ON THE ONTOLOGICAL AND EPISTEMOLOGICAL STATUS OF MATHEMATICAL ENTITIES

UDC 111 Platon; 165 Platon

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Abstract. *By examining the ontological and epistemological image of Plato's doctrine and that of the neo-Platonists Iamblichus and Proclus Diadochus, we explain in more detail the viewpoints of the latter on "mathematical beings", "geometrical beings" and "mathematical methods". We also find that the principles of "the one and the many", of "the limited and the unlimited" etc. are happily discovered "strongholds" reinforced through two-and-half millennia of development, especially in the natural sciences.*

Key words: *Plato, Proclus, ontological, epistemological, mathematical, object.*

PLATO

On the forms

Μηδεὶς ἀγεωμέτρητος εἰσίτω[1] is only a rougher expression of the furthest essence of Plato's thought on the forms – whether these words really belong to him or not. Geometric forms, namely, triangles, squares, circles, have a double nature, both as ideals and realities, incarnating at the same time the intelligible and the sensible, the durable and changeable; with notional as well as visible forms. The latter surpass the former numerically:

"for there are many similar to them, while of (each) form there is only one".[2]

In general, Plato used the word "forms" to denote the special timeless and constant essences which are the patterns or models of particular things and beings. The nature of a form itself is to be a pure *being*:

"that colourless and formless and untouchable being which really exists,"[3]

[1] Id est.: "Let none enter who does not know geometry" - an supposed inscription at the entrance in Academy.
[2] *Metaphysics* 987 *b*.

the *cause* of a thing:

"it is not beautiful according to anything but in that it participates in the beauty with the beautiful",[4]

its *pattern* and *model*:

"these forms of which we speak are proto-images in the eternity of nature",[5]

a *truth*:

"that which represents real knowledge of that which is only itself",[6]

and (even) *beauty*:

"suddenly to see something by its nature miraculously beautiful",[7]

And there is no doubt that Plato was inclined to the Eleatic starting point that all which really exists and is true, is "based" on identity with itself - and is an image of a non-created, non-changeable, motionless and imperishable being, which he "dedicated" to the mentioned essences. Through the forms exist all that really is, it acquires its essence thereby, doing it in a necessary way, so the forms are said in Plato to realize the cosmological, the ontological and the logical function over things, beings, phenomena. Thereby he found a way, for the first time in the course of human thought, to inhabit a space which was once possessed only by the Gods,[8] and the world in which we reside to be just a detailed copy of "the world of ideas". Things are further, per him, a conjunction of one sensible, material, perishable and even "bad" part, as well as of another one, which is insensible, incorporeal, constant and "good". What does possible the knowledge about them to be necessary and general, but it can be realized only if could be "gotten across" an infinite gap (χωρισμός) which separates two mentioned worlds.

Here is a pregnant statement, in the spirit of the previous quotes – from Plato[9] himself, capable of expressing concisely the teachings of this Archon of all idealisms and all that would come later.

"For any particular being, there are three essential things in creating the notion of it, whilst the fourth is the concept itself, and the fifth - what is really comprehensive and what really is [form]. The first is the *name* [ὄνομα], the second *definition* [λόγος], the third *image* [εἴδωλον], and the fourth - the *notion* [ἐπιστήμη]."

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[3] *Phaedrus* 247 *a*.
[4] *Phaedon* 100 *b*.
[5] *Parmenides* 132 *a*.
[6] *Phaedrus* 247 *d-e*.
[7] *Symposium* 210 *e*.
[8] In truth, by words: "with the power of God", "with the power of will" ... these questions are also answered: where from a thing has rather this, than other essence, it is possible its knowledge in the spirit, do it in a necessary way etc., so that it is matter here of a special "pantheism".
[9] Plato: *Letters*, "Rad" Beograd,1978, p. 65.

Or in two larger quotations:

"The same goes for a straight or curved line, for form, colour, for good and beautiful and righteous and for all objects, derived from nature or made by human hand, for fire, water and other things, for all beings and mental properties, for every action and state."[10]

"There are thousands of ways of showing how each of the four cognitive stages is indefinite. But, as stated before, the most important proof is: between two things i.e. a human being and some of its properties, the soul tends to know the essence and not the property whilst all of the four stages provide, through words and facts, what the soul is seeking …".[11]

Because such are "the good, beautiful and righteous" and a sovereign should perceive all of it so that he could be capable of forming a model state, Plato headed, let us recall, to Syracuse (for the second time) to meet the tyrant Dionysus Junior, so he could gain certain knowledge on this matter. The teacher started by lines, circles, forms … in education, so that at one point it seemed that the sovereign really believed in regulations in a society if they should exist.

