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BASIC CONDITIONS OF PROPER PART SURFACE GENERATING WHILE MACHINING ON CONVENTIONAL MACHINE TOOL

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Abstract. *Machining of the part surface on the conventional machine tool sometimes presents geometrical problems that are not envisioned in classical geometry of surfaces. This paper presents definition and illustration of the basic conditions of proper part surface generating while machining on conventional machine tool, i.e. on turning machine tool, on milling machine tool, on machine tool for gear manufacturing operations etc. This result is significant for it substantiates earlier works in planning of part surface machining operations. From a theoretical viewpoint, it is remarkable that one can analytically describe all basic geometrical conditions that are necessary for proper part surface generating while machining on conventional machine tool. It also offers simple practical means of part surface generating without undercutting and overcutting, but within the given tolerance on the part surface accuracy.*

Key words: *part surface, machine tool, undercutting, gauging, tolerance.*

Nomenclature

P – part surface to be machined;
T – machining surface of the cutting tool;
$X_pY_pZ_p$ – Cartesian coordinates of a point on the part surface P;
Ω_i – parameter of the part surface $P(X_p,Y_p,Z_p)$ relative motion (i = 1,2);
$\vec{N}$ – common perpendicular to the surfaces P and T at the point K of their contact;
$\vec{V}$ – velocity vector of the surfaces P and T in their relative motion.

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1. INTRODUCTION

The problem of the form cutting tool design consists in determination of the shape and of the dimensions of a cutting tool intended for machining of a given surface *P* of a workpiece. In machining of metals the part surface *P* to be machined accomplishes a definite motion in reference to the cutting tool. As a result of this motion, the surface *P* occupies a series of consecutive positions. The surface *T*, tangent to the consecutive positions of the machined surface being considered, is called *the machining surface of the tool T*.

Cutting edges of the cutting tool must be located on the machining surface of the tool *T*. If the cutting edges projects beyond the surface *T*, they cuts into machined part surface *P* in the machining operation and excessive amount of stock, more then required, will be cut out of the blank. In this manner, any cutting tool can be regarded as a body bounded by the machining surface of the tool *T*, conjugated to the part surface *P* to be machined, to which cutting properties have being imparted. In other words, the part to be machined and the cutting tool can be conceived as components of a peculiar mechanism consisting of these two conjugate links which contact each other in the course of machining.

Thus, a milling cutter can be considered as a solid of revolution that has being converted into a cutting tool by making of flutes and of relief surfaces behind the cutting edges, for example by relieving operation. In the process of machining this solid of revolution contacts the part surface *P* to be machined.

The machining surface of the tool *T* conjugated to the part surface *P* can be generated in various ways.

Machining of the part surface *P* on conventional machine tool sometime presents geometrical problems that were not envisioned in classical geometry of surfaces. One such a problem is in the area of proper part surface generating. Undercutting and overcutting (i.e. gauging) are one of the critical problems, which unfortunately has not been thoroughly investigated.

Let's assume that the geometry of the part surface *P* to be machined, the geometry of the machining surface of the tool *T* and the parameters of their relative motion are given and are completely determined beforehand. This is necessary and sufficient to give the definition and to develop the approach for analytical description of the basic conditions of *Proper Part Surface Generation* while machining on conventional machine tool (further - *conditions of PPSG*). Dhande, S.G. et all [2], Radzevich, S.P. [3], Rodin P.R. [5] and Wu, D.R. and Luo, J.S. [6] investigated the problem under consideration.

Conditions of *PPSG* may prevail in machining workpiece under which it proves impossible to machine a given part surface *P*, or undercutting or insufficient cutting of a part of material of the blank is observed. In all of these cases, the part surface *P* can't be generated strictly in compliance with the part drawing. It is obviously of prime importance to ascertain the reasons for the deviation of the machined surface *P* from the given dimensions, and to establish the conditions of *PPSG* (i.e. the conditions of the machinability of the part surface *P*) under which the aforesaid deviations do not occur or are within permissible limits (i.e. within the given tolerance).

2. THE FIRST CONDITION OF *PPSG*

The condition of the existence of a machining surface of the tool T is the first and essential condition of in machining PPSG operation.

For the machining of the surface P to be possible, it is necessary that the machining surface of the tool T exists for a given case, i.e. the machining surface of the tool that contacts the part surface P in the process of machining. This condition of *PPSG* requires that contact be provided (simultaneously or at each instant of time) between points of the surface P of the workpiece and of the machining surface T of the cutting tool.

The perpendicular $\vec{N}_P$ to a given part surface P occupies definite position relatively to the part. It is apparent that it is not possible to change its direction without changing the shape of the part surface P. Therefore, if no machining surface of the tool exists for a given case, an effort is made to attain one by changing the direction of the velocity of the relative motion.

