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DISCONTINUOUS DRYER DYNAMICS

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Abstract. *On the basis of really accepted and critically justified assumptions, a non-linear as well as linearized mathematical model of discontinuous vacuum dryer has been developed. Model is in the form of partial differential equations with time-varying coefficients. Using the appropriate numerical methods an adequate simulation has been done. Derived results show a quite well agreement with the real dryers dynamical behavior.*

Key words: *dryers, mathematical modeling, simulation, dynamic analysis*

1. INTRODUCTION

Nowadays drying systems construction is connected with many requirements concerning final product quality, which is very important particularly in the pharmaceutical and food industry. In order to fulfill all these demands, a correct mathematical models of drying process have to be developed. Their aim is to recognize the process features, as well as characteristics, and to help us to construct a suitable control unit system, which will be able to lead the process dynamics, in a satisfactory manner.

2. PROCESS DESCRIPTION

Vacuum drying plant is shown in Fig. 1. Material to be dried is placed on pans, which are warmed-up by vapor, flowing through the shelves. The main advantage of this type of dryers is that a thermolabile and easily oxidable materials can be dried.

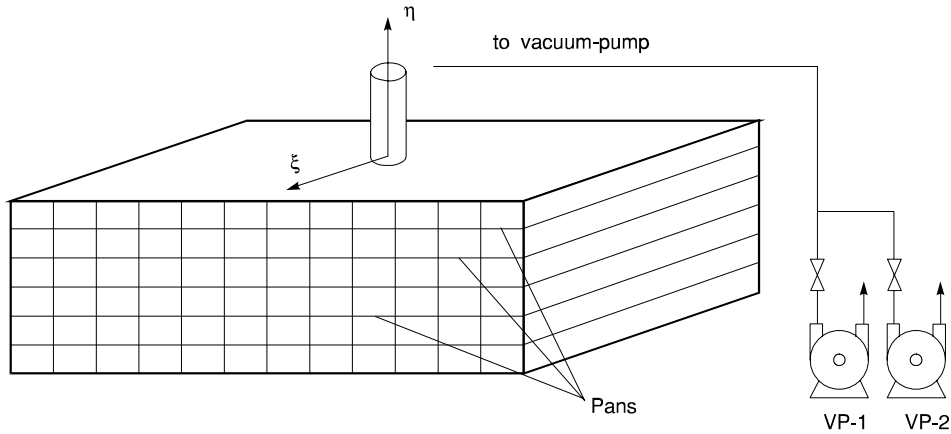


Fig. 1. Drying plant

Wet material (textile paint) is carried in the vacuum chamber, and placed on 72 pans. The pans are made of low-carbon steel, in order to obtain good heat transfer coefficient. Pans dimensions are: $1200 \times 2400 \times 55$ mm. Since they are subjected to aggressive surrounding, and therefore disposed to rust, they must be cleaned short after every drying.

The pans are stacked on shelves, which represent heat exchangers, and are supplied with overheated steam, $\theta_F = 110$ [°C], $p_F = 1,471 \cdot 10^5$ [Pa]. The shelves are made of cast iron, since they are more resistant to corrosion than steel. During the process, 1600 [kg] of paint, which consists of 60 % water on wet basis, has to be dried. At the end, as declared by technological requirements, 1 % water can remain in the product.

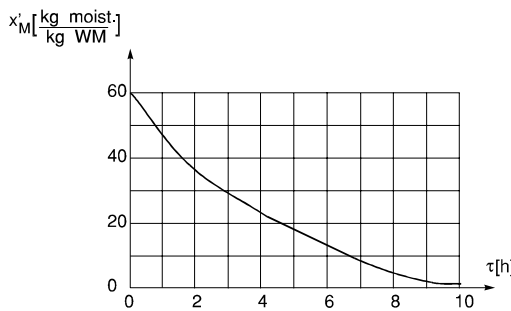
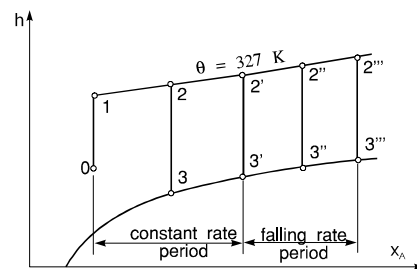


Fig. 2. Drying curve.