This is (only) an excerpt from the "theory of forms, which as the ontology and epistemology of the same theory, leads to a double image of knowledge and of opinion, in the first case, i.e. of the intelligible and visible world, in the second one. *Knowledge* is related thereby to the intelligible world and *opinion* to the world of sensible things and forms. The first of pairs is "above" the other and each of them includes two new instances, equally ordered. They are *reason* and *understanding* (διάνοια) which are "related" to the *proto-principles* and *mathematical objects* (τὰ μαθηματικα)[12], respectively (within the knowledge and the intelligible world), i.e. the *belief* and *premonition* which "grasp" the world of living beings and images, in that order (as referring to opinion and the visible world).

This is illustrated by the quotation:
- "Do you have both worlds: intelligible and visible?
- Yes, I do.
- Thus imagine they are as this line divided into two unequal parts, and every part – visible and intelligible – divided again in the same manner. You shall find images in the visible world that differ by its clearness. And by images I mean, in the first place, images of objects and afterwards, reflections of objects in water …"[13]

The mentioned instances in the ontological semi-plain are ordered thereby - from higher to lower – according to the degree of *reality* they have and in the epistemological one – according to the principle of *clarity* of knowledge which they possess. So the highest place in the hierarchy of beings, according to Plato, will belong to *forms*[14] (as princi-

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[10] *Ibid*, p. 66.

[11] *Ibid*, 67.

[12] Aristotle's expression on it should be: "Plato asserted that apart from sensible and ideas, as something intermediate, do exist mathematical objects also, which differ from the first ones as eternal and unmovable, and from ideas - for they are many similar, and (each) idea is only one". (*Metaphysics* 987 b).

[13] Plato: *Republic*, p. 510 *a*.

[14] On the ideas also it is possible to speak with distinctions. They are either "pure" (*reasonable*, *noetic*), either "intelligible" (*dianoetic*), or sensible ideas, depending on subjective power by which they are "grasped". A pure

ples, proto-principles), at a place lower than that there are *mathematical objects*, the lower *world of animals* and, finally - the mentioned *images*[15].

Homologous to this sequence in all points is that composed of *reason*, *understanding*, *belief* and *premonition* and this is a descendent sequence by the degree to which its members can be known.

"So it suffices, I said, if we nominate as before the first part science, the second conviction, the third belief, and the fourth conjecture; then the latter two, opinion, and the first two knowledge."[16]

A multiple identity of relations takes place here: the intelligible world to the visible one is as knowledge to opinion, and the proto-principles to the world of living beings are as mind to belief, i.e. geometrical forms to images as intellect to conjecture etc.

Or, as Plato said:

"As the forms are to sensible things, so knowledge is to opinion and as knowledge is to opinion, so the mind is to belief, and the intellect to conjecture."[17]

So it remains (from the Academy onwards) the world of sensible things and beings to be linked by means of an "infinite" multitude of quadruple "chains" of relations to the supra-sensible world, what is perishable and material – to the eternal and incorporeal, terrestrial – to divine. And this relation is just the "thing (being) - idea" that performs the logical, ontological, epistemological, moral etc. functions which the other member of the dyad realizes over to the first one to be both necessary and general. In such a way, we have seen, the dominant standpoint that only the Gods (understanding both divine and human nature) realize the mentioned functions in man is disarranged therewith or is made by those schematic "powers" designated as λόγος, νούς and so on; place has been given here to a special "pantheism" which, especially in the sciences, is of use for constructions of notions and derivation of conclusions within them. This time diffuse, this ontological theism would acquire "a more classic" shell through the Neo-Platonists Porfyreos, Iamblichus and Proclus, in the form of special "hypostasies", or "big" ordered schemas of thinking etc.

At the same time, by Syrian, say, the forms are largely distinguishable from the things which they are related to. They are neither abstractions of the sensible data nor are they general notions at all: for they are simple, perfect and inseparable essences, distinct from things themselves[18].

To think in such a way, Plato may have taken Pythagoras as his model who – in truth, by the principle of all existing –foresaw the *abstract* essence, number, in the sense of,

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idea hereafter belongs to the gender of divine essences, as well as a formal, efficient or a final cause, whereas every "intelligible" idea reside in the soul, it supposes a reasonable idea and is subordinated to it, having not creative but only an epistemological power. As number, image or harmony they are impressed in the soul (*Timaeus*), or they are "marks" of noetic ideas (*Republic* 509) etc.

[15] This multiplicity of beings (something different one) Plato supports with Pythagoras.

[16] *Ibid*, p. 534 *a*.

[17] *Ibid*.

