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THE APPROXIMATIVE MODELS OF THE BOX GIRDER IN STRUCTURAL ANALYSIS

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Abstract. *This research work deals with the analytical calculating model of the elastic deformation of the box girder elements of the supporting structure machines. Beam elements, changeable geometry longitudinal cross-section are the reason for regular approximation of calculating models. Introduced approximations of the middle cross-section cause rough deviations of the exact mathematical models. A proposed calculating model for definite elastic deformations of the box girders, changeable geometric cross-section, is developed in the paper. For basic beam types of transporting machines and heavy vehicle, a procedure of defining equivalent geometry, for choosing the best approximation is developed. Approximation introduced in this way simplifies structural analysis and introduce controlled elastic deviation from the linear analytic model.*

Key words: *structural analysis, box girder, approximative models*

1. INTRODUCTION

The reducing of calculating complexity of analytical and numerical models of real supporting machines is done by series of approximations of structure elements. Approximations are obvious because the crane structure, rail vehicles, mining and other machines that need a few hundred thousand degrees of freedom. These machines classes often have supporting elements as closed, box girders. The length of the elements can reach a few tens of meters. Their mass and overall cross-section put them into elements of special importance. The importance of supporting structure at heavy machines is also the reason for careful choice of applied approximation. The research of the quality of approximation is not only guarantee of good machine analysis, but also rational modeling

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approach. The quality of modeling is not increased by its complexity but by choosing the least approximation [2,3].

2. Basic mathematical procedure

General constructive element is a squeezed element of the structure (with all components of the inner connections). That is a closed box girder in console form, length L, linear changeable cross-section of the given geometry of the ends $H_1/B_1/t$ and $H_2/B_2/t$. The thickness of the wall (t) is constant. The console is loaded by concentrated force F at the free end, and the continual changeable longitudinal loading by its own mass (weight) $q_{(Z)}$. By introducing quotient $R = (B_2 - B_1)/L$ and $T = (H_2 - H_1)/L$, it is possible to define cross-section moment of inertia $I_{X\,(Z)}$. At thin walls ($t < H/25$), it is possible to get moment of inertia according to approximate equation (1), with accuracy greater then 0.4%:

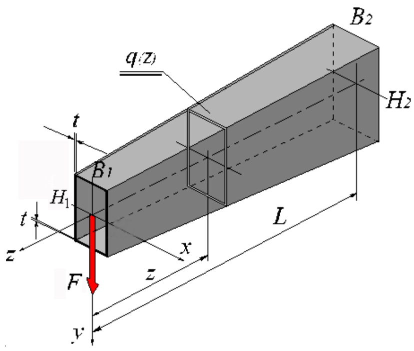


Fig 1. Box girder model of the changeable cross-section

$$I_x \approx 0.5 \cdot B_{(z)} \cdot H_{(z)}^2 \cdot t - B_{(z)} \cdot H_{(z)} \cdot t + 0.166 \cdot H_{(z)}^3 \cdot t - H_{(z)}^2 \cdot t^2 , \tag{1}$$

At girder linear longitudinal geometric change, the basic cross-section dimension can be defined by the relations: $H_{(Z)} = H_1 + R \cdot z$ and $B_{(Z)} = B_1 + T \cdot z$. R and T are linear quotients of the cross-section proportionality. By their introduction at relation (1), moment of inertia at arbitrary cross-section (at position z (2), with following constants C_1–C_4) is got. Continual loading of its own mass $q_{(z)}$ (with constants C_1–C_6), is given in relation (3-4).

$$I_{x(z)} = C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4 \tag{2}$$

$$q_{(z)} = A_{(z)} \cdot L \cdot \gamma \approx 2 \cdot \gamma \cdot (B_{(z)} + H_{(z)}) \cdot t = \gamma \cdot C_6 + \gamma \cdot C_5 \cdot z \tag{3}$$

$$C_1 = 0.5 \cdot R^2 \cdot t + 0.1666 \cdot R^3 \cdot t ,$$

$$C_2 = 0.5 \cdot R^2 \cdot B_1 \cdot t + H_1 \cdot R \cdot T \cdot t - R \cdot T \cdot t^2 + 0.5 \cdot R^2 \cdot H_1 \cdot t - R^2 \cdot t^2 ,$$

$$C_3 = H_1 \cdot B_1 \cdot R \cdot t + 0.5 \cdot H_1^2 \cdot T \cdot t - B_1 \cdot R \cdot t^2 - H_1 \cdot T \cdot t^2 + 0.5 \cdot H_1^2 \cdot R \cdot t - 2 \cdot H_1 R \cdot t^2 ,$$

$$C_4 = 0.5 \cdot H_1^2 \cdot B_1 \cdot t - H_1 \cdot B_1 \cdot t^2 + 0.1666 \cdot H_1^3 \cdot t - H_1^2 \cdot t^2 ,$$

$$C_5 = 2 \cdot (T + R) \cdot t , \quad C_6 = 2 \cdot (H_1 + B_1) \cdot t , \quad q_0 = \gamma \cdot C_5 \tag{4}$$

Console deflection, definite from deformation work of bending and deformation work by transversal forces is sum of the particular deflection y_S and y_T. *Bending* is done by the moment of bending $M_{(Z)}$, (5) at cross-section *z*, from the continual loading by its own mass $q_{(Z)}$ and concentrated force *F*. Bending deflection is defined by the relation (6.a,b,c). I_1 and I_2 present integrals in given limits, *E* is a module of elasticity. The first addend is its own mass influence (marked by index *q*) and second is the influence of the concentrated force *F* (marked by index *F*).

