AN EXPERIMENTAL METHOD FOR DETERMINATION OF NATURAL CIRCULAR FREQUENCY OF HELICAL TORSIONAL SPRINGS

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Nenad T. Pavlović, Nenad D. Pavlović

Faculty of Mechanical Engineering, University of Niš
Beogradska 14, 18000 Niš, Yugoslavia

Abstract. Helical torsional springs are mostly used as driving elements for rotational discontinuous motion of elements or subassemblies. One spring end is connected to the frame whilst the other one to the element to be moved. A helical torsional spring has been fully described with five parameters: the middle diameter of the spring, the diameter of the spring wire, the number of the spring coils, the natural circular frequency and the initial angular displacement. In order to design a helical torsion spring, it can be modeled as a space curved beam being stressed by flexion or as an elastic hollow cylinder being stressed by torsion. The results obtained using these models can be verified experimentally. In this paper an experimental method for determination of natural circular frequency of helical torsional springs has been presented. Also the experimental results have been compared with the results obtained using the theoretical models.

Key words: experiment, natural circular frequency, helical torsional springs

1. INTRODUCTION

Helical torsional springs (Fig. 1) are mostly used as driving elements for realization of rotational discontinuous motion in various elements or subassemblies. They are used in the design of measurement equipment, sport equipment, toys, clocks and electrical devices.

One spring end is connected to the frame whilst the other one to the element to be moved. The fixed spring end can be rigidly clamped (the spring 1 in Fig. 1), joint connected (the spring 2 in Fig. 1) or simply supported. The spring legs can be run out in the tangential (Fig. 2a), radial (Fig. 2b) or axial direction (Fig. 2c).
A helical torsional spring as a driving element is fully specified with five parameters: the middle diameter of the spring, the diameter of the spring wire, the number of the spring coils, the natural circular frequency and the starting angular displacement. The parameters of spring are determined for a given design task, e.g. for the given parameters of the motion and the load, technological requests, geometrical and constructive requests.

In order to design a helical torsion spring, it can be modeled as a space curved beam being stressed by flexion or as an elastic hollow cylinder being stressed by torsion. Greater accuracy is achieved using the model of space curved beam being stressed by flexion, which also considers the influence of the spring type (the legs can be run out in tangential, radial or axial directions) and the way of the spring connections with the frame and the system to be moved (joint connection, simply supporting or rigidly clamping). This procedure offers in addition a possibility to include influence of the supporting thorn.

The state of an element of space curved beam can be fully described by the system of 12 first order linear differential equations. This system can be reduced to 6 first order linear differential equations by introducing some simplifications. In order to simplify the mathematical model, the beam is divided into $n$ segments where the masses of each
segment are concentrated at its ends. As the number of segments increases the error becomes smaller. In such case a matrix method has to be used to calculate natural frequencies. There is no dimensional equation in closed form for this computational model.

Helical torsional springs as driving elements can be designed using another theoretical model. This model represents the spring as an elastic hollow cylinder being stressed by torsion with equal dynamic characteristics as the spring, i.e. the same spring constant, length and moment of inertia per axis unit length. The second order differential equations that describe such a system can be solved explicitly.

The model of hollow cylinder stressed by torsion offers several different design solutions for a given task. However, applicability of such model is somewhat limited because it doesn't consider the effect of different connections of the spring legs. According to this model the cylinder can only be rigidly clamped.

2. NATURAL CIRCULAR FREQUENCY MEASUREMENTS OF HELICAL TORSIONAL SPRINGS

The results obtained using these models can be verified experimentally. For this purpose, a device for measuring the time that the element driven by a helical torsional spring needs to come back from initial angular displacement to its equilibrium position has been constructed.

The measurements have been performed using springs with tangential and radial legs. The springs had different diameters of the wire, middle diameters of the spring and coil numbers.

The measuring device is shown on the Fig 3. It consists of: electrical motor (1); triggering mechanism (2); rotating disc (3); helical torsional spring (4); thorn as a guiding element of the spring (5); optocoupler (6) and electronic circuit (7).

The triggering mechanism (2) has been constructed as a step mechanism and was driven by the electrical motor (1).

Springs have been tensed while the eccentric lever (2A) has been seized by the leg of the helical torsional spring. After running out of the grasp, the spring (4) comes back to its equilibrium position.