[18] Boetius believed that the forms are the same as universals, when the focus is on *sensibiliae* (sensible things) etc.

"proto-principle, at the same time matter of being and the factor of their changes and states"[19]

but its "forms" are real extrapolations of particular things or beings, serving here as patterns or proto-images.[20]

Otherwise in the spirit of Orphism-Pythagoreism "perceived", Plato's forms produce an eternal *mind* which undertakes the course of a divine *number*, from non–decaying *monad* via sanctified *tetrad* – the nurturer of all things[21] – to the (divine) *decade* etc.

On the One

The second supreme instance of Platonism should be the dogma on the *Good* (*Republic*), or on the *One* (τὸ ἕν) (*Parmenides*) and their prerogatives are divine. A supreme (absolute) reality belongs to the latter, and one level lower is the idea of the *Dyad* (δυάς), as exactly a unity of the *limited* (πέρας) and the *unlimited* (ἄπειρον)[22], of the *same* (τὸ ταὐτον) and the *different* (τὸ ἕτερον), *odd* and *even* etc. Beyond the being (and preceding it), ineffable and unnamed, it carries within itself the most primitive principle, so that it may be spoken of (only) in a contradictory way: it is both similar and different, but neither similar nor dissimilar, out of time, as well as in time itself, out of space, but in space etc. It tells the same tale as the "one and all" of Eleates, the other principles originate in it and it is not a principle of anything particular, not even of being. The One is deprived of any essence in itself and any potential, energy or power are absent here.

The idea of the Good is discussed in Simplicius's *Comment on Epictetus*:

"The source and principle of all things is the Good: because to what all things strive is the principle and the end of everything. The good breeds everything: the beginning, the middle and the end. By doing so, it makes them close and similar, beauty breeds beauty … principle many principles. For One, Principle, Good[23] and Divinity are the same …"

Then, an infinite multiplicity of divine natures is necessarily derived from One (as the Sun involuntarily emanates light from itself) and in the case of the One it would be so called "proto-essential" units as "imprints" of an ineffable principle of things. Plato interprets the divine natures themselves in various ways – theologically, dialectically, symbolically, through images, or the causation of things …

But, what makes up "the primitive triad" as something that the original cause initially derives from itself? The answer is *being*, *life*, and *mind*; in that very order, which owe their quintessence to the first cause which makes them incomprehensible. *Philebus* describes the primitive triad as *limited*, *infinite*, and *mixed*[24] and elsewhere it will be: *one*,

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[19] *Metaphysics* 986 a.

[20] Let us point out here that by speaking - further - on "the participation of things in the forms, Plato was near to the definition itself of the mathematical concept of mapping - which we see today can bear (generate) the totality of mathematical beings. Its occurrence followed later through the centuries (1637 - Descartes, 1694 - Leibnitz).

[21] According to the *Pythagorean hymn*.

[22] These principles of "limited" and "unlimited", say, are underlying for mathematical notions of exhaustion, derivation, differential ... or for irrational number itself, that is, every (finite) dyad.

[23] Implies desirability.

[24] Or, by epithets: *symmetry*, *truth*, and *beauty*.

many and *all*; the Haldean mythology defines it as *father*, *power*, and *mind* - so that the limited is compared to the father and the infinite to the power. Or a triple triad is the foundation of the nominal world and it is made up of an obscure union of one, being and power, as an intelligible triad, which clearly distinguishes the three elements and, finally, the third of them as all the wealth of the noumenal multitude.

This transcendental triad is different from the Christian one because it is evolved from the first cause which is "beyond" being, essence and multitude … and does not "share" with them the same nature.

This trinity of Plato's, deprived of any number, was described by Damaskin as follows:

"How can the same nature be one and many? Because many is an infinite potentiality of One. So, how can it be one and all? Since all is One's energy variously dispersed … As we said, there are three ordered principles: father, power and fatherly mind. But they are neither one nor three, nor are they one and at the same time three."

The afore-mentioned "triadity" will finally be replaced in the *Parmenides* dialogue by a special "dialectics of opposites", in the sense of unity of the latter:

"Let it be told also: no matter if the One is or is not, it and other things, in relation to themselves and to it, by any way they are everything and nothing and furthermore, they appear as everything and nothing respectively. That is an absolute truth."[25]

IAMBLICHUS

Iamblichus in many ways corrected Plotinus's ontological image of the world by finding that an infinite multitude of (semi)divine beings mediate between a man and the Trinity, or that besides Plotinus's One, different from the Good, takes place another One. The latter sets free the triads of *intelligible essences* (νοηταί) and *mental beings* (νοεροί), so that the first one consists of *reality* (ὑπάρχις), *force*, and *mind* etc., also there are three genders of gods: the superior twelve - which breed 36 others, which in turn breed 360 new gods – twelve sub–celestial and 21 worldly gods. Next are the god-protectors of people and nations etc. and Iamblichus richly elaborated on this in his work on theology, theurgy and mantics.

