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METHOD OF SYNTHESIS OF CARDANIC MOTION

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Abstract. *This paper deals with the practical method for synthesis of fourbar mechanisms performing approximately cardanic motion. These mechanisms may be applied for the purpose of the straight line motion direction change. In practice, they can be used for both machine and devices design in Mechatronics. Beside the above said, the results of analysis of the mechanisms designed following the proposed method. Due to its simplicity, the proposed method may be interesting for the design practice, are presented.*

Key words: *cardanivc motion, synthesis of mechanisms*

1. INTRODUCTION

Within the design practice introduced at the Institute for Machine Mechanics of Belgrade University, it is apparent that various machines show the need for linear motion to be transformed into another.

That problem is not the new one. Numerous outstanding scientists worked on it, but the reached solutions had major flaws. Thus, the new interesting matter for discussions appeared. The major flaw noticed in these solutions is the necessity of the sliding blocks to be applied.

The discovery of the family of plane mechanisms with turning joints, which only two coupler points generate approximate straight line motion along non-parallel paths, became a significant problem for the research which author started in 1978.

The firs attempt toward the design of a mechanism which could be a member of the mentioned family [1] gave poor results. There appeared the need for more extensive study regarding this problem.

Soon, it was noticed that a plane, which two points move along linear non-parallel paths, performs cardanic motion [2]. Considerable attention was paid by Rauh [3] to the

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study of cardanic motion and research of approximate cardanic motion of the coupler of fourbar [4]. Other than Bottema and Freudenstein, the problem of instantaneous cardan position was not solved in the period of further 20 years [5].

Starting from their idea that cardanic motion can be achieved in four infinitely close positions, a hypothesis is set stating that such motion is also possible in four finitely close positions. This hypothesis proved to be true and achieved, by synthesis, the family of fourbar mechanisms with approximate cardanic motion in a set interval. At the same time, Prof. Freudenstein published his results regarding research on Ball Point [6], in which, as a side-effect, he discovered the possibility to synthesize cardanic motion in four finitely separated positions. A period of 20 years was necessary for problems of cardanic motion of coupler of fourbars to be solved. After the detailed research of this problem, and among the large number of achieved results [7], bearing in mind the limited scope of this paper, presented is just a small section interesting for practice.

2. Foundation of Method

Within the study of the behaviour of Burmestwer's curves in cardanic motion of coupler plane of fourbar mechanisms [7], determined is that the curves of the center point often fall apart on two perpendicular lines. At the same time, the circle point curves always have one branch covered with first design position of the plane. Beside this, concentration point (Fig. 1) is always in the vicinity of the cross-section between the center point curve and circle points curve.

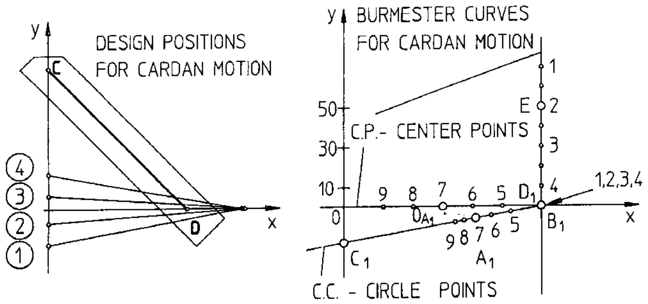


Fig. 1. Symmetric design positions and Burmester curves for cardanic motion.

After the discovery of these facts, there appeared logical need for detailed research regarding formation and fundamental concepts of the plane's cardanic motion. Cardanic motion is defined as the motion reached when a circle revolves within another circle (Fig. 2) twice its size. In such a case, all points of the moving plane in which the small circle is lied (for example G, F, H, E) trace ellipses which center is the same as the one of the stationary circle; in some cases, the points along the perimeter of the smaller circle (C, D, and random S) - also called the Cardan circle - move along straight lines of the same length as the diameter of the larger circle. The center of the Cardan circle A obviously makes a circular path. This concept is the discovery of G. Cardano (1501-1576), thanks to whom the elliptical motion of a plane is also called cardanic motion.

