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KANT, SCHELLING … OR CREATIVISM or On a question in the philosophy of mathematics

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Abstract. *In the philosophy of mathematics, as in its a meta-domain, we find that the words as: consequentialism, implicativity, operationalism, creativism, fertility, … grasp at most of mathematical essence and that the questions of truthfulness, of common sense, or of possible models for (otherwise abstract) mathematical creations, i.e. of ontological status of mathematical entities etc. - of second order.*
Truthfulness of (necessary) succession of consequences from causes in the science of nature is violated yet with Hume, so that some traditional footings of logico-mathematical conclusions should equally be fallen under suspicion in the last century. We have in mind, say, strict-material implication which led the emergence of relevance logics, or the law of excluded middle that denied intuitionists i.e. paraconsistent logical systems where the contradiction is allowed, as well as the quantum logic which doesn't know, say, the definition of implication etc. Kant's beliefs miscarried hereafter that number (arithmetic) and form (geometry) would bring a (finite) truth on space and time, when they revealed relative and curveted, just as it is contradictory essentially understanding of basic phenomena in the nature: of light as an unity of wave – particle, or that both "exist" and "don't exist" numbers as powers of sets between $\aleph_0$ *and c (the independence of continuum hypothesis) etc.*
Mathematical truths are "truths of possible worlds", in which we have only to believe that they will meet once recognizable models in reality.
At last, we argue in favor of thesis that a possible representing "in relief" of mathematical entities and relations in the "noetic matter" (Aristotle) would be of a striking heuristic character for this science.

Key words: *Consequentialism, implicativity, truthfulness, possible worlds, noetic matter.*

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When we used the words from the title "Kant, Schelling, . – or creativism" to ponder on the indicated alternative, we had in mind primarily to ask one kind of questions in this domain the distinct priority over any other. It is a matter of personal conviction, though, whether one's curiosity will occupy itself with one philosophical problem rather than with another. We equally recognize several "paradigms" which, in a sovereign way, dominate this area. These ossified forms often acquire the nature of the very prejudice that some (mathematical) objects exist to shed light on the primacy "as a rule" in the "ontological", "epistemological" or "anthropological" key[1]. This is suited by a certain "laziness of mind" to penetrate beyond the usual problem-solution types, or beyond the hypertrophy of evoking them, and into the field of (possibly) other kinds of questions, and of answering them.[2]

Why is precisely Kant both a paradigm and prejudice in this regard?

He is the latter because his work *Critique of Pure Reason,* within the whole philosophical tradition, has the aureola which is not to be "touched", although it essentially answers here (only) the question - "How can *a priori* synthetical judgements be possible"?, the answer to which is nearly trivial - these are "the forms of sensation',' space and time, as well as the categories of quantity, quality, modality and relation. In addition, the phrase itself, or its parts, is heedlessly repeated throughout the work hundreds of times, without the need for it, and it does not go in favour of the "greatness" of this work.

And he is the former, because Kant saw the mathematical enigma of the highest rank in questions such as "what makes the mathematical objects *exist* at all, and the knowledge of them to be necessary and general?". These are two eminently philosophical questions, the first of which " searches" for the place that necessarily belongs to mathematical entities in the hierarchy of beings, whereas the second one answers to one of the central questions of any gnosiology.

In the first case, we tacitly assume that an object has "more dignity" if it has a higher position in a fixed order, and this fact implies higher degrees of its reality, its necessity, and its truth all the way to justness and beauty. Let us illustrate it with the example of Plato, who "bound" the simplest spatial forms such as geometrical solids - cube, tetrahedron, octahedron and icosahedrons, to one of the (antique) elements - earth, water, fire and air respectively, doing the same with the fifth one, ether, represented by dodecahedron. Here four solids-elements are sufficient to build the world of (finite) things and beings, also making possible the transition of some creations into others, from which only dodecahedron is exempt – the unchangeable element which enters into the composition of immortal souls. But even in such a completed world, the mathematical entities still remain the key to understanding that other world, the only one that really exists, the world of ideas. Their importance lies precisely in this – in the ontological, in the epistemological, as well as in the moral sense.