But finding mathematical relations to have a prominent place in the hierarchy of beings, Iamblichus turned in interpreting mathematical relations rather to the mysticism of Pythagorean numbers, instead of finding them a proper place in the order of ideas. He described the superior One as *monad* and assigned to it the number 1, number 2 to the Mind as *dyad*; number 3 to the Worldly soul etc., and the entirety of all beings was 10, the *decade*.

He spoke of mathematical beings themselves placing them between *ideas* and λόγοι, amongst *material* and *spiritual*, *finite* and *infinite*,[26] but differentiating them from

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[25] *Parmenides* 166 a.
[26] Their "intermediary" feature on the (ascending) path to the truly "monolithic" cognition is object of attention of this neo–Platonist in the work: *De communi mathematica scientia liber*, ed N. Festa, Stuttgart 1975.

souls for being immobile. They owe their principles to the One, they can sanctify, and everything that is beyond the senses, amorphous, and eternal is "grasped" only by them. On the other hand, after Iamblichus, the principle of multiplicity is in the One, which he identifies with matter, so that when it is "joined" to the One, the resulting phenomenon is number. At the same time, a point and an extension are the principles of geometrical forms and so on.

PROCLUS DIADOCHUS

The neo–Platonist Proclus repeated in his learning Plato–Plotinus's mystical idea of the one, the only, identifiable, and ineffable deity beyond any essence or definition, and of three (sub–hypostases) "derived" from it: *One* (*Good*)[27], *Mind,* and *Worldly soul,* but nonetheless*,* he expressed himself more particularly on the specific triad path of evolution of all that exists. This path lies in fact, in the triple aspect of any thing: first in its *residence* (μονή) in the One, next in its *separating* (πρόοδος) from the One, and finally in *returning* (ἐπιστροφή) to the same. The One does not directly emanate from itself the second Mind hypostasis, but it does it first with a multitude of so-called "transcendent" or *absolute units* (ἑνάδες), which should have to express the absoluteness of the principle. Also being called "gods", these units make a harmony, which is named by Proclus as *providence* (πρόνοια), and it would be the first mediation of an ineffable deity on the descending path towards the mental and sensual world.

Proclus's second hypostasis the Mind, as the first one, expresses itself through a multitude of ordered triads of existing minds, each of which brings the (mentioned) moments of unity, separation and returning. And thus is "composed" the whole sensual–reflective world, just in the way of: "one which is thought of (*object*), "one that thinks and that is thought of" (*subject–object*) and "one that thinks" (*subject*). Each instance consists of: first, *being* and *existence*, second, *life* and *force*, third *thinking* and *knowledge*, but by the principle: "everything contains everything", being, life and knowledge can be revealed all three times.[28]

The branching of the first triad begins with one (purely) *cognitive* triad, of the second, as *cognitive life*, and of the third one as *self–alive,* so that the first of the elements are manifested by *limit* (πέρας), *limitless* (ἀπέρας) and by *essence* (οὐσία) - which corresponds to Plato's *mixed* from the *Philebus* dialogue. About the second triad, let us say that it "consists" of "divine numbers", and about the third one - that it is structured like Chronos, or pure mind, Rea as the life giver, and Zeus the demiurge, creator of all that exists etc.

Finally, Proclus's third hypostasis the worldly soul intermediates between the sensual and reflexive – and is "divided into layers" by the same pattern to a series of (subordi-

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[27] Some aspect of Plato–neoplatonic One in cognition tends to express comprehensively in our epoch "theory of all" of Stephen Hawking in the framework of "quantum theory of gravity", as a realized unity of (former) quantum mechanics and theory of relativity.

[28] Say, in the first of them, "life" and "knowledge" are contained in "being" etc.

nated) triads of psychic beings: divine, diabolic, abstract … until the human and animal are reached.

Below is a sketch of the initial segment of learning of this later neo–Platonist. (*see figure*).