Fig. 3. h - x_A diagram

According to drying curve, Fig. 2., which has been acquired by experimental results,

drying period is about 10 hours long. Warming-up period lasts 1 hour, constant-rate period lasts 1,5 hour, and falling-rate period takes 7,5 hours. One hour more has to be evaluated for discharging, after drying, and refilling of the chamber, afterwards, with a wet material. Dry paint bulk density is about 1625 [kg/m³]. Working pressure, within the vacuum chamber is 0,15·10⁵ [Pa]. Pressure, inside the chamber is achieved, before warming the material up, using VP-1 vacuum-pump. Process, which is occurring inside the drying chamber, is represented using the h vs. x_A diagram, shown on Fig. 3. Material is warmed-up, from initial 15 [°C] to final 54 [°C] (point 1). On this temperature, under 0,15·10⁵ [Pa] pressure, water begin to boil inside the material. Using the VP-2 vacuum-pump, pressure remains constant in the vacuum chamber, so it can be represented by straight line 1-2. Air, within the chamber, is saturated by steam, which evaporate from the material. Process variables are summary discussed in [2].

3. MATHEMATICAL MODEL OF DISCONTINUOUS VACUUM DRYER

In order to form a vacuum shelf dryer model, the process balance equations method, has been used. Drying result is represented by four independant process variables, so it is necessary to form an adequate number of equations for drying process dynamics description. Before that, a few necessary assumption, founded on the real basis, will be accepted.

1. *Assumption:* Heating fluid temperature and pressure are constant during the drying process.
2. *Assumption:* Heating fluid flow is one-dimensional.
3. *Assumption:* Piping temperature field is one-dimensional.
4. *Assumption:* Air, inside the drying chamber acts like ideal gas.
5. *Assumption:* Values c_{DA} , c_{DM} , r^0 , r_a , r_{DA} , r_{DM} , λ_{DM} , can be considered constant during the drying process.
6. *Assumption:* Pressure, within the chamber is constant, and equal to 0,15·10⁵ [Pa].
7. *Assumption:* Heat transfer coefficient from heating plate to moisture, and from vapor to environment are constant during the drying process.

All the presented assumptions, as well as those that will be used further on, are detaily clarrified and verificated in [2].

On the basis of accepted model, balance equations can be written, in the following manner:

Material balance equation, for moisture content within the air inside the vacuum chamber:

$$\begin{aligned} \frac{\partial(x_A \cdot \rho_{DA} \cdot l \cdot d\xi \cdot d\eta)}{\partial t} = & x_A \cdot \rho_{DA} \cdot w_{DA} \cdot d\xi \cdot d\eta + \kappa_l \cdot \rho_{DM} \cdot l \cdot d\xi \cdot d\eta - \\ & - \rho_{DA} \cdot (x_A + \frac{\partial x_A}{\partial \eta} \cdot d\eta) \cdot (w_{DA} + \frac{\partial w_{DA}}{\partial \eta} \cdot d\eta) \cdot x_A \cdot l \cdot d\xi, \end{aligned} \quad (1)$$

where are: $x_A = x_A(t, \eta)$, $w_{DA} = w_{DA}(t, \eta)$.

Energy balance equation, for moisture content within the air inside the drying chamber:

$$\begin{aligned}
& \frac{\partial[(c_{DA} + x_A \cdot c_W) \cdot \theta_{DA} + x_A \cdot r^0] \cdot \rho_{DA} \cdot l \cdot d\xi \cdot d\eta}{\partial t} = \\
& = (c_{DA} \cdot \rho_{DA} \cdot \theta_{DA} + x_A \cdot c_W \cdot \rho_{DA} \cdot \theta_{DA} - x_A \cdot r^0 \cdot \rho_{DA}) \cdot w_{DA} \cdot l \cdot d\xi + \\
& + \rho_{DM} \cdot \kappa_1 \cdot [r_m + c_{DA} \cdot (\theta_{DM} - \theta_{DA})] \cdot l \cdot d\xi \cdot d\eta + \\
& + K_{FE} \cdot A_{FE} \cdot (\theta_F - \theta_{DA}) \cdot l \cdot d\xi - K_{AE} \cdot A_{AE} \cdot (\theta_{DA} - \theta_E) \cdot l \cdot d\xi \cdot d\eta + \\
& + \{c_{DA} \cdot \rho_{DA} \cdot (\theta_{DA} + \frac{\partial \theta_{DA}}{\partial \eta} \cdot d\eta) + \rho_{DA} \cdot (x_A + \frac{\partial x_A}{\partial \eta} \cdot d\eta) \cdot (\theta_{DA} + \frac{\partial \theta_{DA}}{\partial \eta} \cdot d\eta) + \\
& + r^0 \cdot \rho_W \cdot (x_A + \frac{\partial x_A}{\partial \eta} \cdot d\eta)\} \cdot (w_{DA} + \frac{\partial w_{DA}}{\partial \eta} \cdot d\eta) \cdot l \cdot d\xi,
\end{aligned} \tag{2}$$