What Kant did in that sense was that he brought number (numbers) into a relation with the succession of moments in time, and the geometrical forms with the extension in space, thus putting the highest ontological claim on these instances (space and time) – that of "the constituent forms of reality". Or, in the terms of transcendental idealism – de-

[1] The so-called "Kuhn's paradigms"

[2] In terms of value, unlike this, precisely constant rules such as moral (and especially religious) dogmas form the identity of an individual within the community of people, contributing to its survival throughout the history, or to its existence in humanity.

prived of the contents in itself, these pure forms of sensation bring a double picture of both transcendental ideality (which is *a priori*) and empirical reality (for they apply to all objects of sensation and to all people). More specifically, the pure notion of number would, for instance, be a synthesis of the sensory phenomenon of quantity and its corresponding intellectual representation, realized in the so-called "transcendental scheme" in pure time. This power has the ability to connect the sensory with the intellectual, etc.

According to this, here too, mathematical objects obtain a high place in the hierarchy of all beings, as they do with Plato, but does this satisfy "best" the real philosophical curiosity in this area? For the idea that numbers are numbering "set" in time, or that geometrical figures are "consequences" of space, is a (rather) naïve notion. And isn't a far "more philosophical" problem, for instance - "where the mathematical deductions are conceived and which rules apply to them", or in other words - "what is this that makes mathematics to be mathematics, above all"? More precisely, Kant seems to have let out of his sight that which Aristotle clearly distinguished, and those are material, formal, efficient and final causes of something, since transcendental idealism "accepts" only the material cause and speaks only superficially about the others.[3] For instance, a piece of marble we have "helps only a little" for Hermes's figure to be sculpted in it, although it is potentially imposed in the mass, without an idea-conception which the sculptor should have in his mind. The objects made of wood, metal,etc. are farthest removed from the matter of wood, or metal, – it *itself* does not "impart" on them the forms that they have, but their creators, their makers do.

By going back, Kant answered the "ultimate" question - "What produces – in truth 'for us' and not 'in itself' – the space and time, the answer to which would be, in short – space is a pure form of external, and time of internal perception". Space and time themselves remain to be *Dinge an Sich*, therefore, unfathomable. Thus Kant approached the Schelling's question - "Why something (being) exists, rather than nothing (non-being)", as the most rare and the most "bold" question of both the layman curiosity, and philosophical reflexion in general.[4] This, without a doubt seeks to penetrate into God's very concept before creating the world– "the sin" that our foremother Eve committed when, dispite the prohibition, she tasted the fruit from the "tree of knowledge" Thus the question of transcendental idealism – to what do mathematical entities, numbers and geometrical figures owe their existence - appears to be of the same (existential) order?

In truth, Kant's interest in the philosophy of mathematics – as well as in the pure sciences and the metaphysics – referred (only) to the knowledge which he designated as the"a priori" and "synthetical", finding the oasis of such knowledge to be in each of the three spheres - sensation, reason and mind. It is brought on, certainly, by the subject-predicate judgements[5] and the only possible case here is that, in one instance, the notion of predicate can (by analysis) be "extracted" from the notion of subject, or not be in the other instance. These are two classes of such propositions, distinguished as "analytic" and "synthetic" notions, while in the epistemological sense they are referred to as those that

[3] When he finds that mathematics is "a sensory reasoned activity in constructing notions in perception"

[4] From this point, for instance, Hegel's absolute spirit begins a spiral path of its self-development, and Hegel "tried to think nothing by "lighting it up" with the equally empty pure something" (Bloch), etc.

[5] Which is not a sufficiently general case - this pattern of propositions is eluded by the statements such as "either . or", "if . then", etc.

do not enlarge the knowledge and those that do, as follows. We have a triangle as a "closed line with three segments", or the proposition whose subject is "triangle", and its three predicates are "to be a line", "to be a closed line" and "to have three segments". Here, the propositions such as "triangle" is a line", or "triangle is a closed line" are analytic (they only say and repeat a part of which is contained in the definition of triangle), but a multitude of other - synthetic – positions (where this is not the case) are also valid, such as "the sum of interior angles is equal to right angles", "the medians are intersected in the same point".[6]

As for the distinction between positions based on *a priori* and *a posteriori* principles, it is made by the question whether they become true (already) in the reason, i.e. before the experience, or after experiment, (only) in experience, as follows. The analytic propositions are (all) *a priori*, whereas truthfulness of the scheme of propositions "the bodies expand in the heat" is owed to verifying this in experience – by going from one of them to another, until we find that this is true for lead, copper but not for water. Thus, the latter proposition is synthetic and *a posteriori*, but, as we said, Kant turned nearly all his "transcendental interest" to the *a priori* propositions in mathematics, as well as to science and metaphysics. The predicate of proposition shall not be contained here in the subject, and such a knowledge will be *a priori* because what we get to know about the notions and their relations is just what "we import ourselves in the reflexion by a construction in the perception". [1]. This is certainly one of "the most philosophical places" in the system of transcendental idealism, but critically speaking, let us at least say the following:

First, the requirement that the definition of a notion should by itself "release" all the knowledge about the object which it refers to, implies the understanding of the logical status of definition - namely, does it serve to distinguish an object from any other, or is it required to bring out its essential properties (some or "all"), i.e. the very object?[7] Rather, let us comply with the conviction that it is primarily the first, for we recognize, for instance, a triangle by three segments and this designation belongs to its definition. Is it necessary to (immediately) require for it to contain that the sum of angles equals two right angles, or that "its four main points" are on the same line? The question "what is a triangle" hides in itself the meaning "what is *all* triangle", while the answers are (logically) separated by way of definitions and theorems. Thus, designations which are not necessarily "essential" and not necessarily "all" can be applied here, so that this epithet alone should be more or less an arbitrary one - the mentioned property of the "four main points" is the essential one for this figure, but the definitions of a triangle avoid it as a rule.

Secondly, here we alone really create the notions of "angle", of "median", of "orthocentre", but the whole geometrical knowledge is owed to such apparent truths, such as that "the objects equal to the same object are equal among themselves".[8] There is the same thing in arithmetic too, where such a truth (this time "provable"[9]) is - "The numbers equal to the same number are also equal among themselves". We recognize twice "the principle of identity" in the basis, or the principle of uniform flow of time – i.e. of such "unfolding" of extension (of space) etc., on which different types of axioms are based" –

[6] Before the new"truths on triangle", one famous mathematician exclaimed *A bas le triangle!* (*Down the triangle!).*
[7] The standpoints known as "nominalism", "conceptualism", "realism", as folows
[8] The first axiom in Euclid's *Elements*
[9] On the grounds of the axioms of Peano

axioms of connexion, order, continuity, movement (in geometry), induction (in arithmetic), etc. And should we have to ask "why these apply", or why by such simple determinations (of numbers, of figures) it is possible to make an abundance of true assertions on them (theorems), the "answer" is perhaps in the idea of a sort of "pre-established harmony", on which Leibniz once spoke.

However, we should not disregard that we are in the domain of mathematical idealities and that the analogy with empirism here is (only) of a sensory-intuitive nature. This is the weak point of Kant's farthest conviction that axioms of geometry bring innermost truths about space, and those of arithmetic about time. For neither time flows uniformly (but does it "with regard to", hence, relatively) nor is space absolute, as the intuition on it tells us. We know that time is dependent on the velocity of the movement of bodies, as well as that the space is (at least) curved – therefore, non-Euclidian, and not even that of Lobachevsky, but rather Riemann's unreachable manifoldness of an arbitrary number of dimensions,[10] whose properties depend on the disposition of the cosmos masses (of the stars) and of the motion that they produce. Therefore, Kant's conviction that by "putting" numbers into time and geometrical figures into space, and concatenating these categories with a (powerfull) "glue" of transcendental idealism, he would undoubtedly show that all the knowledge of space and time is possible exactly as necessary and general, remains an illusion.

Then, could we - from the position of the development of the science of mathematics, two centuries after Kant – bring to a (higher) clarity the circle of philosophical questions he dealt with, and which are, after all, of the highest importance in this area?

The mathematical object is therefore none other than an abstraction, because even if there is a place for intuition on the way to the notion, it appears that it is deceptive, as in the case of space and time. Historically, "the deceptions" began with the irrational numbers of Pythagoras, for infinite number of addends here gives a final sum in the result; Aristotle distinguished between potential infinity and actual infinity, and with Cantor, for instance, it became clear that an enumerable infinity is (only) a real subset of continuum. Associated with it is the result of P. J. Cohen [2], according to which an arbitrary multitude of other infinities, set between the two mentioned, is not incompatible with the axioms of the set theory, although we fail to "see" for now which classes of numbers fit that place.[11]

Or "beyond" the intuition of number that we have, we proceed with symbols $\aleph_0$ (cardinal), ω (ordinal), ε (the highest ordinal), doing the same with the sign $\surd(-1)$ (imaginary unit). Although these are "numbers", the following applies here:

$$1 + \omega \neq \omega,\ \omega + 1 \neq \omega,\ \omega = 2\omega,\ \omega = 1 + 2\omega$$

(ω - ordinal) and so on.

Moreover, in the geometry of XIX century especially (Lobachevsky, Riemann, Klein), a conviction has been confirmed over and over again, one which in the theory of knowledge had been promoted by Descartes, Leibniz, Hume, etc., on the existence of two kinds of truths

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[10] For instance, *string theory* is being developed nowadays in physics, and the world with ten dimensions is studied, and there are also philosophies that promote doctrines about "nothing" as "somethig", etc.