Thus, for instance (Fig. 1.1), in the coordinate system XYZ a plane P travels in a straight-line velocity $\vec{V}_P$ perpendicular to the plane. In its motion, plane P occupies a series of consecutive positions P_1, P_2, P_3, etc. In this case, the condition of contact is not observed, since normal $\vec{N}_P$ and relative velocity $\vec{V}_P$ are parallel to each other. It is impossible to find out the enveloping surface. No such a surface exists.

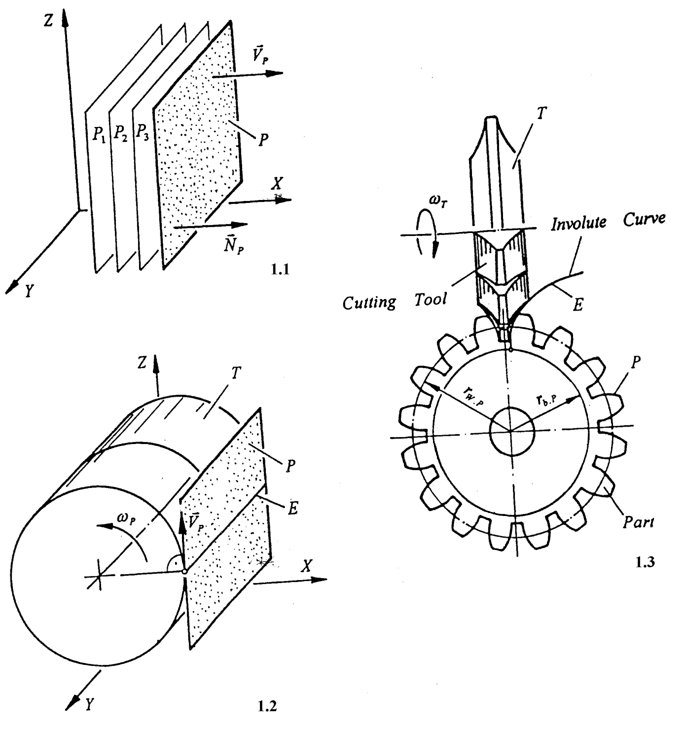


Fig. 1. Generation of the machining surface of the tool T as an envelope of the part surface P to be machined in their one-parametric relative motion.

Let's change the direction of the velocity $\vec{V}_P$. Let $\vec{V}_P$ is the linear velocity of the rotation of the plane P with the angular speed ω_P is parallel to (Fig. 1.2). Then the condition of contact will hold true for all points of the characteristic line E simultaneously. Characteristic line E is the line of contact of the part surface P to be machined and of its envelope surface, say of the machining surface of the tool T. The machining surface of the tool exists and coincides with the surface of the circular cylinder T. In the process of machining, mutual sliding of planes P and T is observed. This case corresponds to machining of the cylindrical surface with flat (slab) broaches.

If the machining surface of the tool T conjugated to the part surface P leads exists it can be generated in various ways.

The first method of generating of the machining surface of the tool T consists in determining of the surface T as an envelope of the surface P of the work in its motion relatively the tool. Usually linear contact of the surfaces P and T along characteristic curve E is observed in such case (for example, in milling toothed gears with the form cutter). As a particular case, surfaces P and T may coincide each other as, for instance, the conjugate helical surfaces of a screw and of a nut. Good example of such a tool is tape, threading die, etc. The coincidence of the surfaces P and T is observed in the case when their relative motion causes the sliding of the surface relatively itself.

Let us consider the elementary example of machining of a spur gear with the form-milling cutter (Fig. 1.3) which rotates about its axis with the angular speed ω_P.

Equation of the part surface P to be machined can be expressed in implicit form

$$P(X_p, Y_p, Z_p) = 0 \qquad (1)$$

The part surface P moves relatively coordinate system that imbedded to the cutting tool, with the parameter of the relative motion Ω and form in such a motion a family of surfaces. The equation of the random position of the surface P in its relative motion can be represented as follows

$$P(X_p, Y_p, Z_p, \Omega) = 0 \qquad (2)$$

As known from analytical and differential geometry [1], the envelope to the family of the surfaces (2) is expressed by the set of two equations

$$\left.\begin{array}{l}\text{The Machining}\\ \text{Surface of the}\\ \text{Tool } T\end{array}\right\} \Leftarrow \begin{cases} P(X_P, Y_P, Z_P, \Omega) = 0; \\ \dfrac{\partial P(X_P, Y_P, Z_P, \Omega)}{\partial \Omega} = 0, \end{cases} \qquad (3)$$

where X_P, Y_P, Z_P – are the Cartesian coordinates of a point on the given part surface P;
Ω – is the parameter of the relative motion of the given part surface P.

The machining surface of the form-cutting tool also can be found out employing kinematical method. Most general results can be obtained employing *Differential Geometric Method of Part Surface Generating* [3] and, in part, tooth surface fundamental forms [4].