$$M_{(z)} = z^2 \cdot (q_0 + q_{(z)}) / 6 + F \cdot z = 0.1666 \cdot \gamma \cdot (3 \cdot C_6 + C_5 \cdot z) \cdot z^2 + F \cdot z \tag{5}$$

$$y_s = \int_0^L \frac{M_{(z)}}{E \cdot I_{x(z)}} \cdot \frac{dM_{(z)}}{df} dz \,, \tag{6-a}$$

$$y_s = \frac{\gamma}{E} \int_0^L \frac{0.5 \cdot C_6 \cdot z^3 + 0.1666 \cdot C_5 \cdot z^4}{C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4} dz + \frac{F}{E} \int_0^L \frac{dz}{C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4} dz \tag{6-b}$$

$$y_s = \frac{\gamma}{E} \cdot I_1 + \frac{F}{E} \cdot I_2 = y_{s(q)} + y_{s(F)} \tag{6-c}$$

Console deflection of transversal forces is defined from the general relation (7), where $F_{T(z)}$ is transversal force (8) and $(S_{x(z)}/\xi)$ is quotient of the static cross-section moment and thickness of the rib. Integral $\int (S_{x(z)}/\xi)^2 \, dA$ is defined for all cross-section according the relation (9). *G* is module of sliding.

$$y_T = \frac{1}{G} \int_0^L \int_A \frac{F_{T(z)}}{E \cdot I_{x(z)}^2} \cdot \left(\frac{S_{x(z)}}{\xi} \right)^2 dA dz \,, \tag{7}$$

$$F_{T(z)} = \gamma \cdot C_6 \cdot z + 0.5 \cdot \gamma \cdot C_5 \cdot z^2 + F \tag{8}$$

$$\int_{(A)} \left(\frac{S_{x(z)}}{\xi} \right)^2 \cdot dA \approx 2 \int_0^{0.5 \cdot H - t} [0.25 \cdot B(z) \cdot (H_{(z)} - t) + 0.5 \cdot (0.5 \cdot H_{(z)} - t)^2 - 0.5 \cdot y^2]^2 \cdot 2 \cdot t \cdot dy \tag{9}$$

Transversal force deflection is in the form of (10-a,b) with following integral function $P_{0(Z)}$, (10-c).

$$y_T = \frac{y}{G} \int_0^L \frac{(C_6 \cdot z + 0.5 \cdot C_5 \cdot z^2) \cdot P_{0(z)} \cdot dz}{(C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4)^2} dz + \frac{F}{G} \int_0^L \frac{P_{0(z)} \cdot dz}{(C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4)^2} dz \,, \tag{10-a}$$

$$y_T = \frac{\gamma}{G} \cdot I_3 + \frac{F}{G} \cdot I_4 = y_{T(q)} + y_{T(F)} \,, \tag{10-b}$$

$$\begin{aligned} P_{0(z)} &= 05333 \cdot t \cdot [0.5 \cdot (H_1 + R \cdot z) - t]^5 + 0.666 \cdot t \cdot (B_1 + T \cdot z) \cdot [0.5 \cdot (H_1 + R \cdot z) - t]^3 \\ &+ 0.25 \cdot t \cdot (B_1 + T \cdot z)^2 \cdot [0.5 \cdot (H_1 + R \cdot z) - t] \cdot (H_1 + R \cdot z - t)^2 \end{aligned} \tag{10-c}$$

Deflection of bending and transversal forces is defined in that way, so the influence of its own mass and concentrated force is analytically defined. According to this the total deflection is the sum of particular deflections $y_{S(q)}$, $y_{S(F)}$, $y_{T(q)}$ and $y_{T(F)}$. The procedure of finding integrals I_1, I_2, I_3 and I_4, is realized by numerical integration, applying the expanded trapezoidal rule [1].

3. COEFFICIENT DEFINING OF EQUIVALENT SECTION

The main task at approximative models developing is defining of calculated accuracy. This analysis led to defining the connection coefficients of the initial geometry (of changeable section) and applied-approximative geometry. The connection is made by corrective coefficient of initial geometry $\boldsymbol{k}_H$, $\boldsymbol{k}_B$, which provide the quality of approximation. The most frequent applied approximation belongs to the item of proportional cross-section (by form), of constant length geometry. So real constructive box girder (a structure part) is replace by its equivalent, and of the same proportional section form. The equivalent box girder is of the constant cross-section $B^*/H^*/t$ and of the same deflection with initial (basic) model. It is necessary to define geometry of equivalent girder. Analysis of the different static influences, shows a remarkable lower influence bending than shearing. So the deformity of shearing work can be neglected at long flexible items. According to the same structural item form, the deflection of the equivalent girder (with constant cross-section $B^*/H^*/t$), is defined by the following relations (11-13):

$$y^*_{(z=L)} = \frac{q^* \cdot L^4}{8 \cdot E \cdot I^*_X} + \frac{F \cdot L^3}{3 \cdot E \cdot I^*_X} \tag{11}$$

$$I_X \approx 0.5 \cdot B^* \cdot H^{*2} \cdot t - B^* \cdot H^* \cdot t^2 + 0.1666 \cdot H^{*3} \cdot t - H^{*2} \cdot t^2 \tag{12}$$

$$q^* = 2 \cdot \gamma \cdot (B^* + H^*) \cdot t \tag{13}$$

By equalizing the deflection of real girder (2.6-c) and equivalent girder (11), it is possible to find functional relation between the initial and equivalent geometry:

$$y_S = y^*_{(z=L)},$$
$$\frac{\gamma}{E} \cdot I_1 + \frac{F}{E} \cdot I_2 = \frac{q^* \cdot L^4}{8 \cdot E \cdot I^*_X} + \frac{F \cdot L^3}{3 \cdot E \cdot I^*_X} \tag{14-a,b}$$