[11] Along with cardinal (one, *unus*) and ordinal (the first, *primus*) numbers, as we know, there are also partitive (once, singuli) and adverbial (by one, semel) numbers in the language. Is it possible to construct a mathematical theory starting from the two latter classes, as it is done in the case of the former two – with the "cardinals" and "ordinals"? It would lead to the "completion" of this infinite gap.

- factual (véritées de fait, matters of fact) and reasonable (véritées de raison, matters of ideas), so that the quest for the latter kinds of truths rather than for the former ones suits the science of numbers and geometrical creations. In his habilitation thesis[12], Riemann saw"hypotheses" at the basis of geometry, where "axiomes" (as the "evident" truths) reigned up to that time; Veierstrass discovered a continuous function on an interval, but without any tangent, so that Peano could construct a "curved line" which fulfills a whole square as a surface. Number, angle, and triangle, still remain (basic) entities, but relation, and mapping acquire a predominance in this science, for they grasp more of its essence, or they bring the "key" for its proceedings and methods to be recognized to the highest extent. A clear example of this is the algebriac notion of "group", expressed in (nearly logical) terms of "null", "neutral", or "inverse", and which are sufficient for the definitions of each of the geometries - euclidian, affine, projective, topology - to be deduced this way. In short, the difference between them is drawn those mapping functions which leave some of their properties invariable - in metric (Euclidian) geometry these are the distances between points, in affine geometry – parallelism of straight lines, and in projective geometry and topology – the projective mappings and homeomorphisms, respectively.[13]

Furthermore, with Lobachevsky (1826) the standpoint once again essentially changed, and the old Euclidian science ceased to be a "science of spatial forms" (Engels); we can't draw two parallel straight lines in the cosmos - if it is curved itself, and the rays of light deflect too, while the Earth rotates around its axe, and also orbits around the Sun in the cosmos which is not still either.

Thus the science of geometry, by ultimately neglecting the nature of objects it deals with in the basis, made possible the authentic productiveness of its theories - to Lobachewsky, Rieman, Hilbert,"points" and "'straight lines" would henceforth be any objects that can be subsumed under some relation, and not only those in the "common sense" of the word (or in Euclidian sense). All this would not happen at the expense of logical impecability (coherency) of the theory itself. Thus relation and order have pointed out their supremacy over number, magnitude and form, thus favouring something that Leibniz once did - giving arithmetic, algebra, and geometry a common name *ars combinatoria.*

Let us formulate it all in a somewhat more succinct way, as follows.

On the one hand, we have a "Kantian" group of questions such as "*how* the synthetical *a priori* judgements are possible", *why* they obtain the necessity of validity, *in what way* mathematical notions are constructed in perception etc., which refer to the purely ontological sense of quest for that "necessary being" (an instance, a "material") that enables them. And essentially close are gnosiological questions related to the sensory-reasonable nature of the mathematical notions, of the certainity of knowledge attained in this science, or the degree of "truth" about the reality that it brings. But these are *general* ontological and *general* gnosiological questions too, whereas, on the other hand, what the mathematical universe is impressed by from the beginning is, above all, its symbolic structure, which by rising on immaterial grounds, achieves the supreme coherency of its creations. Then we find that a (possibly) *basic question* in the philosophy of mathematics would be just this -

[12] Its title is: "On the hypotheses which are at the basis of geometry", 1854.
[13] With Felix Klein in s.c. its *Erlangen's programme*

How are these creations made and what makes the transition from one "space of symbols" to another legitimate?

Thus, the most succinct expression of this idea is realized in the group of words **to follow**, **to ensue**, **the succession**, **the consecution**, in which the mathematics remains "elusory", and the experience only helps it formulate analogously its own means of expression - definitions, axioms, postulates, – and that, as we know, will not be sufficient to exhaust the intuitive domain to which they are referred to (Gödel). We have already expressed ourselves against "the apparency of axioms", as a reliable criterion (of truthfulness) of mathematical theories (Pythagoras, Kant, Lobachevsky), and let us use this this occasion to evoke the opinion of A. Einstein regarding that: "As long as the mathematical propositions refer to reality, they are not certainties and *vice versa*".[14] Indeed, how can the axioms bring this "micro truth" on reality if the latter is what we have yet to discover and it represents an unforeseeable secret? The thruts in mathematics are (rather) approximate, "the truths of this world", conditional but not absolute nor timeless. In the world we live in, for instance, such truths are "on the side" of number constants: e, π, $\hbar$ (Plank's constant), or the equation $E = mc^2$, not disregarding the marvellous ability for all "to be arranged by numbers" (Pythagoras), which we bear witness to today, and not only in the universe of informatics. But, the inherent the power of mathematics is to penetrate (or apply to) other (possible) worlds, which are potentionally given to the existing world – as it has been done once with the four dimensional world.