Let us consider the process of grinding of a plane surface P with a cylindrical grinding wheel as an example (Fig. 2.1). In operation, the wheel has a translatory motion at a velocity $\vec{V}$. At an arbitrary point A of the machining surface of the grinding wheel T, the normal vector $\vec{N}_A$ is not perpendicular to the velocity vector $\vec{V}$. At the conjugate point of contact, say at the point B, the vector $\vec{N}_B$ of the perpendicular to the surface P at B and the velocity vector $\vec{V}_B$ are perpendicular to each other.

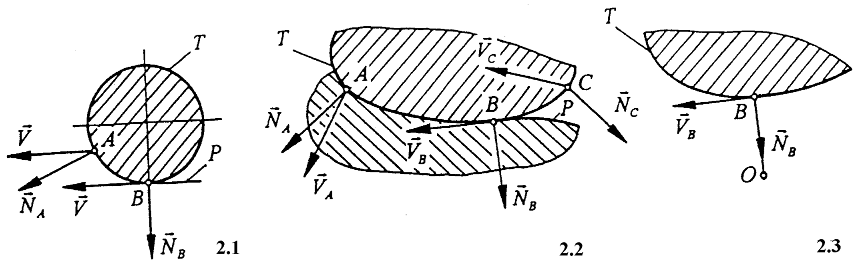


Fig. 2. Generation of the machining surface of the tool T as an envelope of the part surface P to be machined in their two-parametric relative motion.

Similarly, in the motion of an arbitrary surface P (Fig. 2.2) in the vicinity of the point A, portion of the machining surface T of the tool cuts into the body of the blank (vectors $\vec{N}_A$ and $\vec{V}_A$ are not perpendicular to each other).

At the point C there is a departure of a portion of the machining surface of the tool T from the blank (the vectors $\vec{N}_C$ and $\vec{V}_C$ also are not perpendicular to each other).

At the point B the vector $\vec{V}_B$ is located in the tangential plane (vectors $\vec{N}_B$ and $\vec{V}_B$ are perpendicular to each other) so that point B generates its conjugate point on the blank (Fig. 2.3).

The condition of proper contact of the surfaces P and T (i.e. the condition of the perpendicularity of the vectors $\vec{N}$ and $\vec{V}$) can be analytically expressed employing the dot product of these vectors

$$\vec{N} \cdot \vec{V} = 0. \tag{4}$$

This condition enables to determine the points of contact of the conjugate surfaces P and T at a various instants of time. In the coordinate system embedded to the workpiece the set of the points of contact of the surfaces P and T represents the part surface P to be machined. In the coordinate system embedded to the cutting tool the set of points of contact of the surfaces P and T give the machining surface of the cutting tool T.

Thus, if a known surface P has a certain motion in space and generates the enveloping surface, its characteristic curve E can be defined as the line at each point of which the vector $\vec{V}$ of the velocity of the relative motion is tangent to the part surface P.

The condition of contact of the surfaces P and T in a particular case of instantaneous rotary motion reduces to the familiar property of common normal.

Upon instantaneous rotary motion about its axis, the velocity vector of any point of the surface P is perpendicular to the radius connecting the given point of the surface P or surface T with its axis O. It follows that in instantaneous rotary motion, the common perpendicular to the conjugated surfaces at the points of contact must pass through the axis. Point B is such point in its location shown in the Fig. 2.1. Here the vectors $\vec{N}_B$ and the vector $\vec{V}_B$ are perpendicular to each other.

The second method of generating of the machining surface of the tool T involves the use of the auxiliary generating surface A and consists in the following.

In the coordinate system XYZ the motion of the workpiece and of the tool is given. Next, we impart a certain motion to the system XYZ and determine the auxiliary surface A as an enveloping P in its motion relative the coordinate system P. We find the machining surface of the tool T as the envelope of the surface A, which moves together with the coordinate system XYZ relative the tool T. In this case, the point of contact of the surfaces P and T is most often observed.

Thus, in the example being considered a spur gear is machining with the gear hob (Fig. 3). The spur gear to be machined has radius of the base cylinder $r_{b,P}$ and rotates about its axis with angular velocity ω_P. The spur gear and the hob moves relatively each other in such a way that the pitch circle of the gear to be machined with the radius $r_{W.P}$ and with the pitch line W_A of the auxiliary surface A roll relatively each other without sliding. Complicated relative motions of the surfaces P and T can be decomposed in a case on two partial motions:

– a linear motion with the velocity $\vec{V}_A$ and

– a rotational motion with the angular velocity ω_A respectively.

The envelope surface to the moving with the velocity $\vec{V}_A$ part surface P to be machined is the auxiliary surface A (Fig. 3.1).