Found relation (14-b) is the function of two unknown quantities B^* and H^*. If there is only one condition of equalized deflection (deflection in one coordinate direction), it is better to express unknown parameters by quotient θ -(15). So only one unknown cross-section quantity is looked for,

$$\theta = \frac{H^*}{B^*} \ . \tag{15}$$

Since medium dimensions of the box girder ends are known (B_{SR}, H_{SR}), unnamed coefficient of equivalent cross-section k_B and k_H, can be made. The average quotients of equivalent box girder geometry (16) are:

$$k_B = \frac{B^*}{B_{SR}}, \qquad k_H = \frac{H^*}{H_{SR}} \tag{16}$$

By replacing H^* from (15) in equation (14-b) and by arranging it, the equation of third degree by unknown k_B, is got and defined by relation (17) and constants (18):

$$k_B^3 - \frac{C_8}{B_{SR}} \cdot k_B^2 - \frac{C_9}{B_{SR}^2} \cdot k_B - \frac{C_{10}}{B_{SR}^3} = 0 \tag{17}$$

$$C_7 = \frac{\gamma \cdot I_1 + F \cdot I_2}{0.08333}, \qquad C_9 = \frac{3 \cdot \gamma \cdot L^4 \cdot (1+\theta)}{0.5 \cdot C_7 \cdot \theta^2 + 0.1666 \cdot C_7 \cdot \theta^3},$$
$$C_8 = \frac{C_7 \cdot t \cdot (1+\theta)}{0.5 \cdot C_7 \cdot \theta + 0.1666 \cdot C_7 \cdot \theta^2}, \qquad C_{10} = \frac{4 \cdot F \cdot L^3}{0.5 \cdot C_7 \cdot \theta^2 \cdot t + 0.1666 \cdot C_7 \cdot t \cdot \theta^3}, \tag{18}$$

Very similar to the previous by treating (14-b) and according to the H^*, it is possible to find equivalent coefficient equation of the girder's high:

$$k_H^3 - \frac{C_8 \cdot \theta}{H_{SR}} \cdot k_H^2 - \frac{C_9 \cdot \theta^2}{H_{SR}^2} \cdot k_H - \frac{C_{10} \cdot \theta^3}{H_{SR}^3} = 0, \tag{19}$$

Relation (17) and (19) can be directly applied for coefficient defining of equivalency $\boldsymbol{k_H}$ and $\boldsymbol{k_B}$. In this way, it is possible to get equivalent geometric girders (B^*, H^*). The procedure of finding integral $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ can be automatized by using same numerical integrational treatment such as algorithm of the expanded trapezoidal rule (used here), or *Romberg's* algorithm. Solving the polynomial is done (17, 19) by modified *Bairstowa* method from *Platzman* [1]. Researches have shown that (at completely used stress constructions), the influence of its own mass on geometrical girders is very low. That can be proved by numerical example, typical for cranes.

Example 1:

Changeable geometric cross-section is: B_1=0.50 m, B_2=1.50 m, H_1=0.50 m, H_2=1.00 m, the thickness of the wall are t = 0.006 m, and the length is L = 4.00 m. If θ = 0.5÷3.0 is changed and influences $\gamma = 78 \cdot 10^3$ N/m^3, $F = 200 \cdot 10^3$ N, equivalent coefficient of the cross-section high is range $\boldsymbol{k_H}$= 1.024÷1.552, and coefficient $\boldsymbol{k_B}$ = 1.536÷0.388. Parallel analyze of the influence of its own mass on final results, shows that coefficients $\boldsymbol{k_H}$ and $\boldsymbol{k_B}$ are lower with maximal deviation up to 0.25 (%). At small and thin wall girders, these influences are of minor importance. From these, it can be concluded in the category of short girders, the influence of its own mass for choosing equivalent geometry, is neglected. This approximation eliminates the need for defining integral $\boldsymbol{I_1}$ and simplifies mathematical procedure for calculating coefficient of equivalent section.

Example 2:

Box girder of changeable cross-section with features: B_1=0.10 m, B_2=0.25 m, H_1=0.20 m, H_2=0.50 m, t = 0.005 m, L=0.50 m (B_{SR}=0.175 m, H_{SR}=0.35 m), figure one, is observed.

Coefficient solutions k_H and k_B are shown in figure 2. This example shows that application of approximation middle cross-section geometry is unappropriate, that is inaccurate. Found equivalent coefficients show the connection complexities with initial box girder geometry.

Example 2: H_1=0.2 m, H_2=0.5 m, B_1=0.1 m, B_2=0.25 m, L=0.5 m, t=0.005 m.

