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EIGENFREQUENCIES ANALYSIS FOR THE DEEP DRILLING MACHINE GEAR SET

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Abstract. *This paper contains an overview of the procedures that can be used for eigenvectors and eigenfrequency analysis of deep drilling machine gear set, that is produced in "ZASTAVA-MAŠINE", d.o.o, using a Finite Elements Method. By using a rapid development concept, procedure for finite elements mesh generation of the gear body for either a gears with straight or oblique tooth profile, is described. For each and every element from the system, eigenvectors and eigenfrequencies calculations are done. Software development for 3D gear geometry generation is produced. This software is generalized for oblique tooth profile gear geometry generation.*
Key words: eigenfrequency analysis, deep drilling machine gear set

1. INTRODUCTION

One of the basic tasks in dynamic analysis of the various constructions is to evaluate the displacements of the construction as the time dependent functions when the time varying loads are given. These functions can be defined either as the continual functions (when there is continual mass distribution in the system) either as a discrete functions (when the system has a known number of the degrees of freedom). Mathematical description for determination of the dynamic displacements is known as *equation of motion*.

Before the equations of motion are defined one should assume that the system for which those equations are to be defined is *linear* (it's dynamic forces are bonded with acceleration, velocity or displacement vectors by means of linear coefficients) or *nonlinear* (mechanical properties of the system are not constants).

Oscillations of the linear systems have a possibility of elongation separation on free oscillations mode shapes. This feature means a great advantage for practical applications. Systems with very large number of degrees of freedom can be described through n independent equations-one for each degree of freedom. This means that the system is

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replaced with analogous system of n partial oscillations.

In nonlinear systems case, total time interval is divided into a very small, equal time steps. During each of the time step system is assumed to be linear with characteristics that it had at the beginning of each time step. So, nonlinear system is to be replaced with series of continually changeable linear systems.

Since equations of motion of nonlinear systems, for most practical problems, can not be solved in closed form, one has to apply numerical methods to solve them. There are 3 categories of such problems:

- At first, construction is simplified through appropriate dynamic model. Methods are:
  a) reduced mass method,
  b) method of substitution of the continual mass with discrete concentrated masses and
  c) substitution of the given system with equivalent system.

  First two methods are known as *methods for solving physically discrete* systems.
- Second is approximation of the differential equations of motion of the problem with a model of finite number of degrees of freedom by means of certain numerical procedures. These methods are:
  a) finite differences method,
  b) iteration method,
  c) finite elements method etc.
- Third category can be applied to variation methods like Rayleigh-Ritz and Galerkin method.

Finite elements method (FEM) is developed as the response on growing need to solve complex constructive systems for which there is no possibility of finding the solution in closed form. Application of this method assumes that system is divided on elements with finite dimensions, finite elements. For each element, approximated solution can be found, and afterwards simply by adding all the elements of the system, system of simple algebraic equations is developed. On such a way a really complex system of differential equations leads to the system of algebraic equations which represents body physically divided into finite elements.

2. EIGENVECTORS AND EIGENFREQUENCIES PROBLEM

Equations of motion for undamped systems, under the assumptions of constant stiffness, mass and free oscillations (there are no time dependent external force, displacements, pressure or temperature), can be represented in form:

$$[\mathrm{M}]\{\ddot{u}\}+[\mathrm{K}]\{u\}=\{0\}\,, \tag{1}$$

where: [K] - stiffness matrix (known) and [M]- mass matrix (known).

Linear system will have free harmonic oscillations, if

$$\{u\}=\{\phi_i\}\cos\omega_i t\,, \tag{2}$$

with: $\{\phi_i\}$- eigenvector (unknown),
ω_i - i-th eigenfrequency and
t - time.