The auxiliary surface A can be employed as a machining surface of the tool for rack cutter (Fig. 3.1) as well. In machining operation such a cutter moves in a straight direction with the velocity $\vec{V}_A$. The envelope surface to the consecutive positions of the auxiliary surface A that in rotational motion moves with a velocity ω_T is the machining surface of the tool T. In addition in machining operation (Fig. 3.2) the gear hob and the workpiece move relatively each other along the gear axis with the velocity S_P.

The example which has been considered above relates to the one parametric relative motion of the surfaces P and T. Very often the surfaces P and T relative motion is two parametric.

As known from analytical and from differential geometry [1], the envelope surface to the two parametric family of surfaces

$$P(X_P, Y_P, Z_P, \Omega_1, \Omega_2) = 0 \tag{5}$$

is represented by the set of three equations

$$\left.\begin{array}{l}\text{The Machining Surface}\\ \text{of the Tool } T\\ \text{as an Envelop of the}\\ \text{Part Surface } P\end{array}\right\} \Leftarrow \left\{\begin{array}{l} P(X_P, Y_P, Z_P, \Omega_1, \Omega_2) = 0;\\ \dfrac{\partial P(X_P, Y_P, Z_P, \Omega_1, \Omega_2)}{\partial \Omega_1} = 0;\\ \dfrac{\partial P(X_P, Y_P, Z_P, \Omega_1, \Omega_2)}{\partial \Omega_2} = 0,\end{array}\right. \tag{6}$$

where Ω_i – are the parameters of the given two parametric relative motion of the surface $P(X_P, Y_P, Z_P) = 0$ and $(i = 1,2)$.

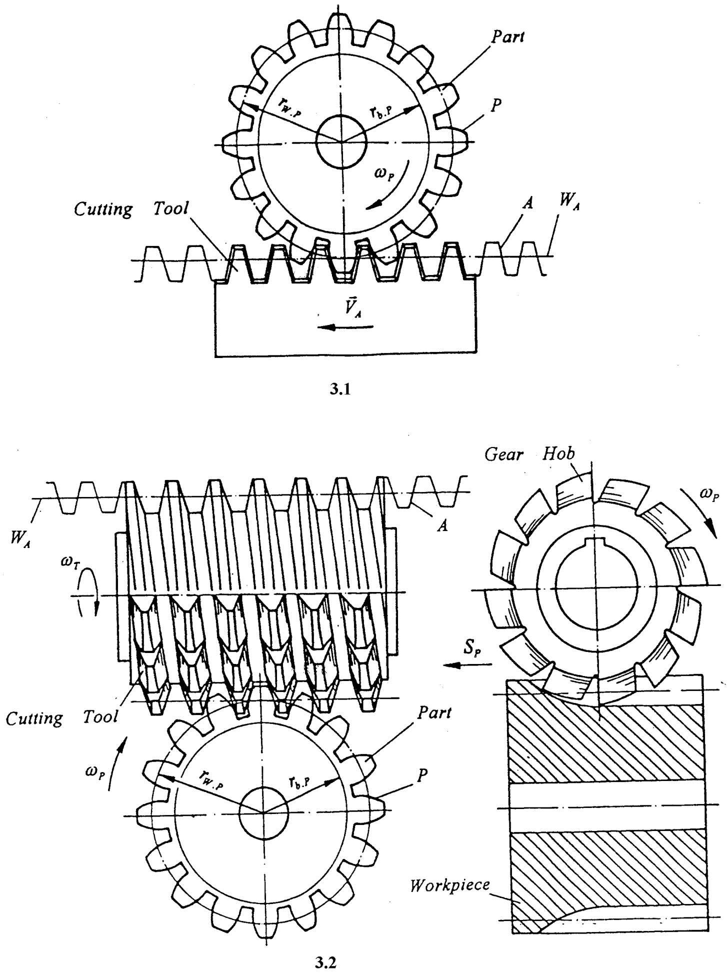
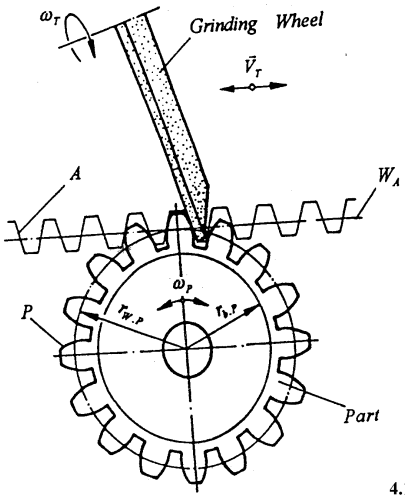


Fig. 3. The examples which illustrates that the first condition of the *PPSG* can satisfy (1) and can not satisfy (2).