	One			
				limit
			mental being	unlimited
		object	mental life	essence
			self–life	
			divine numbers	
God	Mind	subject–object	guardian gods	
			executive gods	
			Chronos	
		subject	Rea	
			Zeus	
	World soul			

On the Mathematical Being

Plato's division is the basis for the conviction that mathematical entities have the power to make knowledge of what really exists, for in the hierarchy of beings (we have seen) they are a "bridge" between the sensible world and the realm of the forms. The words of Proclus himself about this would be:

"Mathematical beings necessarily belong neither among the first nor among the last and least simple of the kinds of being, but occupy the middle ground between the partless realities - simple, incomposite, and indivisible - and divisible things characterized by every variety of composition and differentiation."[29]

Thus these beings share the double nature of incorporal essences and terrestrial things, so that - for in both cases it is matter of extremities – not even a discourse on them can be identified either as based on the "principles or the things".[30] With its bower neither in heaven nor on the Earth, such an entity resides in the human spirit in the form of a "notion" acquired about it and, at least by it, it brings some constant and "impenetrable" whole, different from any changeability, divisibility, fluidity ... It would be a dianoetic (intelligible) image made by this power of spirit over the sensible in which, again, the noetic (reasonable) essence of being is reflected so that it is up to it to find the way and "reach" what is a principle, one and indivisible, just as diversity, divisibility and multitude are. This is so since the essence has an original power to exteriorize itself in the sensible, in the material as well as all that is reached on that path we grasp by *mathesis*. But if it cannot be realized over the Plato - Plotinus - Proclus' One - being "beyond" of ineffable and as not "reached" by the dianoetic power – although we recognize here as the furthest

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[29] *A Commentary on the first Book of Euclid's Elements*, with introduction and notes by R. G. Morrow. Princeton [NY]: Princeton Univ. Press, 1970.

[30] See: Hartman: *Principes philosophiques des mathématiques* (published in "Philosophie et mathématique de Proclus", par Stanislas Breton, éd. Beauchesne, Paris 1969), p. 189.

principles of being *limited* and *unlimited* on which, since Pythagoras, Plato[31], ... numbers[32] and all intuitive knowledge that exists are based. Say the arithmetical unity is a manifestation of the principle of limit, and infinite succession of integers - of the principle of the unlimited. Equally when considering the divisibility of continuous values that can be divided without end etc.

From the "interaction" of two principles - mathematical and metaphysical - follows the procession (πρόοδος) of ordered realities where, by the former, the idealities of Reason should reach unity and their constancy and by the latter – diversity, changeability etc. But with these two, more "secondary" principles, different for arithmetic and geometry, found their place. By making number possible, these two principles are "prolonged", first, to the visible images and, after that, to the beings themselves[33]. In the manner of Plotinus's same emanation, all that is hidden in principles is manifested once "above" them and in that way what is initial and identical passes into the different and the other.

That "extraction" is for Proclus the "order" itself, or the sequence (διάκοσμος), for what is primordial, identical and one and is penetrated by the principle of limited, precedes the multiple and coming to be - which the principle of the unlimited is related to. It is also said that the latter, by acting on the former, translates it from rest to movement, or from unity to development, bringing to the light of day only mathematical concepts and discursive thinking, in general. The unity is that finite starting point on the unlimited way of generations of always new and new numbers (finite as well) in arithmetic and such an amalgam "limit - unlimited", finite - infinite", ... is such a "dialectic" moment that never ceased to mark Proclus' thought in this area.[34]

It is the same when the division of a quantity in geometry proceeds to the infinite and Proclus here resorts to the (peripatetic) terms of "actual - potential", finding that if the parts "dominate" over an actual existence, the whole as infinite persists (only) in a potential way. On that infinity, a further, irrational number is based, or the incommensurability of magnitudes - and this is exactly "the limit, according to Hartman, that separates arithmetic from geometry"[35] etc. - but a constant "lack in terms of Neo-Platonism[36], this author observes, will not allow Proclus to bring the two infinities to a concept."

But together with these principles, the mathematics shares particular "theorems" with being, and they are based on the general concepts of *equality* and *inequality*, of *analogy* and *division*, and so to the *analysis*, *symmetry*, *synthesis* ... as to all that is common to a

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[31] By Plato, say, after the idea of One to which belongs the highest (absolute) reality, comes the idea of Dyad, just as unity of limited and unlimited, of the same and different, odd and even etc.

[32] A binary representation of numbers is in the basis of powerful computer software of latter decades. Namely, on the double language "there is electricity (1) - "there isn't electricity " (0), that only computers "understand", is based its high operative power, which is bordered with the possibility of "artificial intelligence" itself. So is created a special "virtual reality", when both color, sound and motion are "produced" by only the sequences of symbols 0 and 1 etc.

[33] Plotinus will not think up to the end this generalized relation of its "primary" proto-principle (*One*) and these later which, certainly, come after it.

[34] The notions of "actual" and "potential", say, are derived from there, but, as Hartman remarks, "a constant poverty of neoplatonics in terms", will not allow Proclus to bring up different infinities to a concept. It would be made by Georg Cantor when he found, e.g. that the cardinality of denumerable sets is strictly lesser than the power of continuum, and the latter is lesser than a denumerable sequence of "transfinite cardinals" etc.

