira kojoj se priklanjamo danas. Imamo u vidu kretanje planeta oko Sunca po eliptičnim putanjama, izražene njegovim zakonima, kao i hipotezu da to čine pod njegovim uticajem. Bilo je to rodno mesto Njutnovog univerzalnog zakona gravitacije. Tako se potvrdila antička ideja o harmonički sazdanom univerzumu - jer idu tome u prilog i Pitagorejska zamisao broja, kojim su uređene sve stvari u svemiru, i Platonovih pet tela pravilnog oblika, jednako kao i Aristotelovo učenje o nebeskim sferama itd., sve do Kopernikanskog heliocentričnog sistema.*

Ključne reči: *harmonija, eliptična putanja, zakon, Pitagora, Platon, Kopernik, Njutn.*