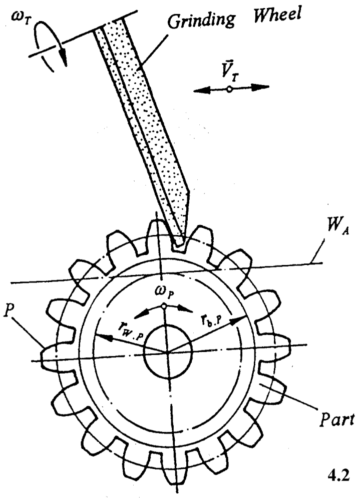
The similar approach of generating of the machining surface of the tool T is available in the cases of multi parametric relative motion of the surface $P(X_P, Y_P, Z_P) = 0$, i.e. when $(i > 2)$ [3].

For multi-axis NC machining of sculptured part surface *P* Radzevich, S.P. [3] has investigated other method of generating of the machining surface of the tool *T*.

To complete the consideration of the first condition of *PPSG* let us consider the gear grinding operation (Fig. 4). In the case under consideration the first condition of *PPSG* is satisfied (Fig. 4.1) if the radius of the base cylinder $r_{b.P}$ of the gear to be machined is less then the radius of the gear pitch cylinder $r_{W.P}$, i.e. $(r_{b.P} < r_{W.P})$. The first condition of *PPSG* is not satisfied (Fig. 4.2) if the radius of the base cylinder $r_{b.P}$ of the gear to be machined is larger then the radius of the gear pitch cylinder $r_{b.P}$, i.e. $(r_{b.P} > r_{W.P})$.



4.1



4.2

Fig. 4. To the derivation of the equation of contact of the surfaces P and T in machining operation on conventional machine tool.

Accordingly, various types of the machining surfaces of the tool T can be obtained for a given machining procedure and the various designs of cutting tools can be designed on the basis of these surfaces for machining of the given part surface P.

All of the machining surfaces of the tool with point contact touch the machining surface of the tool T that has linear contact with the part surface P to be machined. Many of different methods can be restored to in order to determine the dimensions of the machining surface of the cutting tool. According to the foregoing methods, this amounts to finding the enveloping surfaces.

3. THE SECOND CONDITION OF *PPSG*

The condition of proper contact of the part surface P to be machined and of the machining surface of the tool T without their mutual penetration, i.e. without their interference, is the second condition of PPSG.

It is known that the machining surface of the tool *T* and the part surface *P* to be machined contact each other along the characteristic curve *E* (see above). Surface *P* bounds the body of the workpiece, and the surface *T* is reproduced by the cutting edges of the tool. The contact of the surfaces *P* and *T* can be or external or internal.

In external contact, surface *T* is outside the body of the workpiece in the vicinity of point of contact. Thus, there will be no cutting of the surface *T* into the surface *P* and, consequently, a part of body *P* will not be cut out.

With internal contact, surface *T* will pass into the body of the workpiece in the contact vicinity. Therefore, in machining, the form-generating surface *T* will penetrate into the body of the workpiece and cut out the corresponding parts of it. As a result, it proves practically impossible to machine the workpiece in compliance with the drawing.

The method of making sections can be used to investigate the character of contact of surfaces *P* and *T*. This method consists in passing a series of planes, arranged along the characteristic curve, through the surfaces *P* and *T*, and revealing the nature of contact of the surfaces in the section.

Great many different kinds of curves and types of their contact in the sectional planes can be observed. Thus, a convex profile may contact to another convex profile. In this case, there will be no penetration at any radii of curvature at the point of contact of the profiles.

With internal contact, mutual penetration of the profiles occurs at any radius of curvature. This also concerns concave profiles.

A convex surface may also be in contact with a concave surface. There will be no mutual penetration of the profiles in the vicinity of the point of thir contact if the radius of curvature of the concave surface is equal to or larger than that of the convex surface. Otherwise, there will be mutual penetration of the profiles and it will prove impossible to form the given part surface *P*.

Two fundamentally different kinds of contact are possible between the convex and concave surfaces:

(1) the concave surface corresponds to part surface *P* and the convex surface – to the machining surface of the tool *T*;
(2) the concave surface corresponds to the machining surface of the tool *T* and the convex surface – to the part surface *P* to be machined.

In the first case, if penetration of the profiles is observed, it is necessary to reduce the radius of curvature of the curved section of the machining surface of the tool *T*. As a rule, decreasing diametrical dimensions of the cutting tool can reduce the radius of curvature of a curved section. In this case, the maximum permissible tool diameter can be reached for which there is still no penetration of the profiles.

In the second case, on the contrary, it is necessary to increase the radius of curvature of the curved section of the machining surface of the tool *T* if any penetration of the profiles is observed. This can be usually be achieved by increasing the diametrical dimensions of the cutting tool. In this case, the minimum permissible tool diameter can be determined for which there is still no penetration of the profiles.