Eigenfrequencies represent characteristic of the system and do not depend on the choice of the exact coordinates which helps to describe the system.
Equation (2.1) becomes:

$$(-\omega_i^2[\mathrm{M}]+[\mathrm{K}])\{\phi\}_i=\{0\}\,. \quad (3)$$

Equation above is satisfied if $\{\phi\}_i=\{0\}$ or $([\mathrm{K}]-\omega_i^2[\mathrm{M}])$ is zero. First option is trivial, so we can obtain frequency equation:

$$|[\mathrm{K}]-\omega_i^2[\mathrm{M}]|=0\,. \quad (4)$$

This is eigenvalue problem which can be solved for n square values of the eigenfrequencies and n values of the eigenvectors that satisfies equation (3), where n-is the number of degrees of freedom (DOF) of the system.

For output, one can show eigenfrequencies instead of circular frequencies in form:

$$f_i=\frac{\omega_i}{2\pi}\,, \quad (5)$$

where: f_i - i-th eigenfrequency in Hz.

2.1 An overview of the finite elements method procedures used for the problem solving

Determination of eigenvalues and eigenvectors is popular for the equations obtained during the evaluation of mode shapes for the system presented by finite elements. Generalized form of the system of algebraic equations is:

$$[\mathrm{K}]\{\phi_i\}=\lambda_i[\mathrm{M}]\{\phi_i\}\,, \quad (6)$$

where: $\lambda_i=\omega_i^2$ - eigenvalue (unknown).

Procedures used for the extraction of eigenvalues and eigenvectors from the system of equations can be used for solving buckling problems also. Table 1 presents an overview of the methods and solvers developed until nowdays for this class of the finite elements problem.

Table 1. Eigenvalues and eigenvectors problem solving procedures

Type of procedure	**Full extraction out of the reduced matrices**	**Partial extraction**			
Method	Reduction method	Subspace	Block Lanzos	Unsymmetric	Damped
Applications	Any	Symmetric	Symmetric	Unsymmetric	Symmetric or unsymmetric with dumping
Guy reduction	Yes	No	No	No	No
Extraction technique	HBI	Subspace which internally uses Jacobi	Lanczos which internally uses QL algorithm	Lanczos which internally uses QR iterations method	

3. AN EXAMPLE OF MODE SHAPES AND EIGENFREQUENCIES CALCULATIONS FOR THE GEAR SET

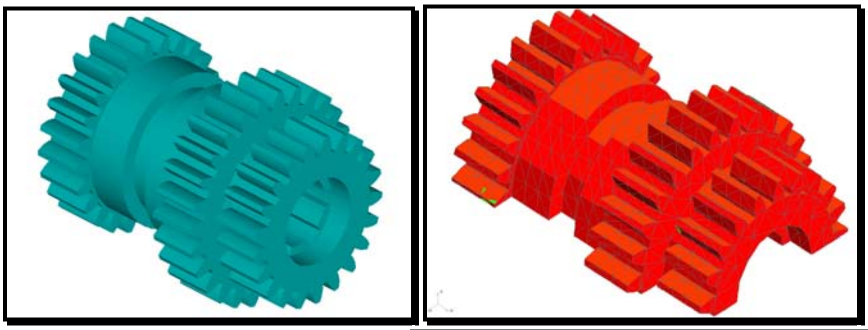

Table 2. Material properties

PROPERTY	VALUE
Young's modul	20 E6 N/cm^2
Poisson's coefficient:	0.32
Shear modul :	3.6 E6 N/cm^2
Material density:	0.0078 kg/cm^3
Material:	Č. 4320