[35] Hartman, op. cit. p. 195.

[36] *Ibid,* p. 196.

number, to a form, to a motion ... The proportion is, say, "for" a number the same as "for" quantity, and Proclus will express himself in Aristotle's terms of the possibility of a primitive ("mathematical") science on "being as such"[37]. It will bring the principles to geometry and arithmetic and would "justify" knowledge in them, but should they *in ultima linea* be based on sensibility or on reason? A sensible basis is not contained in the notions of "equal", "indivisible", nor in the notions of point, circle, line, although we do not cease to "visualize" when dealing with them. What remains is that they have roots in some ideas innate to the soul, such as patterns, models, paradigms, which the soul contains from eternity, turning them into *a priori* and logical necessities. All this is never the case with any particular which is regularly both contingent and arbitrary.

Otherwise, the *representation* (ἡ φαντασία, *imaginatio*), in Proclus, just as intellect which it helps "are in the middle of sensual perception and pure mind",[38] so that the "role assigned to representation would be Proclus's main supplement to Platonist theory"[39]. This notion was first elaborated by Aristotle, making a distinction between "deliberate representation" (ἡ φαντασία βουλευτικὴ) and "intelligent representation" (ἡ φαντασία λογιστική), or the "representation of comprehension" (ὑπόληψις) etc., on the ascending path from the sensual to the intelligible, just as the Stoics, Epicureans, and other neo–Platonists (Plotinus, Porphyries, Syrian) claimed.

Then how is the "mathematical" created in the soul or how does the soul "discover it within"? Proclus's answer is that it is achieved in two ways: through a – never empty – soul as a fullness (πλήρωμα) of ideas, or when those intelligible (noetic) contents find an echo in it. For the soul is just a degree (the initial step) in the "self-unwinding" of potentialities of mind which, in turn, replaces its non–ethical contents with a spiritual one. Thus "self–movable numbers" precede arithmetic ones, and the signs of the zodiac are preceded by geometrical ones, and so the soul is populated with mathematical beings in their πλήρωμα to the degree where they can serve as *definiens*. In other words, the ideas are non extensive, compact essences … but the power of representation makes of them extensive and divisible wholes, by providing a specific "intelligible matter", to keep them growing. For example, before being projected[40] on the screen of representation, the circle doesn't have either a circumference or a centre, but when transported into the "sensual matter" it gains visual features.

Proclus insists that, by following the archetypes, the soul *produces* from itself mathematical concepts in abundance, which creates a logical structure, whereas the role of the unlimited is to be the potentially in Aristotle's sense etc.

On the Geometrical Being

Roughly speaking, what separates the arithmetical from the geometrical has its roots in the concept of space, being differently interpreted by Pythagoras, Democritus, Plato, ...

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[37] In the contemporary science, say, the hidden structure of elementary particles really became (sufficiently) "replaceable" with the relation of prime numbers in the sequence, such that two universes - of material reality and number set - should be by it mutually "isomorphs". Or: there is today "theoretic biology", founded of course on the number.

[38] Glen R. Morrow: *Introduction*, p. 62.

[39] *Ibid.*, p. 59.

[40] This essential feature of Proclus's teaching is known as "projectionism".

"Unclear and difficult, according to Plato, this notion had the meaning of "receptacle" (χώρα) for corporal and incorporeal natures and Aristotle translated them into "sensible matter" (ὕλη αἰσθητή) and "matter of reason" (ὕλη νοητή)[41]. The latter bears geometric forms in itself and it was changed by Proclus by "geometrical matter", because with the difference of the former, it would incorporate the properties of divisibility, extensionality and it would not resort, in knowing them, to the sensibility, nor would the dianoetic character of the science of geometry be disturbed.

We find here as well an extremely subtle contemplation of what is in the basis of all that is general, but it differs from the pure notions on multitude which we know. It would be their *logic unity* based on the representation and receive an ontologic character and incarnate a double relation: toward sensible and noetic, to matter and knowledge, to passive and active.

"The representation, says Proclus, is excited by itself to move the object which is realized to exteriority",[42] on the path of indivisible, identic (living) to the diverse, extensible, to a form. For geometrical forms on the supreme noetic level - inextensibles and invisibles - residing in the reason are, by the force of representation, "mapped" from here onto "intelligible matter", in the soul. They receive both extensibility and visibility if they are, by force of representation equally, "traced" in the sensible matter, giving themselves to the science of geometry thereby.