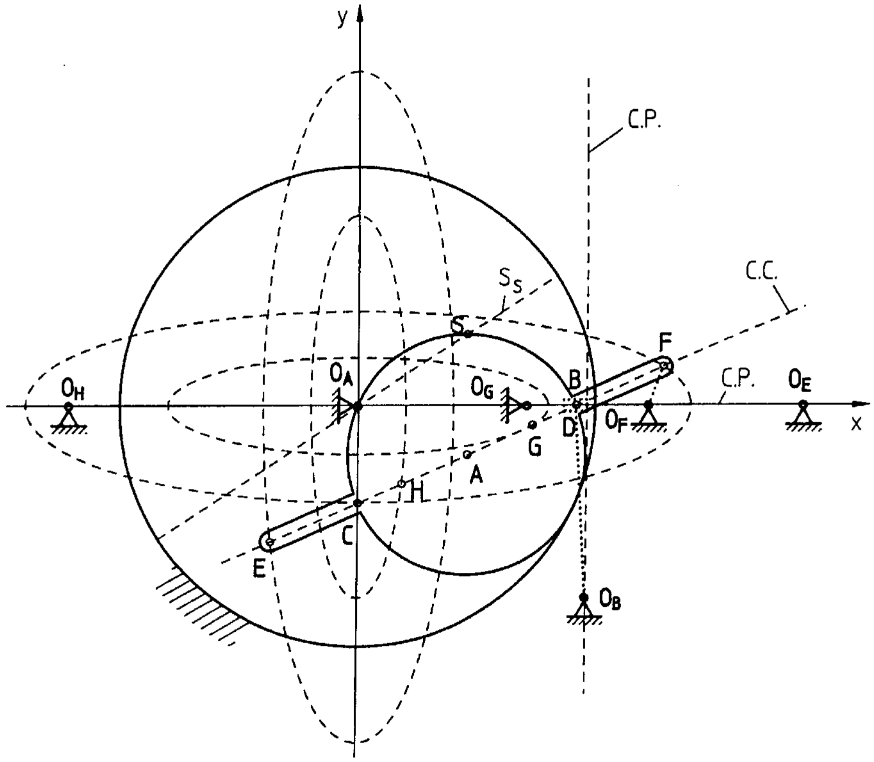


Fig. 2. Cardanic motion and circles.

On the basis of this theory, at the end of the 18th century, designed were three-member planetary gear mechanisms, guiding a point along a straight line path. Bearing in mind the above explained concept, using the straight line path of point B of the Cardan circle in which a slider was put, and the circular paths of point A, which motion was realized by the crank O_AA, V. Freemantle [8], an English clockmaker, has, even before 1803, designed the isosceles slider crank mechanism.

The plane of the coupler of this mechanism performs the exact cardanic motion, being guided over the turning joints in points A and B, providing paths identical to the ones these points have during elliptical motion. This results from the well-known fact that the motion of a rigid body is defined by the motion of two points of the points B and S of the Cardan circle and with a slider placed in them, acquired the mechanism with the elliptical motion of the coupler. The detailed description of these mechanisms may be found in the reference [9] under numbers 1542 and 1540. Regarding the mentioned mechanisms for guiding the plane of the Cardan circle, used were the perimeter of the circle, points on the perimeter and the center point circle A, but not any other point with an elliptical path was used in the motion. According to Rauh [3], the use of the path of other points within the Cardan circle was not considered until the 1930's. However, the elliptical paths of those other points, especially their convenient parts, such as the circles of the curves by vertex can be used in the design of the mechanism for the straight line motion transformation.