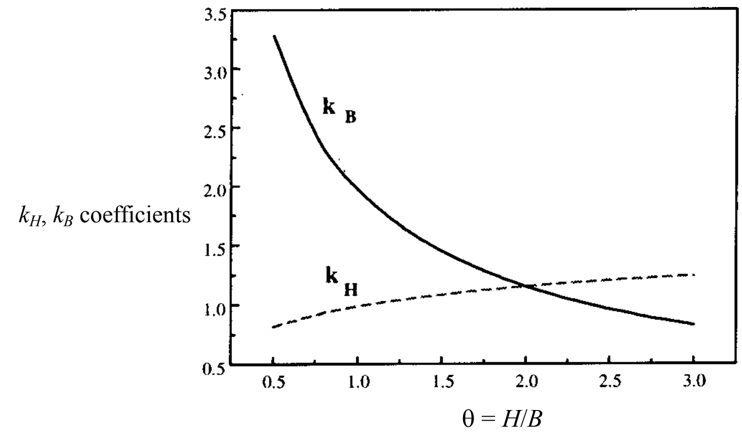


Fig. 2. Coefficients of equivalent cross-section k_H and k_B in example two.

4. Equivalent Box Girder at Structural Analysis of Heavy Vehicles

The constructions of heavy vehicles analyzed by finite element method, super-elements and substructures require few thousand degrees of freedom. It is an important limitation for choosing the computer and processor's time for calculating. On the other hand a great number of results (data) is not relevant for analyze, it is only "passing" control, when it is dealt with solid continuum. For these constructions long box girder in the form of "I" profile are used, with geometric reduction at the ends (see figure 3). The same designation marks for height and width is applied. For sheet metals of "I" profile, designation marks for the vertical rib thickness δ_2 and horizontal plate δ_1 are introduced. The inertia moment of cross-section can be defined by approximative relation (20-a,b), with error lower then 0.5%:

$$I_{X(Z)} \approx 0.5 \cdot B_{(Z)} \cdot H_{(Z)}^2 \cdot \delta_1 - B_{(Z)} \cdot H_{(Z)} \cdot \delta_1^2 + 0.08333 \cdot H_{(Z)}^3 \cdot \delta_2 - \\ - 0.5 \cdot H_{(Z)}^2 \cdot \delta_1 \cdot \delta_2 + H_{(Z)} \cdot \delta_1^2 \cdot \delta_2 \tag{20-a}$$

$$I_{X(Z)} = C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4 \tag{20-b}$$

The inertia moment (20-b) is of identical form with (2) with new coefficients C_1-C_4, according the relation (21):

$$
\begin{aligned}
C_1 &= 0.5\cdot R^2\cdot T\cdot \delta_1 + 0.08333\cdot R^3\cdot \delta_2\\
C_2 &= 0.5\cdot R^2\cdot B_1\cdot \delta_1 + H_1\cdot R\cdot T\cdot \delta_1{}^2 + 0.25\cdot H_1\cdot R^2\cdot \delta_2 - 0.5\cdot R^2\cdot \delta_1\cdot \delta_2\\
C_3 &= H_1\cdot B_1\cdot R\cdot \delta_1 + 0.5\cdot H_1{}^2\cdot T\cdot \delta_1 - B_1\cdot R\cdot \delta_1{}^2 - H_1\cdot T\cdot \delta_1{}^2 + 0.25\cdot H_1{}^2\cdot R^2\cdot \delta_2 -\\
&\quad - H_1\cdot R\cdot \delta_1\cdot \delta_2 + R\cdot \delta_2\cdot \delta_1{}^2\\
C_4 &= 0.5\cdot H_1{}^2\cdot B_1\cdot \delta_1 - H_1\cdot B_1\cdot \delta_1{}^2 + 0.08333\cdot H_1{}^3\cdot \delta_2 - 0.5\cdot H_1{}^2\cdot \delta_1\cdot \delta_2 +\\
&\quad + H_1\cdot R\cdot \delta_2\cdot \delta_1{}^2
\end{aligned}
\qquad (21)
$$

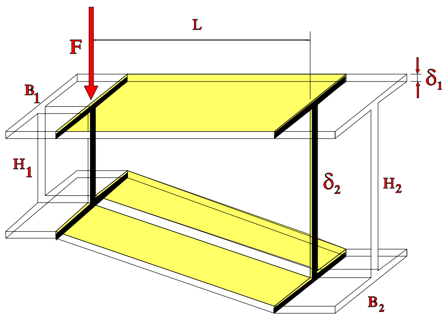


Fig. 3. Changeable cross-section of "I" profile for rail-vehicle construction

Using previously defined quotient θ (15), identical general solutions (17 and 19) for equivalent coefficient k_H and k_B (16), with new constants C_8, C_9 and C_{10} (22), can be obtained. For this "I" profile, they are:

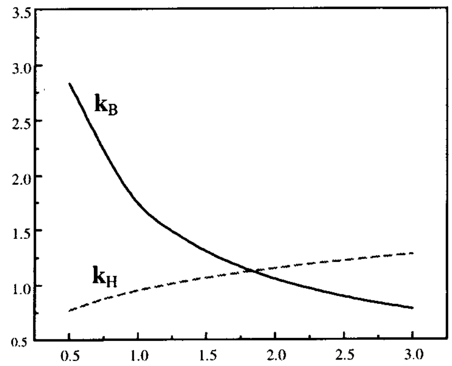
$$
C_8 = \frac{\delta_1^2\cdot\theta + 0.5\cdot\delta_1\cdot\delta_2\cdot\theta^2}{0.5\cdot\delta_1\cdot\theta^2 + 0.08333\cdot\delta_2\cdot\theta^3},\quad C_9 = -\frac{\delta_1^2\cdot\delta_2\cdot\theta}{0.5\cdot\delta_1\cdot\theta^2 + 0.08333\cdot\delta_2\cdot\theta^3},
$$
$$
C_{10} = \frac{4\cdot F\cdot L^3}{3\cdot I_2\cdot(0.5\cdot\delta_1\cdot\theta^2 + 0.08333\cdot\delta_2\cdot\theta^3)}. \qquad (22)
$$

Example 3:

In the case a real girder, with geometry: $B_1 = B_2 = 0.30$ m, $H_1 = 0.30$ m, $H_2 = 0.80$ m, $\delta_1 = 0.01$ m, $\delta_2 = 0.008$ m, $L = 2.00$ m and $\theta = 0.5 \div 3.0$, equivalent coefficients k_H and k_B (for changeable "I" profile of equivalent profile with constant cross-section) are obtained. Figure 4 shows the results of equivalent coefficients k_H and k_B, calculation. So, by previous analysis coefficients k_H and k_B (for chosen geometry quotient θ) are defined. By these coefficients the geometry of equivalent girder is found with analytical accuracy. That is why approximation with equivalent girder is the best.

Example 3: H_1=0.3 m, H_2=0.8 m, B_1=B_2=0.3 m, L=2.0 m, δ_1=0.01 m, δ_2=0.008 m,

k_H, k_B coefficients



$\theta = H/B$

Figure 4. Equivalent coefficients k_H and k_B for wagon's "I" profile.

5. The analysis of particular approximations at heavy machines

The presence of particular elastic deformations (in the total deformations) can be extracted by calculating of their percentage participation. The equation (23) defines the influence of its own mass on the total end-deflection girders. The equation (24) defines the influence of transversal force on the total end-deflection. If the influence of its own mass or transversal force are neglected, then relations defined by equations (23-24), show the approximations $\varepsilon_{(q)}$ and $\varepsilon_{(T)}$ of calculating model.

$$\varepsilon_{(q)} = \frac{y_{S(q)} + y_{T(q)}}{y_{S(q)} + y_{S(F)} + y_{T(q)} + y_{T(F)}} \cdot 100\ \% \tag{23}$$

$$\varepsilon_{(T)} = \frac{y_{T(q)} + y_{T(F)}}{y_{S(q)} + y_{S(F)} + y_{T(q)} + y_{T(F)}} \cdot 100\ \% \tag{24}$$

Example 4:

Crane's box girders made of construction steel which is characterized by the biggest mass are observed. The box-girder of arbitrary cross-section B_1 = 0.894 m, B_2 = 1.720 m, H_1 = 0.612 m, H_2 = 1.42 m, thickness of the wall t = 0.01 m relatively short L = 5.00 m, ($L/H_{SR} \approx 5$), under the force of $500\cdot10^3$ N, has the total deflection $y_{S(q)}+y_{S(F)}+y_{T(q)}+y_{T(F)}$= 0.00008+0.008186+0.000022+0.001632 = 0.00992 m. Bending deflection from force $y_{S(F)}$ (at maximal cross-section efficiency), makes 82% of the total deflection. The influence of continual loading on total deflection is approximately 1%.

The found results of geometric analysis of cranes box-girders with features: B/H=0.5÷3.0, middle high H_{SR} < 1.00 m, t < 2. m and range L < 4.50 m, show the influence of the own mass on the total deflection under 1%. These influences are got at maximal stiffness efficiency at console root. After this, it is questionable which girder geometry it is, where the influence of its own mass on total elastic deflection is dominating.

Example 5:

In order to find the answer, girder used for construction of long jib at portal-sleeving cranes is analyzed. The girder of the same geometric cross-section from the previous example but of the length L=26.40 m, (L/H_{SR}≈26), under concentrating force F=100·10^3 N has end-deflection $y_{S(q)}+y_{S(F)}+y_{T(q)}+y_{T(F)}$ = 0.062025+0.240985+0.000618+0.00172 m = 0.305348 m. The influence of its own mass is 20.5%, and influence of transversal force is 0.76 %. The calculation on bending force done by middle geometric approximation (on the end cross-section) gave bigger deflection for about 60%. At these long girders it is obvious that transversal force can be neglected but not the bending deflection caused by its own mass. Figure 5, according equation (23-24), show the deviation of the calculating girder model in the function of length L (at approximation of its own mass and girder transversal force). Box girders geometry H/B=0.5÷3.0, middle highs H_{SR}=1.0 m, are observed at the some time.

By analyzing the box girder of the high H_{SR} = 1.0 m, area of minimal error caused by applying middle (average) approximation is L/H_{SR} ≈ 8÷12, depending of quotient θ. This is the area of the best approximation, figure 5-b. The influences of contact and local deformation are excluded.