So in matter, incarnated geometric forms (circle, triangle), stay on the one side, by receiving their existence from them and, on the other side, the geometric forms themselves created in an intelligible and uncorporeal matter. In the latter the notions of geometry are conceived where, by an effort of imagination, a rational thought is dialectically directed to the dianoetic essence. It is about a "creative trip" of thought to the exteriority from it itself, which Proclus is said to be an "anticipation", and it is possible to say about a geometric being that it is conditioned threefold: by thinking itself, by its dianoetic form, and by the Platonic "receptacle" (χώρα) - just as by that where something is coming to. Finally, from the viewpoint of the achievements of the scientists of the New age (Descartes, *analitic geometry*), here the mutual relations of numbers are designated as dianoetic, which with an accurate preciseness bring the relation of quantities and positions, so Proclus has (dialectically) anticipated this later discovery. And this is none other than a way for what is divisible (form) to be represented by the indivisible (number), the extendable – by the un-extendable, or that which has a form – by the formless.

A comparative aspect

Which mathematical entities are more prone to cognitive or educational effects - the arithmetical or geometrical?

First, Plotinus compares the duties of a musician and of a lover on the ascending path of the soul toward the mental. The former goes along with the melody, a tone on its ascendance through musical forms, and the latter with a visible beauty to a moral purity, to a

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[41] "There is, from one side, sense perceptible matter, and from the other one - that is reached in the reason. The first, for example: bronze, tree or each matter in motion, and second - that is in a sense perceptible, but not as such; say, mathematical objects" (*Metaphysics*, 1037 *a*)
[42] Proclus: *A Commentary ...*, 52, 52.

beauty of art and so on. The philosopher would as well take advantage of something which would help him find the way amongst the abstract beings and deliver finally the dialectical "truth about the nature and relation of things".[43]

Next, Numenius of Apamea (only) repeats Plato's teaching of mathematical objects and learning by words:

"to study numbers with a young man's enthusiasm in order to reach the goal of the highest science, and it is the One,"[44]

Nicomach of Gerasa also names these objects as "bridges", "ladders" and so on – on the same "ascending" path of purification of the "eye of the soul" from the sensual world to the "pure being". His division of sciences places arithmetic on the top because it "deals" with numbers and relations between them, the second position is for geometry which relies on arithmetic, the third - for music which is founded on arithmetic etc., so that they all precede the philosopher's science–dialectics the highest expression of which is to be found in arithmetic (number).

Iamblichus would incline to arithmetic and numbers on the same ascending path of the soul to the intelligible, when criticizing Euclid's geometrical approach, so that finally, the Syrian would prove that the soul builds up mathematical objects according to a certain order. The two principles, monad and dyad, constitute all kinds of numbers, while some extended essences form geometrical forms. But, the latter get a secondary character and so on.

On the other hand, Proclus would make an "abundant" come back to Plotinus, in a different way than Iamblichus and Nichomachus ... and not in the spirit of Pythagoreism, by postulating geometrical forms in the beginning of the second part of his *Introduction*:

"while the soul manifests its power of cognition, it reflects ideas of geometrical forms upon the representation, as in the mirror. Having received in picture form these impressions of ideas, gives the soul the occasion to turn from images to itself …"[45]

On Mathematical Methods

As διάνοια, or rethinking, we saw, the science of mathematics incarnates a double path: "upwards" and "downwards" - the former being from the sensual to the intelligible and to the first principles, and inversely the latter, so that it corresponds to an "ascending" and a "descending" method of research. The first is qualified as "reasoning" and terminates in the timeless essences – ideas, the other as "sensuality", which is governed by the "external" and doesn't set off any continuity in starting points. Plato's *Menon* ἡ ἀνάμνησις takes place here in the sense that (only) sensations energize a certain "self–remembrance" of what is primitive and logically precedes things, whereas the overbalance in that path belongs to "self–realization of *thought* … in the sense that it turns to itself" (45, 10 – 12). So mathematics doesn't begin and don't end in itself, but needs an external stimulus; this was the case throughout history – from Pythagoras, say, who ascribed to

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[43] *Eneads* I 3, 1 – 3.
[44] *On the Good*, fr. 2, 17 – 23.
[45] *Commentary* …, p. 141.

numbers a part of space … On the ascending path of anamnesis, next, what is conceived as mathematical serves to incite the cognition of the noetic in it, and then to the blind eye is open "the vision of the genuine being" (20, 21), such as it really is. When mathematical thought passes a descending path – from "what is searched to its basis" (18, 23, etc.) – and ends in its "immanent principles": *one*, *many*, *limited*, *unlimited*, in both cases – ascending and descending – it realizes a "dialectical unity" of *identification* and *distinction*, originating from Plato himself. Turned over to the principles, the mathematical becomes something initial or propaedeutic for the science of philosophy, as well as of nature, because we discern in it the order of primitive ideas as the principles at the basis.