The evolution of the mechanisms in which, together with the exact path of point B, another point with a similar path which it would have in elliptical motion, was used for the cardanic motion generation, described is in detail in [3]. The use of the paths of points F and G (Fig. 2), and by installation of the turning joints of the crank, slidecrank mechanisms with approximate cardanic motion were obtained. The description of these mechanisms may be found in [9] under numbers 1544 and 1543. With the use of the paths of points E and H to guide the planes of the Cardan circle, oscillating slider mechanisms can be obtained, the same which is described in [9] numbers 1537 and 1538. The mentioned mechanisms, in which points F, G, E and H were used for guiding, were designed to obtain the approximate straight line path of point C. Professor A.E.R. de Jong (1883-1970) reached the conclusion that for generating straight line paths, other points along the perimeter of the Cardan circle may be used, such as the case with randomly chosen point S (Fig. 2). Based on the mentioned mechanisms, de Jong designed a family of straight line mechanisms described in [9] under numbers 1550, 1551, and 1552. It is characteristic that in all the mentioned mechanisms with approximate cardanic motion, the path point B would have in exact cardanic motion was used. Therefore, there appeared structural error caused by the deviation of the circular paths of points G, F, H and E over which guided were planes of the couplers of mentioned mechanisms, from the elliptical paths these points would have in the exact cardanic motion.

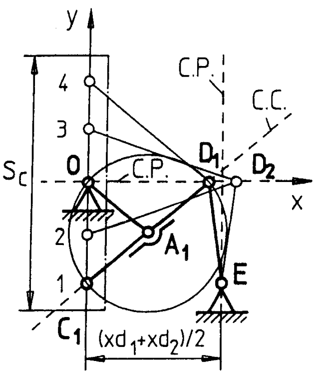
Based on the detailed research Rauh [3] showed that it is not possible to design a fourbar mechanism in which, for the purpose to guide the coupler, one might use those points on the plane of the Cardan circle which simultaneously trace those parts of the ellipses which could be satisfactory approximated with circles. That means that the circular and elliptical paths cross in four points. This fact is obvious in (Fig. 2). Making any combination between the two among points A, F, G, H and E, we may try to make a fourbar mechanism. It is obvious that, at the same time, guiding points of the coupler pass through the vertex of the ellipse. At such an instant, the mechanism takes the change-point position, because the axes of all its members coincide.

Based on series of synthesis of fourbar mechanisms for different sets of Cardan positions, measuring the behaviour of the Burmester curves of the same, as well as bearing in mind facts obtained by Rauh, the following idea is presented.

For the approximate cardanic motion of the plane of the fourbar mechanisms, one should use the exact path of point A, and approximate the path of point B. Since the path at point B in cardanic motion is a line, the problem is the approximation of the straight-line path of point B with circular path. The best approximation of the path of point B along a circular path is possible in a symmetrical interval of the motion of the coupler when the circular path of point B of the fourbar mechanism crosses the line four times and thus generates the smallest error.

3. THE PROPOSED METHOD

Since the each new set of design positions requires the search for Burmester's curves, we tried to simplify the synthesis for the symmetrical set of planes positions and the position of the fixed bearing of the crank in the origin. It is obvious that this simplification will allow a certain error in the design. We assume that in practice this simplification may show satisfying results. The simplification is as follows: even though the results of the numeric synthesis do not prove it accurately enough, in the symmetrical setup of the plane, point D can be taken as a point of the circle and together with point C it defines the design position (Fig. 1). This assumption brings us to a simple expression to define the position of the other branch of the center point curve (Fig. 3) and the circle point curve. Any other center point on the other branch corresponds to the circle point D_1. Even though this simplification seemed inaccurate from the very start, excellent results were achieved, based on which it was decided that the method should be extended for designing mechanisms with different dimensions of the crank O_AA.



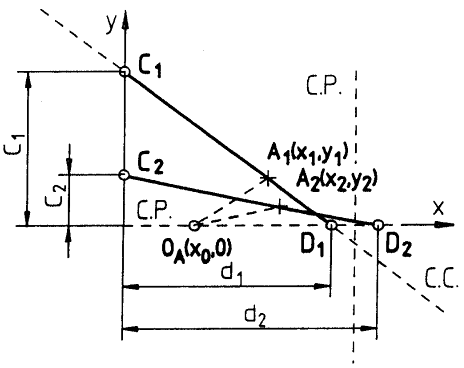


Fig. 3. Design position of mechanism. Fig. 4. Burmester's curves for symmetric positions.