Finely, the profiles may have singular points and, in a particular case, a cusp of the first kind. In the latter case two types of profiles are conceivable: a convex profile and a concave profile. In the first case, interference of the profiles will be observed if the cusp contacts point of the conjugate profile which is not a singular point.

Next we shall consider an example showing how the dimensions of a grinding wheel for sharpening round broaches are determined. Fig. 5 shows an internal conical surface *P* corresponding to the tooth face of a broach, and an external conical surface *T* corresponding to the surface of the grinding wheel. In such a machining operation the broach with a rake angle γ and a clearance angle α rotates about its axis with an angular velocity ω_P and the grinding wheel rotates about its axis with an angular velocity ω_T.

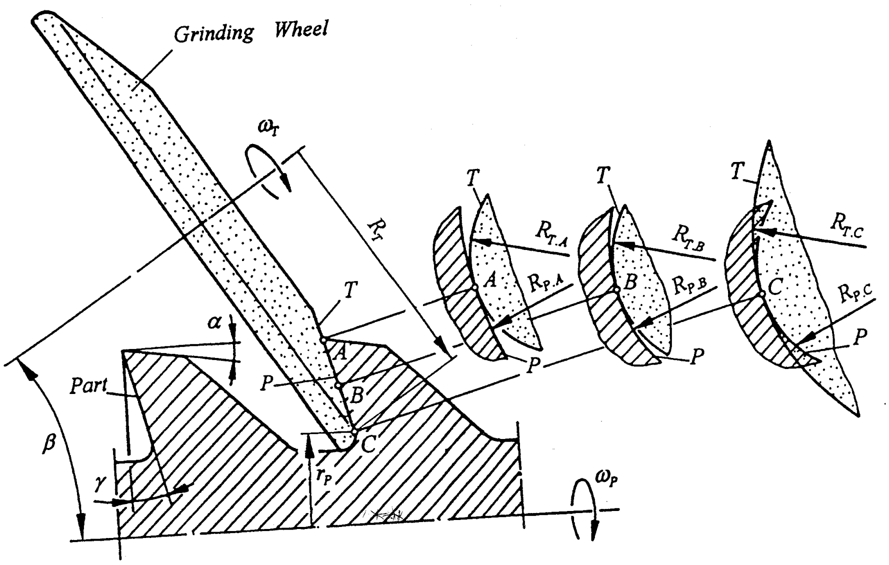


Fig. 5. The examples which illustrates that the second condition of the *PPSG* can satisfies (at the points *A* and *B*) and can not satisfies (at the point *C*).

Through the arbitrary points *A*, *B*, *C* of the straight generatrix of the internal conical face surface of the broach several plane sections are passing. All of them are perpendicular to the straight line *ABC*. At each section through the points *A*, *B* and *C* radii of curvature of the surfaces *P* and *T* are equal to $R_{P.A}$ and $R_{T.A}$, $R_{P.B}$ and $R_{T.B}$, $R_{P.C}$ and $R_{T.C}$ respectively. To regrind the broach in a proper way radii of curvature of the machining surface of the tool *T* at each plane section through the points *A*, *B*, *C* must be less the radii of curvature of the part surface *P* to be machined, i.e. $R_{T.A} < R_{P.A}$, $R_{T.B} < R_{P.B}$, $R_{T.C} < R_{P.C}$, respectively. More over, it is obvious that the following inequalities take place: $R_{T.A} < R_{T.B} < R_{T.C}$, and $R_{P.A} > R_{P.B} > R_{P.C}$. If the parameters of the grinding wheel, say wheel diameter and the angle of cone β, are chosen not in a right way this condition of *PPSG* may be not observed, for example, as at a point *C* (Fig.5). In this case the second condition of *PPSG* not satisfies and the machining of the part surface *P* in compliance with the drawing becomes impossible.

The parameters of the design of the grinding wheel can be determined as a result of the following procedure.

The radius of curvature $R_{P.C}$ of the internal conical surface P in the normal plane section $N - N$ is determined by the Meusner's theorem

$$R_{P.C} = \frac{r_P}{\sin\gamma}, \tag{7}$$

where r_P – is the radius of a cylinder through the point C of the broach (Fig. 5);
γ – is the rake angle of the broach.

The radius of curvature $R_{T.C}$ of the external conical surface T in the same plane section is determined from the similar relationship

$$R_{T.C} = \frac{R_T}{\sin(\beta - \gamma)}, \tag{8}$$

where R_T – is the radius of a cylinder through the point C of the grinding wheel (Fig. 5);
β – is the angle between the axis of the broach to be regrind and the axis of the grinding wheel C.