Poencaré said that the axioms were conventions, but this is certainly the case with definitions and inference rules, too. They are formulated to be as productive as possible, or to accomplish a higher volume of inferences possible. This is referred to not only by the "always renewed" logics, as the constantly different ways of converting one groups of symbols into another; the logic of Aristotle of Kant's age is surpassed, first by dialectic logic (Hegel) and recently by the different "non-classic" logics – modal, relevance, dialetheistic, paraconsistent etc. Therefore, we shall eliminate the possibility to deduce conclusions in mathematics as long as this is done "by following rules" (Vittgenstein) and they remain non-trivial, even if the contradictions were allowed, as in the case of paraconsistent logics.[15] And as the latter are being accepted as "solutions", as in the science of physics, for instance, in the case of (dual) wave and corpuscular nature of light, would it not be possible to allow the same in the case of the discrete-continuous duality in the set theory? Likewise, the antique contrast between"one" and "many" (Pythagoras) would, if accepted, possibly refute the axiom of choice in this theory, for every set would be an "impenetrable" whole (one) from which it is not possible to separate the parts. And in "quantum logic", unlike in classic logic, the function of conjunction is only partially defined, while the implication is not defined at all, like in the so-called fuzzy logics where the belonging of an element to a set is determined (only) by probability, and so on.

This is why the groups of words which describe the nature of mathematical creations most accurately, would be precisely **creativism**, *operationalism*, *implicatibility*, *consequentialism*, *productiveness*, where as we said, every reference to the sensory reality, "common sense", or the other sciences, etc., should have (only) a heuristic character, or the character of "ideas", which would afterwards"extend", in a symbolic space, to the coherent creations.[16]

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[14] Albert Einstein: *La géométrie et l'expérience*, 1921

[15] Vitgenstein also gave right to it when he said: *I predict a time when there will be mathematical investigations of calculi containing contradictions.*

[16] And then these ideas – regarding any (possible) application - would only "draw near", with more or less success,

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KANT, ŠELING, … ILI KREATIVIZAM ili O osnovnom pitanju u filozofiji matematike

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U filozofiji matematike, kao jedne meta-oblasti njene, mi nalazimo da reči kao implikativnost, konsekvencijalizam, operacionalizam, kreativizam, plodotvornost, … zahvataju najviše od matematičke suštine, a da su pitanja istinitosti, zdravog razuma, ili mogućih modela za (inače apstraktne) matematičke tvorevine – drugorazredna. Istinitost (nužnog) sleđenja posledica iz uzroka u nauci o prirodi narušena je još s Hjumom, da bi nekolika tradicionalna uporišta logičko-matematičkog zaključivanja bila jednako dovedena pod sumnju u prošlom stoleću. Imamo u vidu, recimo, protivnost striktna – materijalna implikacija koja je dovela do nastanka relevantnih logika, ili zakon isključivog trećeg koji su poricali intuicionisti, odnosno paraneprotivurečne logičke sisteme gde se protivurečnost dopušta, jednako kao i kvantnu logiku koja ne poznaje, recimo, odredbu implikacije itd. Potom, izjalovila su se i Kantova uverenja da će broj (aritmetika) i oblik (geometrija) doneti (konačnu) istinu o prostoru i vremenu, onda kada su se vreme i prostor "pokazali" relativnim i iskrivljenim, kao što je protivurečno i suštinsko razumevanje osnovnih fenomena u prirodi: svetlosti kao "jedinstva" talasa-čestice, ili to da i "postoje" i "ne postoje" brojevi kao moći skupova između $\aleph_0$ i c (nezavisnost hipoteze kontinuuma) itd.

Istine u matematici su "istine mogućih svetova", za koje treba samo verovati da će naići jednom na prepoznatljive modele u stvarnosti. Najzad mi argumentišemo u prilog teze da bi moguće "reljefno" predstavljanje matematičkih entiteta i odnosa u "umnoj materiji" (Aristotel) bilo od izrazito heurističkog karaktera po ovu nauku.

Ključne reči: *Konsekvencijalizam, implikativnost, kreativizam, istinitost, mogući svetovi, umna materija.*

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to the existing reality, or to the other kinds of realities that would (potentially) be discovered again.