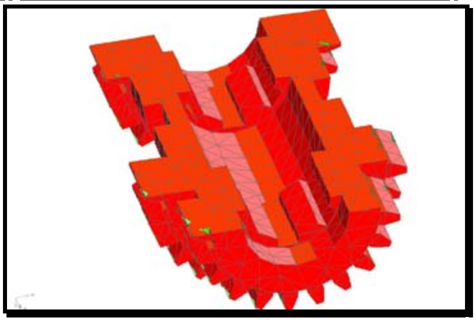

Fig. 1. Model and finite element mesh for the gear set

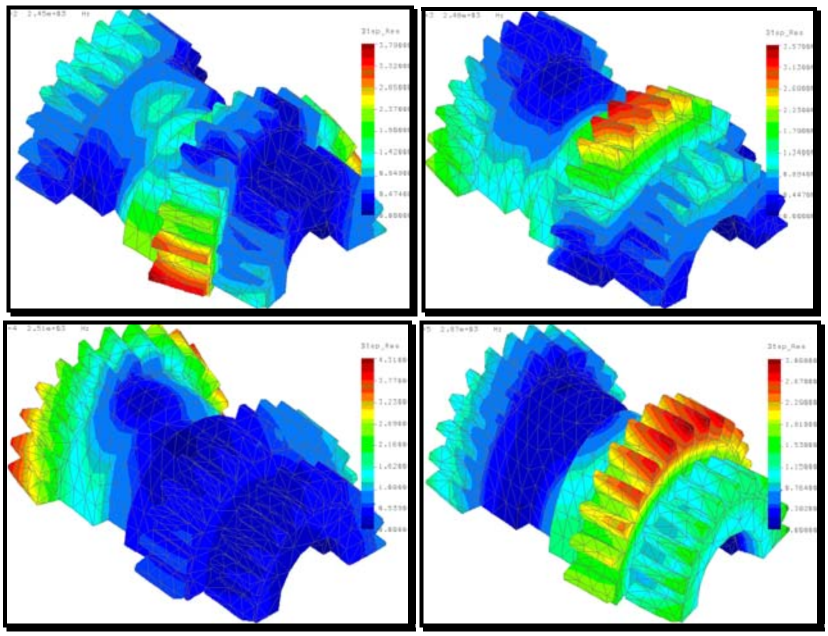

Fig. 2. First four mode shapes of the model

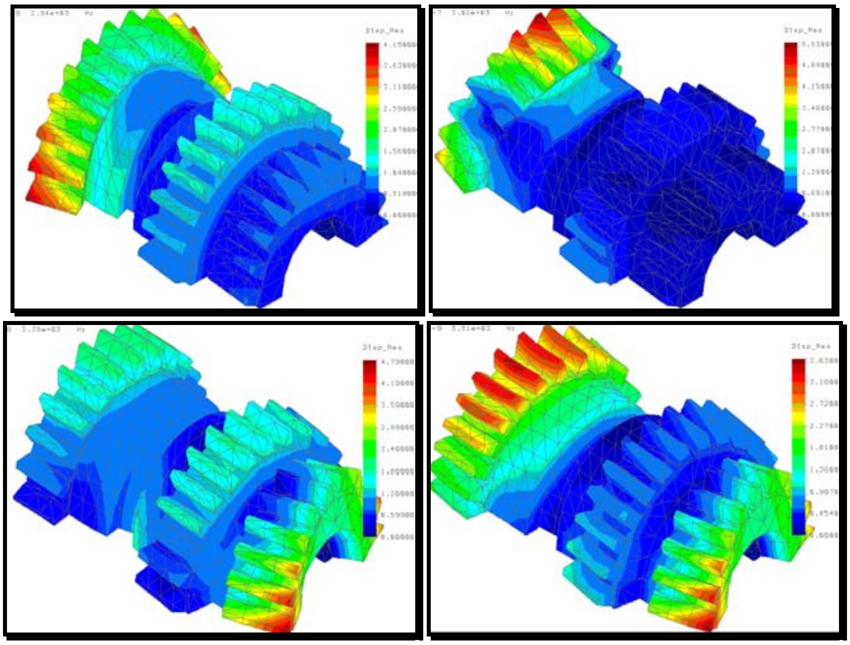

Fig. 3. Second four mode shapes of the model.