At the boundary point C (Fig. 5) the radius of curvature $R_{T.C}$ must be equal to or less than the radius of curvature $R_{P.C}$. Therefore

$$\frac{r_P}{\sin\gamma} \geq \frac{R_T}{\sin(\beta - \gamma)}. \tag{9}$$

Contact of the profiles without penetration will be observed over the portion AC of the characteristic curve E. Consequently, this portion of the surface P can be machined with a conical grinding wheel the maximum radius R_T of which is equal to

$$R_T = \frac{r_P \sin(\beta - \gamma)}{\sin\gamma}. \tag{10}$$

This formula is employed the maximum permissible radius of a grinding wheel that can be used to grind a round broach.

Thus, interference of this kind may be observed in forming all possible types of surfaces. This interference consists in mutual penetration of the conjugate surfaces within the velocity of the point of their contact.

Interference of the second kind may also occur. It consists in mutual penetration of conjugate surfaces beyond the limits of their contact.

Interference of the third kind is also possible. In this case, the mutual penetration of the conjugate surfaces occurs as they approach each other before they come into full contact. To machine the part surface P in strict compliance with the drawing, there must be no mutual penetration whatsoever of the conjugate surfaces P of the part to be machined and the machining surface of the tool T.

4. THE THIRD CONDITION OF *PPSG*

The absence of transitional surfaces of the part surface P is the third condition for generating of the part surface P in machining operation.

The workpiece is bounded by various surfaces with either a sharp or smooth transition from a portion of one surface to a portion of another surface. The machining surface of the tool must also consist of a whole series of parts of various surfaces that are conjugate to the portions of the part surface *P* in a corresponding manner. The various portions of the tool surface can occupy a great diversity of positions to each other.

The following cases of mutual arrangements of the various portions of the machining surface of the tool *T* are possible:

(a) the portions of the machining surface of the tool *T* intersects each other,
(b) the boundary points of the portions of the machining surface of the tool *T* are in contact with each other,
(c) the portions of the tool surface *T* are separated from each other.

In the first case when various portions of the machining surface of the tool *T* intersects, it proves impossible to completely realize them in actual practice. Therefore, those portions of the part surface *P* profile, corresponding to the portions of the profile of the machining surface of the tool *T* that were not realized in metal, will not be machined, and so-called transitional surfaces will be produced at the boundaries of the parts of the workpiece *P*.

In the second and the third cases, all portions of the machining surface of the tool *T* can be realized completely in metal. Therefore, in these cases the workpiece surface *P* can be machined without producing transitional surfaces (without transitional curves) at the boundaries of the various portions of the part surface *P* to be machined. The intersection of adjacent portions of the machining surface of the tool *T* and formation of the transitional curves on the part surface *P* to be machined are most often met with in milling grooves on workpiece. It is found, for example, in milling threads with multiple-threaded cutters. Here a radius curve is formed at the root of the thread, notwithstanding the fact that the cutter has a thread with a sharp crest.

The line of intersection of adjacent parts of the machining surfaces of the tool produces a transitional surface *T* and the part surface to be machined *P* is observed on the transitional surface.

The size of the transitional surface depends upon the shape of the workpiece and the nature of the relative motion of the tool and the work in machining operation. By changing the parameters of this relative motion, the size of the transitional surfaces can be varied and, in special cases, they can reduce to zero.

No transitional curves are produced in cases when the points located at the boundaries of the portions of the surfaces *P* are, at the same instance of time, profiling points for both adjacent portions of the part surface *P* to be machined. In this case, the boundary points of the conjugate portions of the surface *T* will also coincide and, consequently, adjacent portions of the machining surface of the tool *T* will be in contact. For this purpose it is necessary that the velocity $\vec{V}$ of the points on the boundary curve at the moment of profiling be directed tangentially to the boundary curve or be equal to zero.

Let's assume that it is necessary to machine a workpiece comprising a number of portions of surfaces (Fig. 6). The boundary of the first P_1 and of the second P_2 portions is the boundary curve *AB*.

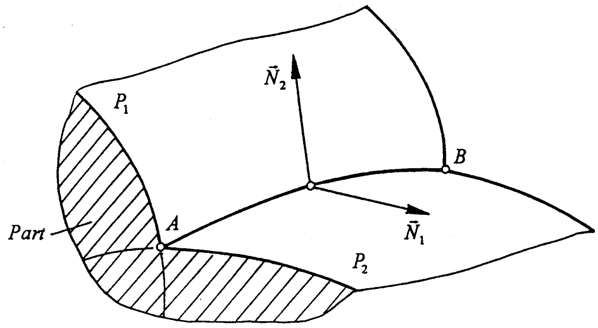


Fig. 6. Diagram of a workpiece surface *P*, that is bounded by two its conjugate portions.