As to mathematical cognition, it is, first, multiple: on the *numbers* and *quantities* (Pythagoreans), then *pure* and *practical*, or *arithmetical* and *geometrical* to which corresponds in application *musical* and *astronomic* cognition. A second division would imply *sensual* and *intelligible* cognition, the former being mechanics, astrology, optics, geodesy, canonics and logistics, the latter - arithmetic and geometry. Here again, arithmetic precedes geometry, because the latter would have to consider beside the unit, its "place" also, but they share a number of common principles like: *one, many, same, different, limited, and unlimited.*

It remains that "pure multitude" in arithmetic and "pure quantity" in geometry are still a place of delimitation between the two mathematical areas, and their "irreducibility", Proclus finds, is due to the "limit" disparate principles of "the border" and "indefinite multiplication of forms", according to Proclus. These principles "set free" in motion a series of - *dyadic* - confronted moments: variable and constant; mobile and immobile; finite and infinite … so that their relation gave birth to Proclus's dialectics.[46] Nikolai Hartmann[47] uses Proclus's systematization of methods in the form of *thesis–antithesis* scheme which is divided into three parts according to the three principles: *i*) by direction, *ii*) by content and *iii*) as a synthesis of the first two cases.

First, by direction, the matter would here be of an ascending path and its two aspects: "analytical" and "horistic", as well as of a "descending" path with "dieritic" and "apodictic" approach. The *analytical* approach refers to assumptions, *horistic* to definitions, *dieritic* to divisions; and *apodictic* to proofs. In the second case, the matter is of a "static" and "dynamic" approach which enclose, first, "horistic" and "dieritic" in itself, and second - "analytical" and "apodictic", and in the third case – of their four combinations: 1) " static – ascending" (horistic), 2) "dynamic – ascending" (analytical), 3) "static – descending" (dieritic) and 4) "dynamic – descending" (apodictic).

According to Hartmann, Proclus "didn't deduce his thought on the descending path"[48], but on the ascending path, namely, on the side of principles, he found a perfect unification, "the *synthesis* of mathematical proceedings"[49]. Or a little later: by "ascending still

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[46] Proclus stays at the contrariety: thesis–antithesis without finding a way to ascend to the triadic scheme: thesis–antithesis–synthesis (Hegel) - even less so to a (possible) tetradic and al. opinion - as it is known, say, by ancient Indian logic (metaphysics), which allows four possibilities: truth, lie, truth and lie, neither truth nor lie to which the Buddhist logicians add: none of it. Nowadays, also, so called "para-consistent logics" (Da Costa, 1963) and other (formal) logics allow contradictory statements, without being trivial.
[47] Hartmann, op. cit., p. 234 – 5.
[48] *Ibid*, p. 236.
[49] *Ibid*, p. 236.

higher, the dialectics proves to be a link and a *crown* of all the mathematical edifice. Just as it closes the whole, by leading the thought to the mind by its own method. From its side, the mind itself enables the unity to a certain higher degree. It is the ultimate and highest unity, where all dialectical methods are overtaken, and that embrace in a unique way, by joining *one* and *many*. Here is committed the *return* of what is dialectically *externalized* towards its foremost unity and whose creative force, we have seen, provides the *opinion* with its "basic" anticipations. This unity combines 'from above' all the achievements from mathematical reasons, bringing the ultimate and highest goal on the ascending path, as well as on the path of cognition alone" (p. 236 – 7).

So the "dialectics" in Proclus's sense of the word and the mind are two central notions of his systematization of mathematical methods, on which relation are based abundant deliberations of the (total) sphere of mathematical entities.

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O ONTOLOŠKO-GNOSEOLOŠKOM STATUSU MATEMATIČKIH ENTITETA: PLATON, JAMBLIH, PROKL DIJADOH

Milan Tasić

Ostvarivši unekoliko uvid u ontološko-gnoseološku sliku učenja Platona, kao i novoplatoničara Jambliha, Prokla i dr., obrazlažemo posebnije gledišta ovog poslednjeg na "matematička bića", "geometrijska bića", "matematičke metode" i sl. Nalazimo da su jednom dogledana načela "jednog i mnoštva", "granice i neograničenog" i sl., srećno iznađena uporišta koja je dva i po milenijumski razvoj (naročito prirodnih) nauka iznova potvrđivao.

Ključne reči: *Platon, Prokl, ontološki, gnoseološki, matematički, objekt.*