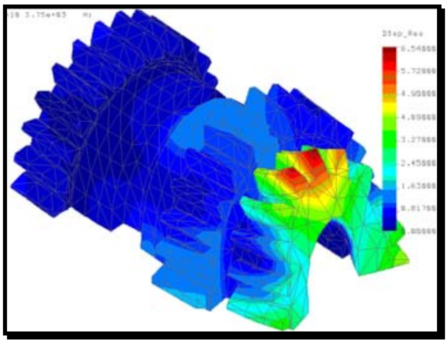

Fig. 4. Ninth mode shape of the model

Table 3. Constructive parameters of the gears

	PARAMETER	GEAR. 1	GEAR. 2	GEAR. 3
1.	Number of teeth:	24 mm	27 mm	21 mm
2.	Modul :	3 mm	3 mm	3 mm
3.	Standard profile:	JUS. M. C1. 016	JUS. M. C1. 016	JUS. M. C1. 016
4.	Factor of profile displacement:	0	0	0
5.	Pitch circle diameter:	72 mm	81 mm	63 mm
6.	Addendum circle diameter:	78 mm	87 mm	69 mm
7.	Base circle diameter:	64.8 mm	73.8 mm	55.8 mm
8.	Measurement teeth number:	3 mm	4 mm	3 mm
9.	Control distance over the teeth:	23.1495 mm	32.1318 mm	23.0235 mm
10.	Standard angle:	20	20	20
11.	Axial distance:	81.005 mm	81.005 mm	81.005 mm
12.	No. of teeth for the second gear:	30	27	33

Table 4. General output data

Total equations	(NEQ) =	4467
Stiffness matrix elem. num	(NWK) =	1096269
Max half band width	(MK) =	492
Mean half band width	(MM) =	245
Number of elements	(NUME) =	4680
Number of nodes	(NUMNP) =	1623
Stiffness matrix block size	(MTBLK)=	1830806
Number of blocks	(NBLK) =	1

Max. diag. value from the stiffness matrix = 0.100895E+11 (4355)
Min. diag. value from the stiffness matrix = 0.331322E+07 (3192)

Table 5. Mass and moment inertia

MASS (kg)	0.106967E+01	VOLUME (cm3)	0.137114E+03		
AXIAL MOMENTS OF INERTIA (cm4)-Global coordinates					
IX	0.490037E+01	IY	0.155763E+02	IZ	0.124879E+02
PRODUCT OF INERTIA IN GLOBAL CARTESIAN COORDINATE SYSTEM (cm4)					
PXY	0.490976E-01	PXZ	-0.552977E-03	PYZ	0.172809E-02
RADIUS OF INERTIA					
RX	0.214037E+01	RY	0.381598E+01	RZ	0.341680E+01
CENTER OF GRAVITY					
CGx	0.245583E+02	CGy	0.166997E+02	CGz	-0.265013E-03
MAIN AXIAL INERTIA MOMENTS					
P1	0.155765E+02	P2	0.124879E+02	P3	0.490014E+01
MAIN AXIAL INERTIA RADIUS					
R1	0.381601E+01	R2	0.341680E+01	R3	0.214032E+01
MAIN AXIS COSINE					
N_11	0.459881E-02	N_12	-0.999989E+00	N_13	0.560324E-03
N_21	0.708559E-04	N_22	0.560674E-03	N_23	0.100000E+01
N_31	-0.999989E+00	N_32	-0.459877E-02	N_33	0.734336E-04

Table 6. Mode shapes and eigenfrequencies of the model

MODE NUMBER	***FREQUENCY (RAD/SEC)***	***FREQUENCY (1/SEC)***	***PERIOD (S)***
1	0.1181485E+05	0.1880392E+04	0.5318040E-03
2	0.1541410E+05	0.2453231E+04	0.4076257E-03
3	0.1556162E+05	0.2476709E+04	0.4037616E-03
4	0.1575632E+05	0.2507696E+04	0.3987725E-03
5	0.1800228E+05	0.2865152E+04	0.3490216E-03
6	0.1849528E+05	0.2943616E+04	0.3397183E-03
7	0.1897922E+05	0.3020637E+04	0.3310560E-03
8	0.2130641E+05	0.3391020E+04	0.2948965E-03
9	0.2206406E+05	0.3511604E+04	0.2847701E-03
10	0.2355739E+05	0.3749274E+04	0.2667183E-03

Mode shapes calculation for gear set described above are done by means of SUBSPACE iteration method. Material properties are shown in Table 2. Table 3 contains constructive parameters needed to develop the software model of the gears in the set. Gear geometry generation is done in the same manner as in [8]. Program in C-language is developed under the platform of AutoCAD release 12. Function developed is used to generate the 3D gear model both with flat or oblique tooth profile. Model prepared in such a manner is transferred in IGES format and afterwards simply imported into a software that does eigenfrequencies and mode shape calculations.