where are: $x_A = x_A(t, \eta)$, $w_{DA} = w_{DA}(t, \eta)$,
 $\theta_{DA} = \theta_{DA}(t, \eta)$, $\theta_{DM} = \theta_{DM}(t, \eta)$, $\theta_F = \theta_F(t)$, $\theta_E = \theta_E(t)$,

Energy balance equation for wet material:

$$A_M \cdot c_{DM} \cdot \rho_{DM} \cdot \frac{\partial \theta_{DM}(t, \eta)}{\partial t} \cdot d\eta = A_M \cdot q_\eta - Q_{VM} - A_M \cdot (q_\eta + \frac{\partial q(t, \eta)}{\partial \eta} \cdot d\eta). \tag{3}$$

Combining the upper equation with a Fourier's Heat Transfer Law:

$$q = -\lambda \cdot \left(\frac{\partial \theta}{\partial n} \right), \tag{4}$$

next equation can be acquired:

$$\frac{\partial \theta_{DM}(t, \eta)}{\partial t} = \frac{\lambda_{DM}}{(c_{DM} + x_M \cdot c_W) \cdot \rho_{DA}} \cdot \frac{\partial^2 \theta_{DM}(t, \eta)}{\partial \eta^2}. \tag{5}$$

with boundary conditions: $\theta_{DM}(t, 0) = \theta_{FN}$, $\theta_{DM}(t, \delta_M) = \theta_{DAN}$

Material balance equation for wet material:

$$\frac{\partial x_M(t, \eta)}{\partial t} = G_M - G_{VM} - \left(G_M + \frac{\partial G_M(t, \eta)}{\partial \eta} \cdot d\eta \right). \tag{6}$$

Using the material transfer equation:

$$i = -\mu \cdot \left(\frac{\partial \theta}{\partial n} \right), \tag{7}$$

with the condition that most of heat transfer fall within the conductive heating of material (because the convective heat transfer is irrelevant in dilute atmosphere), next equation can be acquired:

$$\frac{\partial x_M(t, \eta)}{\partial t} = a_W \cdot \frac{\partial^2 x_M(t, \eta)}{\partial \eta^2} + a_W \cdot \delta_W \cdot \frac{\partial^2 \theta_{VM}(t, \eta)}{\partial \eta^2}. \tag{8}$$

Equations (1)–(8) represent a non-linear mathematical model, in absolute coordinates.

8. *Assumption*: All non-linear members, except $x_A \cdot (\partial w_{DA} / \partial \eta)$, being low order, can be neglected.
9. *Assumption*: Air velocity profile can be considered as linear, within the vacuum chamber.
10. *Assumption*: Nominal proces outlet values x_A , x_M , θ_A , θ_M , are time-variable during the drying process.