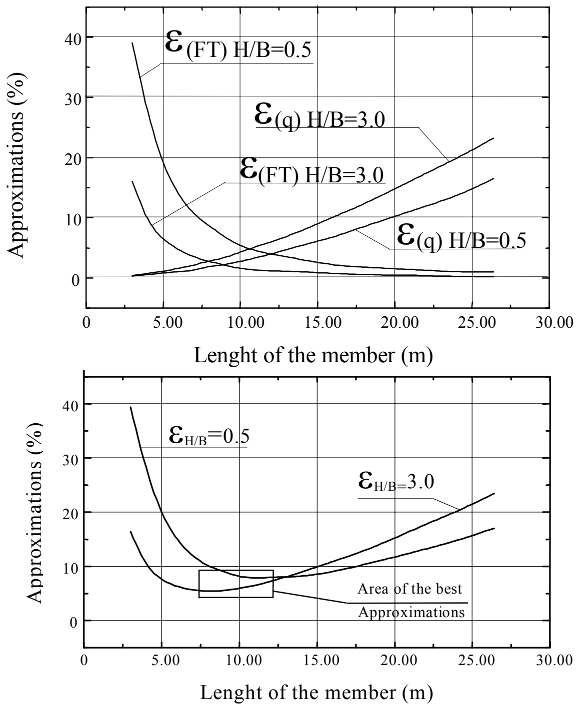


Fig. 5.a,b The relative (percentage) approximations of elastic deformations at different girder length

The proposed procedure can be applied for influence defining of certain approximation of the observed girder and choosing of the best approximation. The given examples show that only geometry on long girders need introducing the influence **of its own mass** in the calculation, but at short box-girders ($L/H < 4$), the approximation is under 1%. On the contrary, the biggest **approximation of transversal force** is made at short girders, small quotient L/H. Approximation done by the middle values of the box-girder geometric cross-section, can provoke unpredictable error, so it is not recommended.

6. THE COMPLEX GIRDER APPROXIMATION OF CHANGABLE GEOMETRY

For choosing the best approximation, complex condition for finding equivalent geometry are important. These are combined the cases when from great number of elastic conditions. Equivalent geometry is defined according to procedure in item 3.0 of this paper. Closed box-girder in the console form, the length L, the linear changeable cross-section, and initial end-geometry $H_1/B_1/t$ and $H_2/B_2/t$, figure 6. Forces at plane cross-section F_X and F_Y at the free end, load the girder. The inertia moment of the cross-section I_x, I_y can be defined according coefficients $R = (B_2 - B_1)/L$ and $T = (H_2 - H_1)/L$. At thin wales ($t < H/25$), the inertia moments can be taken with error lower than 0.4%, according approximation of relations (25):

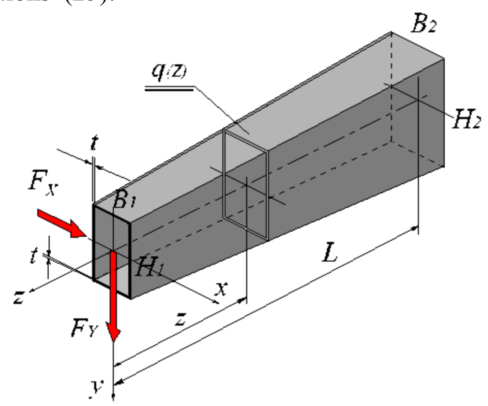


Fig. 6. The box-girder model of changeable cross-section with combined conditions for defining the equivalent geometry

$$I_X \approx 0.5 \cdot B_{(Z)} \cdot H_{(Z)}^2 \cdot t - B_{(Z)} \cdot H_{(Z)} \cdot t^2 + 0.1666 \cdot H_{(Z)}^3 \cdot t$$
$$I_Y \approx 0.5 \cdot H_{(Z)} \cdot B_{(Z)}^2 \cdot t - H_{(Z)} \cdot B_{(Z)} \cdot t^2 + 0.1666 \cdot B_{(Z)}^3 \cdot t - B_{(Z)}^2 \cdot t^2 \quad (25)$$

Using the identical replacements $H_{(z)} = H_1 + R \cdot z$ and $B_{(z)} = B_1 + T \cdot z$, to relations (25), the general inertia moment at arbitrary cross-section z (26) with constants $C_1 \div C_4$, $C_6 \div C_9$ according to the relations (27) and (28) are defined:

$$I_{(X)} \approx C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4$$
$$I_{(Y)} \approx C_6 \cdot z^3 + C_7 \cdot z^2 + C_8 \cdot z + C_9 \quad (26)$$

$$
\begin{aligned}
C_1 &= 0.5 \cdot R^2 \cdot t + 0.1666 \cdot R^3 \cdot t \\
C_2 &= 0.5 \cdot R^2 \cdot B_1 \cdot t + H_1 \cdot R \cdot T \cdot t - R \cdot T \cdot t^2 + 0.5 \cdot R^2 \cdot H_1 \cdot t - R^2 \cdot t^2 \\
C_3 &= H_1 \cdot B_1 \cdot R \cdot t + 0.5 \cdot H_1^2 \cdot T \cdot t - B_1 \cdot R \cdot t^2 - H_1 \cdot T \cdot t^2 + 0.5 \cdot H_1^2 \cdot R \cdot t - 2H_1 \cdot R \cdot t^2 \\
C_4 &= 0.5 \cdot H_1^2 \cdot B_1 \cdot t - H_1 \cdot B_1 \cdot t^2 + 0.1666 \cdot H_1^3 \cdot t - H_1^2 \cdot t^2
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
C_6 &= 0.5 \cdot R^2 \cdot t + 0.1666 \cdot R^3 \cdot t \\
C_7 &= 0.5 \cdot R^2 \cdot H_1 \cdot t + B_1 \cdot R \cdot T \cdot t - R \cdot T \cdot t^2 + 0.5 \cdot R^2 \cdot B_1 \cdot t - R^2 \cdot t^2 \\
C_8 &= H_1 \cdot B_1 \cdot R \cdot t + 0.5 \cdot B_1^2 \cdot T \cdot t - B_1 \cdot R \cdot t^2 - H_1 \cdot T \cdot t^2 + 0.5 \cdot B_1^2 \cdot R \cdot t - 2B_1 \cdot R \cdot t^2 \\
C_9 &= 0.5 \cdot B_1^2 \cdot H_1 \cdot t - H_1 \cdot B_1 \cdot t^2 + 0.1666 \cdot B_1^3 \cdot t - B_1^2 \cdot t^2
\end{aligned}
\tag{28}
$$