Fig. 3 and 4 show the approximate curves of the center C.P. and the circle point C.C. Based on the suggested idea on the simplification of the synthesis, there are two design positions of the coupler plane, defined by the coordinated of points C and D. Compared to the x-axis, the third and fourth position of the guiding plane are symmetrical.

3.1. Analytics of the Method

Equations of the center points curve C.P. are:

$$y = 0 \quad \text{and} \quad y = (d_1 + d_2)/2 \tag{1}$$

The equation of the circle points curve C.C. is:

$$y = c_1 \cdot x/d_1 + c_1 \tag{2}$$

In order to set required coordinates of the points of the circle $A_1(x_1,y_1)$, and for a randomly chosen position of the fixed bearing O_A $(x_0 0)$ we set up a relation between the center of rotation and the pinpoint of joint A in the first and second position. Since that distance is the length of crank O_AA, it must be the same in all positions. According to (Fig. 4), the equation of that relation is:

$$(x_1 - x_0)^2 + y_1^2 = (x_2 - x_0)^2 + y_2^2 \tag{3}$$

Regarding the relations:

$$y_2 = y_1 \cdot c_2 / c_1 = (d_1 - x_1) \cdot c_2/d_1 \qquad (4)$$
$$x_2 = x_1 \cdot d_2 / d_1$$

and their exchange in (3) we come to a simple formula for determining the points of the circle for the desired position of the bearing O_A.

$$x_1 = \frac{d_1}{2 \cdot \left[\frac{(d_2 - d_1) \cdot x_0}{(c_2^2 - c_1^2)} + 1 \right]} \qquad (5)$$
$$y_1 = (d_1 - x_1) \cdot c_1 / d_1$$

Fig. 4 shows the constants in the above equations.

3.2. Procedure of the Synthesis

1. Based on the necessary length of motion for the design of interesting points of the Cardan circle we can determine the length of the path of coupler point C and the diameter of the Cardan circle CD.
2. Using Chebishev's spacing for points of approximation in the interval Sc we choose the design positions C_1D_1 and C_2D_2 (Fig. 4).
3. Based on (1) we can determine x_B, while for the ordinate y_B we take the largest possible one based on the design demands (Fig. 3).
4. The coordinates of joint D are $(d_1,0)$.
5. Let's choose the position of bearing O_A $(x_0,0)$ closest to point O in accordance with demands for the design.
6. Based on the formula (5) we may calculate the coordinates of the turning joint $A(x_1,y_1)$ in the first position. In such a way, we obtain completely defined mechanism and we may make further analysis in order to test the quality of approximation of cardanic motion.

3.3. Results

Fig. 5 shows the results derived from the presented method for 3 designed mechanisms. The length of the paths of 8 chosen points of the Cardan circle were symbolically represented by bars. The first number represents the length of the path in [mm] while the other represents the maximum absolute error of the corresponding point of linear path.

Symbol I gives the error of the mechanism with symmetrically arranged rocker bearing in O_{BI}. II is for the error of the mechanism with the crank bearing O_A moved into O and with the rocker bearing in O_{BI}.

The differences between the deviation of the path of point C of mechanisms given in (Fig. 5) under II and Burmester's synthesis of the acquired mechanism for the same design positions of approximately the same dimensions is in the sixth figure after the decimal point.