It is also assumed that wet material mass, inside the drying chamber is constant during the process. Moisture content, which evaporates during the time (Δt), will be designated as m_W . Using the VP-2 vacuum-pump, quantity equalisation occure in t and $(t + \Delta t)$:

$$\frac{\partial x_A(t)}{\partial t} = - \frac{m_{DM}}{m_{DA}(t)} \cdot \frac{dx_M(t)}{dt} . \quad (9)$$

Change of air quantity, inside the drying chamber, can be assumed as linear time-varying function. Based on the boundary values, next equations can be written:

$$m_{DA}(0) = 1,9 \text{ kg} , \quad x_A(0) = 0 , \quad x_A(t) = 250 \% . \quad (10)$$

$$\Rightarrow m_{DA}(t) = 7,5697 \cdot 10^{-3} \text{ kg} , \quad \Rightarrow m_W(t) = 1,8924 \text{ kg} . \quad (11)$$

Based on preceeding equations, another can be written:

$$m_{DA}(t) = -5,841 \cdot 10^{-5} \cdot t + 1,9 . \quad (12)$$

$$\frac{\partial x_A(t)}{\partial t} = - \frac{954}{-5,841 \cdot 10^{-5} \cdot t + 1,9} \cdot \frac{dx_M(t)}{dt} \quad (13)$$

Next expression is experimentaly acquired:

$$x_{MN}(t) = 1,5 - 4,6 \cdot 10^{-5} \cdot t . \quad (14)$$

Based on wet air dynamics, next equations can be written:

$$x_{AN}(t) = 5 \cdot 10^{-4} + 7,716 \cdot 10^{-3} \cdot t , \quad (15)$$

$$\kappa_{1N}(t) = 4,92 \cdot 10^{-7} + 7,594 \cdot 10^{-6} \cdot t , \quad (16)$$

It should be notified that these expressions are not linear, but can be considered in that way.

Model can be transfered to apsolute deviations, by introducing $\Delta x_A(t, \eta) = x_A(t, \eta) - x_{AN}(t, \eta)$. It is not possible to introduce relative deviations in the form of: introducing $\Delta x_A(t, \eta) = [x_A(t, \eta) - x_{AN}(t, \eta)] / x_{AN}(t, \eta)$, since $x_{AN}(t, \eta)$ is time variable. Coefficients within the partial differential equations are also time variable, so it is unpractical to use Laplace transformation. For that reason, interval $[0, \tau]$, at which drying process is taking place, is divided in n , mutually equal intervals, what enables one to consider the coefficients in partial differential equations as - constants.

Therefore, relative deviations introduce for every one of n intervals, are formed in the following way:

$$\overline{\Delta x_A}(t, \eta) = \frac{x_A(t, \eta) - x_{ANi}(\eta)}{x_{ANi}(\eta)} = \frac{x_A(t, \eta) - x_{ANi}(\eta)}{x_{ANiaver.}}, \quad (17)$$

where: $x_{ANiaver.} = \text{const.}$, as usually for the distributed parameters processes. Outlet average value can be found like :

$$x_{ANiaver.} = (x_{ANi} + x_{ANi+1})/2, \quad t_i < t < t_{i+1}. \quad (18)$$

In this way, constant coefficients are acquired in partial differential equations system, for every one of n intervals.

$$\frac{\partial \overline{\Delta x_A}(t, \eta)}{\partial t} + C_1 \cdot \overline{\Delta x_A}(t, \eta) = \frac{\rho_{DM} \cdot k_F \cdot G_{FN}}{\rho_{DA} \cdot x_{ANiaver.}} \cdot \overline{\Delta G_F}(t). \quad (19)$$

$$\begin{aligned} c_{DA} \cdot \theta_{DAN} \cdot \frac{\partial \overline{\Delta \theta_{DA}}(t, \eta)}{\partial t} + x_{ANiaver.} \cdot r^0 \cdot \rho_{DA} \cdot \frac{\partial \overline{\Delta x_A}(t, \eta)}{\partial t} &= \frac{\rho_{DM} \cdot r_a \cdot k_F \cdot G_{FN}}{\rho_{DA}} \cdot \overline{\Delta G_F}(t) + \\ &+ \frac{c_{DA} \cdot \kappa_{1Na}}{\rho_{DA}} \cdot [\theta_{DMN} \cdot \overline{\Delta \theta_{DM}}(t, \eta) - \theta_{DAN} \cdot \overline{\Delta \theta_{DA}}(t, \eta)] + \\ &+ \frac{K_{FA} \cdot A_{FA}}{\rho_{DA}} \cdot [\theta_{FN} \cdot \overline{\Delta \theta_F}(t) - \theta_{DAN} \cdot \overline{\Delta \theta_{DA}}(t, \eta)] - \\ &- \frac{K_{AE} \cdot A_{AE}}{\rho_{DA}} \cdot [\theta_{DAN} \cdot \overline{\Delta \theta_{DA}}(t, \eta) - \theta_{EN} \cdot \overline{\Delta \theta_E}(t)] - \\ &- \frac{x_{AN} \cdot r^0 \cdot \rho_W \cdot C_1 \cdot \overline{\Delta x_A}(t, \eta)}{\rho_{DA}}. \end{aligned} \quad (20)$$