At the points of the boundary curve *AB* one can draw two perpendiculars $\vec{N}_1$ and $\vec{N}_2$ to the portions P_1 and P_2 of the part surface *P* to be machined. The perpendiculars $\vec{N}_1$ and $\vec{N}_2$ are not aligned with each other if there is a sharp transition from one portion P_1 of the part surface *P* to another it portion P_2. Therefore, for simultaneous compliance with the contact conditions, $\vec{N}_1\vec{V} = 0$ and $\vec{N}_2\vec{V} = 0$ it is necessary that $\vec{V} = 0$ or that the velocity vector $\vec{V}$ be perpendicular to the both vectors $\vec{N}_1$ and $\vec{N}_2$. The latter is true only when the velocity $\vec{V}$ is tangent to the boundary AB of the portions P_1 and P_2.

As an example of the third condition of *PPSG* let's consider the operation of machining of a spur gear with a rack cutter (Fig. 7). While machining of a spur gear with rack cutter pitch line W_T of the rack cutter rolls without sliding about pitch circle of radii $r_{W.T}$ of the spur gear to be machined. For machining of the gear the tooth surface A_PC_P, say the surface P_1, the machining surface of the tool A_TC_T, says the surface T_1, of the rack cutter is necessary. The similar statement is also true to the part surfaces P_2 and P_3: for the tooth surface D_PH_P of the gear to be machined, say the surface P_2, the machining surface of the tool D_TH_T, say, the surface T_2, of the rack cutter is necessary. As well the similar statement is true for the tooth surface E_PG_P of the gear to be machined, say the surface P_3, the machining surface of the tool E_TG_T, say, the surface T_3, of the rack cutter is necessary. But the machining surfaces of the tool P_1 and P_2, P_2 and P_3 intersects each other at the points B_T and F_T correspondingly. That's why the portions B_TC_T, B_TD_T, F_TG_T and F_TH_T of the machining surfaces T_1, T_2 and T_3 of the cutting tool can not be produced and the portions B'_PC_P, B''_PD_P, F'_PG_P and F''_PH_P of the part surfaces P_1, P_2 and P_3 can not be machined in correct way. Instead of these portions of the part surface to be machined the transitional curves $B'_PB'_P$ and $F'_PF'_P$ will be obtained. The dimensions of these transitional curves $B'_PB''_P$ and $F'_PF''_P$ must be within the tolerance on the precision of the part surface to be machined.

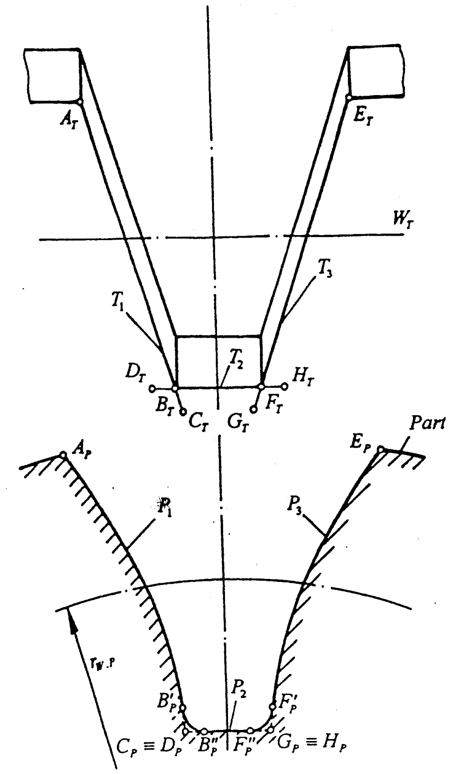


Fig. 7. Transitional curves after machining of the spur gear with the hob-milling cutter.

CONCLUSIONS

Principle geometrical and kinematical problems of part surface generating while machining on conventional machine tool has been considered. This paper presents the concept of basic conditions of cutting processes. Three basic necessary and sufficient conditions of the proper part surface generating have been established and are discussed. Applications of the basic conditions are also discussed in the paper. Further investigations of the process of machining part on conventional machine tool must be enhanced and include physical parameters of process of cutting such as speed of cutting, feed rate, cutting tool angles (rake angle, clearance angle, angle of inclination etc.) and others.

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OSNOVNI USLOVI TAČNOG GENERISANJA POVRŠINE DELA PRI OBRADI NA KONVENCIONALNIM ALATNIM MAŠINAMA

Stepan P. Radzevich

Obrada površina delova na konvencionalnim alatnim mašinama nekiput nameće geometrijske probleme koji nisu predočeni u klasičnoj geometriji površina. U ovom radu se daje definicija i ilustracija osnovnih uslova za ispravno generisanje površina dela na konvencionalnim alatnim mašinama. Ovo je važno u procesu propisivanja tehnologije obrade. Sa teorijskog aspekta značajno je da se mogu analitički opisati svi osnovni geometrijski uslovi potrebni za tačno generisanje površina dela pri njegovoj obradi na konvencionalnim mašinama. Od praktičnog je značaja da generisane površine budu u granicama tolerancije.