The repetition of triggering of the spring and independence from the external influences have been provided due to the triggering mechanism.

Optocoupler (6) together with electronic circuit (7) was used to generate electrical impulses.

To capture spring's motion, a rotational disk with holes equally spaced along its circumference was used. The light that illuminates the optocoupler when each hole passes by, generates the voltage of \(0V\). When the optocoupler is not illuminated, the output voltage is \(10V\).

Instead of an optoelectronic transducer, we could have used a Hall’s transducer (conversion of magnetic field into voltage) or transducer working on the principle of making electrical connection. We have chosen an optoelectronic transducer because of its reliability and fast response time which enabled acceptable time resolution of spring’s motion.

The electronic circuit (7) was powered by a \(12V\) DC source.
Analog signal from the optocoupler has been digitized and then acquired by a PC computer. A special computer program has been written which enabled sampling of 10000 impulses per second, e.g. the sampling frequency of 10kHz.

Fig. 3. The device for displacement measurements of the element driven by a helical torsional spring.

Helical torsional springs with radial and tangential legs have been measured. The fixing spring end was connected with the frame in three different ways: rigidly clamped (Fig. 4a), joint connected (Fig. 4b) or simply supported (Fig. 4c). The movable spring end was always simply supported on the rotating disc.
Fig. 4. A constructive solution of the different types of connection between the fixed spring end and the frame.

Output of the computer program was a file with numerical values of the voltage. The files are presented as graphs for easier understanding (Fig. 5). The numerical values of time have been read from the measurement diagram to get the motion diagram (Fig. 6).

A typical result of a measurement is shown on Figure 5. The spring’s parameters are given in the upper right corner of the figure: number of spring coils \( n = 8 \), diameter of the spring wire \( d = 2\text{mm} \), middle diameter of the spring \( D_m = 22\text{mm} \), length of the rigidly clamped leg \( l_1 = 2\text{mm} \) and length of the movable leg \( l_2 = 23\text{mm} \).
A motion diagram of an element driven by a helical torsion spring is shown on Fig. 6. Parameters of the spring used were: number of the spring coils $n = 4$, diameter of the spring wire $d = 1.7\text{mm}$, middle diameter of the spring $D_m = 18\text{mm}$, length of the rigidly clamped tangential leg $l_1 = 2\text{mm}$ and length of the movable tangential leg $l_2 = 28\text{mm}$.

Fig. 5. Time dependence of the voltage for a typical measurement.

Fig. 6. Time dependence of the angular motion for an element driven by a helical torsional spring.
It is obvious that there is a difference between the theoretical and experimental results. The difference is caused by friction appearing between the moving element and the spring, the spring and the frame, the spring and the guiding element, the neighboring spring coils and the inner friction of the spring material.

The friction losses are greater when the fixed spring leg is rigidly clamped than when the fixed spring leg is jointly connected or simply supported.

It was observed that difference between theoretical and experimental results increases as the ratio between angular displacement and initial angular displacement, $\phi_k/\phi_p$, becomes larger.

The difference is however smaller if the mass and moment of inertia of the helical torsion spring are smaller, because the losses caused by friction are smaller.

### 3. CONCLUSION

In this paper an experimental method for determination of natural circular frequencies of helical torsional springs has been presented. Also experimental results have been compared with results obtained from theoretical models. It has been found that there is a difference between these results, probably due to friction in the system.

The main disadvantage of these theoretical models is in the fact that they do not consider the influence of friction. That is the cause of difference between theoretical and experimental values for natural circular frequency of helical torsional springs.

The differences are smaller if springs with smaller mass and moment of inertia are used and if the angular displacement is considerably smaller than initial angular displacement, which appears very often in practical use. Therefore, the declared theoretical models give results very similar to experimental ones only if the springs with guaranteed values of stretching strength and stretch modulus are used and if angular displacement is considerably smaller than initial angular displacement.

### REFERENCES

POSTUPAK EKSPERIMENTALNOG ODREĐIVANJA SOPSTVENE KRUŽNE FREKVENCIJE UVRTNIH ZAVOJNIH OPRUGA

Nenad T. Pavlović, Nenad D. Pavlović