$$\frac{\partial \overline{\Delta \theta_{DM}}(t, \eta)}{\partial t} = \frac{\lambda_{DM}}{c_{DM} \cdot \rho_{DA}} \cdot \frac{\partial^2 \overline{\Delta \theta_{DM}}(t, \eta)}{\partial \eta^2}. \quad (21)$$

$$\frac{\partial \overline{\Delta x_M}(t, \eta)}{\partial t} = a_W \cdot \frac{\partial^2 \overline{\Delta x_M}(t, \eta)}{\partial^2 \eta} + a_W \cdot \delta_W \cdot \frac{\theta_{DMN}}{x_{MNiaver.}} \cdot \frac{\partial^2 \overline{\theta_{DM}}(t, \eta)}{\partial^2 \eta}. \quad (22)$$

Introducing the next variables the derived mathematical model can be additionally simplified:

$$\begin{aligned} b_{1i} &= \frac{\rho_{DM} \cdot k_F \cdot G_{FN}}{\rho_{DA} \cdot x_{ANiaver.}}, \quad b_2 = \frac{\rho_{DM} \cdot k_F \cdot G_{FN}}{\rho_{DA}} \cdot \frac{(r_a - r^0)}{\theta_{DAN} \cdot c_{DA}}, \quad b_{3i} = \frac{\kappa_{1Na} \cdot \theta_{DMN}}{\theta_{DAN} \cdot \rho_{DA}}, \\ b_{4i} &= \frac{c_{DA} \cdot \kappa_{1Na} + K_{FA} \cdot A_{FA} + K_{AE} \cdot A_{AE}}{\rho_{DA} \cdot x_{ANiaver.}}, \quad b_5 = \frac{K_{FA} \cdot A_{FA} \cdot \theta_{FN}}{\theta_{DAN} \cdot c_{DA}}, \\ b_6 &= \frac{K_{AE} \cdot A_{AE} \cdot \theta_{FN}}{\theta_{DAN} \cdot c_{DA}}, \quad b_7 = \frac{\lambda_{DM}}{c_{DM} \cdot \rho_{DA}}, \quad b_8 = a_W, \\ b_{9i} &= \frac{\theta_{DAN} \cdot a_W \cdot \delta_{DA}}{x_{ANiaver.}}, \quad b_{10i} = \frac{r^0 \cdot x_{ANiaver.} \cdot C_1 \cdot \left(1 - \frac{\rho_W}{\rho_{DA}}\right)}{\theta_{DAN} \cdot c_{DA}}, \end{aligned} \quad (23)$$

Finally:

$$\frac{\partial \overline{\Delta x_A}(t, \eta)}{\partial t} + C_1 \cdot \overline{\Delta x_A}(t, \eta) = b_{1i} \cdot \overline{\Delta G_F}(t), \tag{24}$$

$$\begin{aligned} \frac{\partial \overline{\Delta \theta_{DA}}(t, \eta)}{\partial t} + b_{4i} \cdot \overline{\Delta \theta_{DA}}(t, \eta) = & b_{3i} \cdot \overline{\Delta \theta_{DM}}(t, \eta) + b_2 \cdot \overline{\Delta G_F}(t) + b_5 \cdot \overline{\Delta \theta_F}(t) + \\ & + b_6 \cdot \overline{\Delta \theta_E}(t) + b_{10i} \cdot \overline{\Delta x_A}(t, \eta), \end{aligned} \tag{25}$$

$$\frac{\partial \overline{\Delta \theta_{DM}}(t, \eta)}{\partial t} = b_7 \cdot \frac{\partial^2 \overline{\Delta \theta_{DM}}(t, \eta)}{\partial \eta^2}. \tag{26}$$

$$\frac{\partial \overline{\Delta x_M}(t, \eta)}{\partial t} = b_8 \cdot \frac{\partial^2 \overline{\Delta x_M}(t, \eta)}{\partial \eta^2} + b_{9i} \cdot \frac{\partial^2 \overline{\Delta \theta_{DM}}(t, \eta)}{\partial \eta^2}. \tag{27}$$