Table 4 is used to present general data of the finite elements model for the gear set. Total number of elements is 4680 (tetrahedral); number of nodes is 1623. Table 5 has the calculation results for the mass, inertia effects and geometric inertia properties.

Calculation results for first ten mode shapes can be found in table 6; those results show that the gear set described above can have oscillations in range of higher frequencies (1880-3749 Hz). It is very interesting that each and every gear in the model (from left) can have "it's own" frequency. So, gear 1. can have oscillations at frequencies 2507, 2865, 2943 and 3391 Hz, what is exactly IV, VI, VII and IX mode shape frequency of the gear; this leads to conclusion that the gear has the highest possibility to go in the critical area of frequencies during it's operating time. Gear 2. has the least sensitivity to oscillations, because it is between the gears 1 and 3. Eigenfrequencies for the second gear are 2453, 2476 and 2865 Hz, and they represent the III, IV and VI mode shape. Gear 3. can have higher frequency range: 3391, 3511 and 3749 Hz, what is exactly VIII, IX and X mode shape of the gear.

Boundary conditions are defined (for translation and rotation) for displacement nodes that lies on the connection of the gear and the shaft. Since there is only one symmetry plane in the model, symmetry of the stiffness and mass matrix was used to develop the simplified half of the model. This property enabled the highest accuracy and larger number of nodes and elements to be included in the model.

4. CONCLUSIONS

Most of the mechanical systems nowdays has dynamic loads that cause shortening of the exploatation time, crack, noise and vibrations; in general, total effect of work for the mechanical system is lowered. Reasons for such behaviour are in type of loading, construction and conditions of work where the mechanical system operate. All mentioned causes are bonded and they demand compromises to be done that are very differrent in their nature.

Methodologies used to analyze the behaviour of the mechanical system are mostly dependent on the complicated laboratory testing and very simplified analytic models which can not obtain real picture of what is really happening in the system. Laboratory testing, nowdays, are mostly used to verify prototype of the sistem and they are eleminatory-they can give the answer on wheather the model satisfies working conditions or not. This is because the laboratory testing are mostly expensive and they demand very heavy equipement if one wishes to analyze the very large scale of parameters that influence the functioning of the mechanical system in reality.

In absence of the adequate analytic methods, numeric methods found their place in engineering practice. One of the methods is Finite Elements Method. It is used in this paper to develop the real model of the gear set and to find out about the behaviour of the eigenfrequencies and mode shapes of the gear set.

Calculated model is very flexibile due to developed software. Concept for rapid development is fully applicated, which means that the research can continue in following directions:

- definition of dynamic loading in presumed most critical points of gear contact,
- analysis choice, depending on force state, for example: modal analysis, harmonic analysis, random vibration analysis etc. and
- varying the frequencies parameters to obtain distribution of frequencies on each element; afterwards comparing the results for each contact separately.

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ANALIZA SOPSTVENIH UČESTANOSTI VIŠEVENČANOG ZUPČANIKA DUBINSKE BUŠILICE

Dejan K. Dimitrijević, Vera Nikolić

U radu se daje pregled procedura koje se mogu koristiti za analizu glavnih oblika oscilovanja i sopstvenih učestanosti elemenata složenog sistema jednovretene dubinske bušilice proizvođača "ZASTAVA-MAŠINE", d.o.o, metodom konačnih elemenata. Takođe je na bazi koncepta rapid development-a opisana procedura na osnovu koje se može generisati mreža konačnih elemenata za telo zupčanika, bez obzira da li se radi o zupčanicima sa pravim ili zupčanicima sa kosim zupcima. Za svaki od elemenata je urađen proračun sopstvenih učestanosti i glavnih oblika oscilovanja. Urađen je software za generisanje geometrije zupčanika u prostoru, koji je uopšten na generisanje geometrije zupčanika sa kosim zupcima.