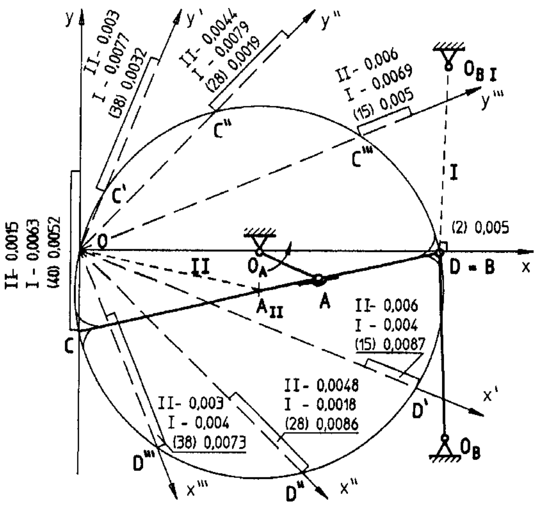


Fig. 5. Maximum absolute errors of 3 Evans type mechanisms.

Shortening the crank O_AA makes the suggested method less accurate. Mechanism given under I has 0.002 [mm] larger absolute error than the mechanism of similar dimensions designed by the Burmester's method.

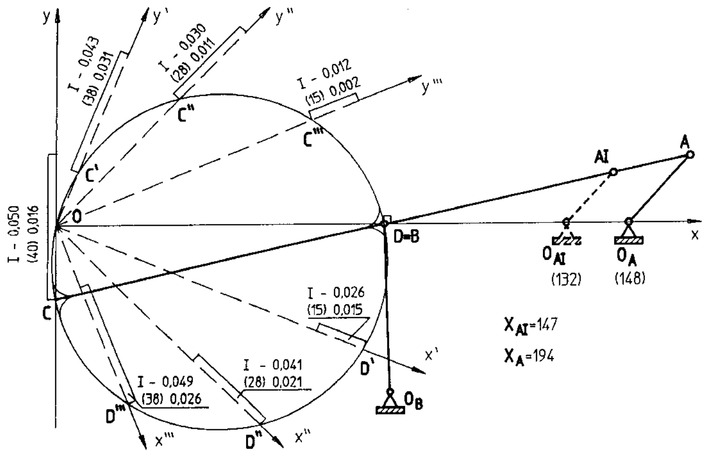


Fig. 6. Maximum absolute errors of 2 mechanisms with crank bearings outside of Cardan circle.

For the design reasons, it is often necessary to place the fixed bearing of the crank outside the perimeter of the Cardan circle. In (Fig. 6) there are two designed mechanisms and the maximum absolute errors of the paths of the guided points of the lines. The mechanism O_AABO_B has the same distance between the fixed bearing as the mechanism

O_AABO_B (Fig. 5), while mechanism $O_{AI}AIBO_B$ has the same length of the crank as the mentioned mechanism in (Fig. 5). Such dimensions of the mechanism are chosen for comparison with the previously designed one. It is noticed that errors from exact cardanic motion of the plane of the coupler in mechanisms in (Fig. 6) are bigger that those in mechanism O_AABO_B in (Fig. 5). To enable comparison, in all five mechanisms the same design positions of the guided plane were used. However, the analysis of the curve of structural errors of the mechanisms in (Fig. 6) showed that the change of the design positions of the plane could significantly change the maximum errors.

The results show the proposed method is useful in designing the mechanisms with symmetrical design Cardan positions for a wide range of positions of the fixed bearing of the crank. This expands the possibility for the synthesis of the adequate design solution.

4. CONCLUSION

Based on the comparative analysis of the quality of approximation of cardanic motion between the mechanisms acquired by numeric synthesis based on Burmester's theory and mechanisms designed by the suggested simplified method for symmetrical design positions, we believe that in a series of practical situations the simplified method is useful for the design practice.

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METOD SINTEZE KARDANSKOG KRETANJA

Aleksandar Sekulić

U ovom radu je izložen jedan praktičan metod sinteze četvoropolužnog mehanizma koji obavlja približno kardansko kretanje. Ovi mehanizmi se primenjuju za promenu smera pri pravolinijskom kretanju. Koriste se pri projektovanju mašina i uređaja u mehatronici. Prikazana je i analiza mehanizama konstruisanih prema navedenoj metodi. Zbog svoje jednostavnosti metod je interesantan i za inženjersku praksu pri projektovanju uređaja i mašina.