Equations (24) - (27) forms a linear drying process mathematical model, in the form of $4 \times n$ partial differential equations system, with constant coefficients. Using the Laplace transformation, first by time, then by space coordinate, with all the initial conditions equal to zero, solutions of this partial differential system can be found. Control, controlled and disturbance variables has been accepted in the following manner:

$$x_{O1}(t, \eta) = \Delta x_A(t, \eta), \quad x_{O2}(t, \eta) = \Delta x_{DA}(t, \eta), \tag{28}$$

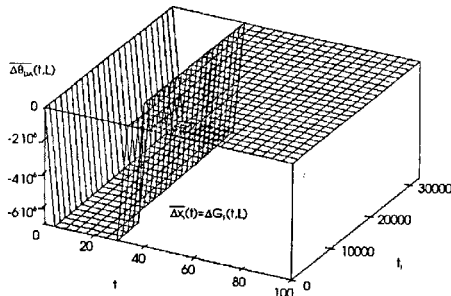
$$x_{O3}(t, \eta) = \Delta x_M(t, \eta), \quad x_{O4}(t, \eta) = \Delta x_{DM}(t, \eta),$$

$$y(t) = \Delta G_F(t), \tag{29}$$

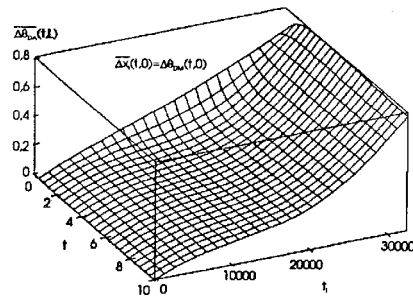
$$z_1(t) = \Delta \theta_F(t), \quad z_2(t) = \Delta \theta_E(t), \quad z_3(t) = \Delta x_A(t, 0), \tag{30}$$

$$z_4(t) = \Delta \theta_A(t, 0), \quad z_5(t) = \Delta x_M(t, 0), \quad z_6(t) = \Delta \theta_{DM}(t, 0).$$

Air temperature change, caused by step change in heating fluid rate, and in wet material temperature, have been shown on Fig. 4.



Air temperature change at the outlet, caused by step change in heating fluid flow rate

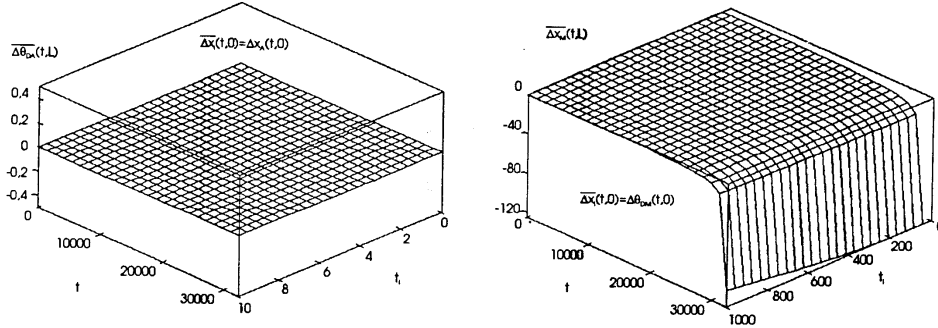


Air temperature change at the outlet, caused by step change in wet material temperature

Fig. 4.

Air temperature change, caused by step change in air moisture content, and material moisture content change, caused by step change in wet material temperature, have been

shown on Fig. 5.