The deflection of the end-console (29, 30) produced by bending can be defined by using the method of bending deformational work, because of bending moments M_x, M_y (from concentrated forces F_X and F_Y). The influence of continual loading of its own mass can be neglected. Researches [3,8] have shown that, at highly loaded cross-section, the influence of its own mass on the total deflection is under 0.25 (%). Relations (29-30):

$$
\begin{aligned}
y_S &= \int_0^L \frac{M_{X(Z)}}{E \cdot I_{X(Z)}} \cdot \frac{dM_{X(Z)}}{dy} \cdot dz = \frac{F_Y}{E} \int_0^L \frac{z^2 \cdot dz}{C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + C_4} = \frac{F_Y}{E} \cdot C_5 \\
x_S &= \int_0^L \frac{M_{Y(Z)}}{E \cdot I_{Y(Z)}} \cdot \frac{dM_{Y(Z)}}{dx} \cdot dz = \frac{F_X}{E} \int_0^L \frac{z^2 \cdot dz}{C_6 \cdot z^3 + C_7 \cdot z^2 + C_8 \cdot z + C_9} = \frac{F_Y}{E} \cdot C_{10}
\end{aligned}
\tag{29-30}
$$

In the relation (29 and 30) C_5 and C_{10} are definite integrals at given limits. E is the *Young's* elastic module. Unknown geometry of equivalent girder can be marked with $B^*/H^*/t$. The deflections of equivalent girder (according of the same statical form) can be defined by relations (31) with appropriate inertia moments (32). By equaliying of appropriate real girder's deflection (29-30) and equivalent deflection (31) one can obtain the relation between initial and equivalent geometry in the form of relations (34):

$$
y^*_{(Z=L)} = \frac{F_Y \cdot L^3}{3 \cdot E \cdot I_X^*}, \qquad x^*_{(Z=L)} = \frac{F_X \cdot L^3}{3 \cdot E \cdot I_Y^*}, \tag{31}
$$

$$
\begin{aligned}
I_X^* &= 0.5 \cdot B^* \cdot H^{*2} \cdot t - B^* \cdot H^* \cdot t^2 + 0.1666 \cdot H^{*3} \cdot t - H^{*2} \cdot t^2, \\
I_Y^* &= 0.5 \cdot H^* \cdot B^{*2} \cdot t - B^* \cdot H^* \cdot t^2 + 0.1666 \cdot B^{*3} \cdot t - B^{*2} \cdot t^2
\end{aligned}
\tag{32}
$$

$$
x^*_{(Z=L)} = x_M, \qquad y^*_{(Z=L)} = y_M \tag{33}
$$

$$
\frac{F_X}{E} \cdot C_{10} = \frac{F_X \cdot L^3}{3 \cdot E \cdot I_Y^*}, \qquad \frac{F_Y}{E} \cdot C_5 = \frac{F_Y \cdot L^3}{3 \cdot E \cdot I_X^*}, \tag{34}
$$

The obtained relations (34) are nonlinear functions of unknown values $\boldsymbol{B}^*$ and $\boldsymbol{H}^*$. They can be solved not only numerically *Newton-Raphson's* method [1], but also by reducing the system to only one equation and its tabulation. The defined concept of changeable box-girder with equivalent box-girder, can be valuated and observed at few examples from the table:

Table 1.

EXAMPLE	H_1/H_2 (m/m)	B_1/B_2 (m/m)	L/t (m/m)	H^*/B^* (m/m)
6.	0.50/ 1.00	0.50/ 1.50	4.00/ 0.006	0.9245 / 0.9185
7.	0.45/ 0.90	0.65/ 1.40	4.00/ 0.006	0.8015 / 1.0240
8.	0.40/ 0.80	0.40/ 0.40.	0.80/ 0.008	0.6053 / 0.5972
9.	1.20/ 1.80	2.00/ 3.00	4.00/ 0.016	1.6780 / 2.5250
10.	0.60/ 1.20	1.00/ 2.00	8.00/ 0.012	1.0640 / 1.5070
11.	1.00/ 1.50	0.50/ 0.50	2.50/ 0.010	1.2170 / 0.7885

In these examples two conditions for equations of elastic deflections are used. These concept are appropriate for searching of equivalent cross-sections of prismatic and box-girder cross-section. One elastic condition is appropriate at light-construction analysis, at truss of a small, usually symmetric cross-section. Three elastic conditions can be used for thin-wall box girders, where basic geometry ***B*** and ***H*** is looked for as well as equivalent thickness of the wall ***t***. Overview of the basic combinations of conditions for defining of equivalent geometry is given in Figure 7:

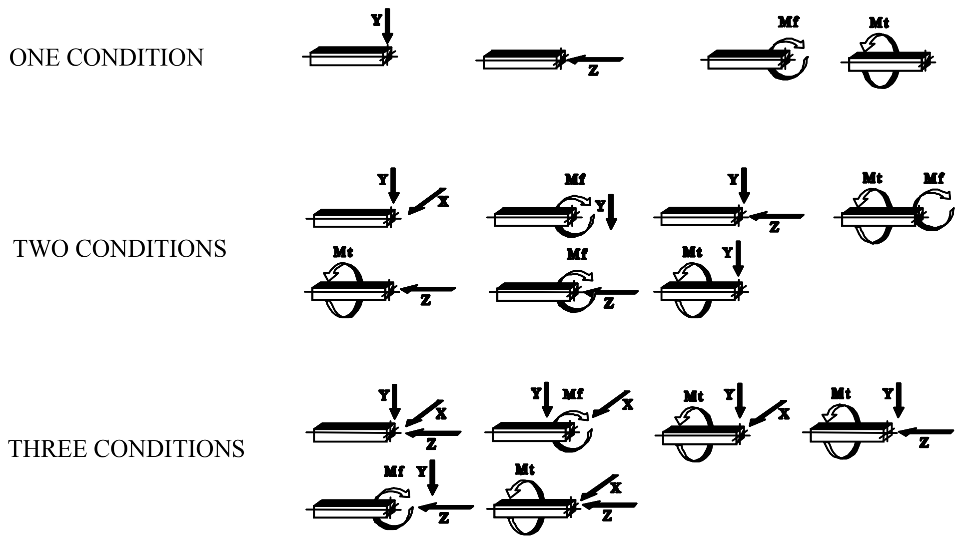


Fig. 7. The basic types of elastic condition for searching of equivalent girders.

7. CONCLUSION

The defined concept can be applied for analytical definition of deformations of linear changeable cross-section of opened and closed box-girders. According to these models the qualities of certain approximations of real girders can be checked. The choice of approximation of continual girder by the discrete form girder, supporting structures can be modeled very effectively and qualitatively. That leads to simple mathematical structure model. The final effect of approximation on continual girders should give simple models for structural analysis. All calculations that use approximative box-girders models will be simple and smaller. This procedure can be effectively realized by using the "small" computers. The researches can be directed towards generalization according to different cross-section and combinations of different loading. It is also interesting to find the equivalent geometry conditions at dynamical tasks. This procedure is recommended for researching different approximation categories and degree of complexities.

REFERENCES

1. Mendjaš, I., Milutinović, P., Ignjatović, D., *100-the best useful fortran's program*, Micro book, Beograd,1991,
2. Sekulović, M., *The Matrix Construction Analysis*, Civil book, Beograd 1991,
3. Jovanović, M, *Equivalent support as a basis for rational Structural Analysis of Railway carriages*, Scientific-professional Railway meeting, Niš 1992, *Železnice* br.10/92,
4. Jovanović, M., *Compound Approximations of the Variable Geometry Box-Girder*, 12. International Conference for material handling in industry, Mechanical faculty Beograd 1992,
5. Dedijer, S., Petković, Z., *The Approximations degree Equivalent dynamical Models of Supporting structures Transportation's and Mine's machines*, Symposium for Transportation machines, SMIETS Beograd 1988,
6. Rašković, D., *Strenght of materials*, Civil book, Beograd 1967,
7. Jovanović, M., *Theoretical basis of the computer-aided structural design*, Mechanical faculty University of Niš, 1994,
8. Jovanović, M., *The supporting structures for Level-change and mechanism's Resistances at Sleeving-jib cranes Optimisation*, - Dissertation, Mechanical faculty University of Niš, 1989,
9. Jovanović, M., Brkljač, N., *The Approximative models of the line's Structures and their accuracy*, The first International Conference for weight machine-building, TM'93, Mechanical faculty Kraljevo, Vrnjačka Banja 1993,
10. Jovanović, M., *The choice of approximate geometrical models of crane supporting structures*, XXI YU Congress of Theoretical and Applied mechanic - JUMEH'95, Mechanical faculty and Civil faculty University of Niš, 1995.

APROKSIMATIVNI MODELI KUTIJASTIH NOSAČA U STRUKTURNOJ ANALIZI

Miomir Jovanović

Rad se bavi istraživanjima analitičkog modela određivanja elastičnih deformacija kutijastih elemenata nosećih struktura mašina. Gredni elementi promenljive geometrije poprečnog preseka po dužini, su razlog redovnih aproksimacija u računskim modelima. Uvođenje aproksimacija srednjih preseka vodi odstupanju tačnih i matematičkih modela. U radu je predložen računski model za određivanje elastičnih deformacija kutijastih nosača promenljive geometrije poprečnog

preseka. Za osnovni nosač, tipičan za transportne mašine i teška vozila, razvijena je procedura definisanja geometrije ekvivalentnog modela na bazi najboljih (najmanjih) aproksimacija. Uvedene aproksimacije na ovaj način pojednostavljuju strukturnu analizu i uvode kontrolisane elastične deformacije u linearan analitički model.