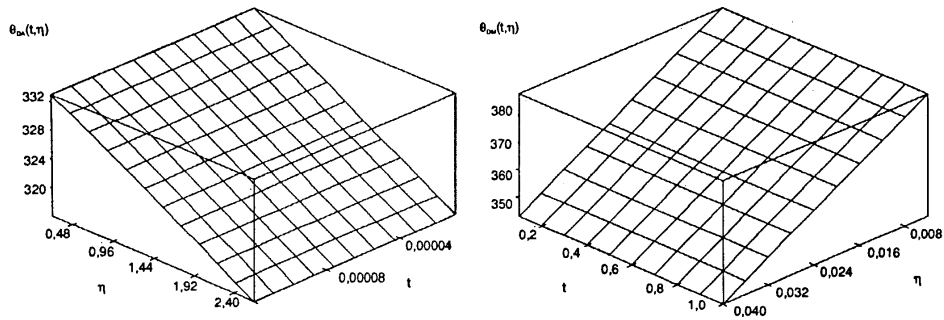


Air temperature change at the outlet, caused by step change in air moisture content

Material moisture content change at the outlet, caused by step change in wet material temperature

Fig. 5.

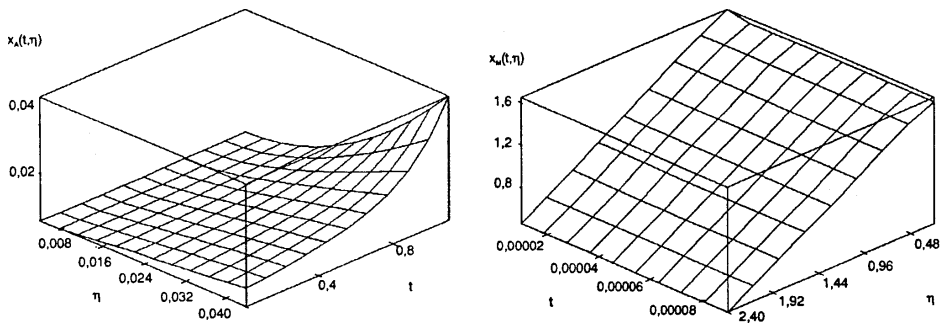
Non-stationary air and material temperature, as well as air and material moisture content profiles have been shown on Fig. 6. and Fig. 7. respectively.



Non-stationary air temperature profile

Non-stationary material temperature profile

Fig. 6.



Non-stationary air moisture content profile

Non-stationary material moisture content profile

Fig. 7.

4. CONCLUSIONS

On the basis of really accepted assumptions, a sufficiently correct vacuum shelf dryer mathematical model has been derived. This model is based upon the material and energy balance equations applied, for non-stationary working conditions of the drying plant, also including drying process kinetics. Introduction the relative deviations of all relevant physical variables, using the "Maple" programming language a drying process computer simulation has been performed on Amiga 1200 computer.

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NOTATIONS

A – cross-section area,	s – complex variable,
a – moisture potential spreading conductivity,	t – time,
b – coefficient,	$W(s)$ – transfer function,
C – coefficient,	w – velocity,
G – total mass flow rate,	x – absolute moisture,
h – specific entalpia,	δ – wet material thermogradient coeff.,
i – moisture flux density,	ξ – space coordinate,
K – heat-transfer coefficient,	θ – temperature,
k – coefficient,	κ – coefficient,
l – plate length,	λ – thermal conductivity,
m – mass,	μ – moisture transfer coefficient,
n – time intervals number,	η – space coordinate,
p – pressure,	ρ – density, bulk density,
q – specific heat quantity,	τ – drying process time,
r – evaporasing heat,	$\Delta(\cdot)$ – relative deviations.

LOWER INDEXES:

$aver.$, a – average	i – inlet,
A – air,	M – material,
AE – air –environment,	N – nominal,
DA – dry air,	O – outlet,
DM – dry material,	VM – vaporized moisture,
E – environment,	W – moisture,
F – heating fluid,	η – toward η – coordinate.
FE – heating fluid – environment,	

DINAMIKA DISKONTINUALNIH SUŠARA**L. L. Pezo, D. Lj. Debeljković, D. Voronjec, V. P. Ilić**

Na osnovu realno prihvaćenih i kritički verifikovanih pretpostavki, razvijen je nelinearni i linearizovani matematički model diskontinualne vakuumske sušare. Model je u obliku parcijalnih diferencijalnih jednačina sa vremenski promenljivim koeficijentima. Korišćenjem odgovarajućih numeričkih metoda urađena je odgovarajuća simulacija. Dobijeni rezultati su u veoma dobrom slaganju sa dinamičkim ponašanjem realnih sušara.