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# ONE OF THE POSSIBILITIES FOR DETERMINING THE HEAT CONDUCTION COEFFICIENT

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**Abstract.** *The paper presents a method for calculating the effective coefficient of temperature diffusiveness in a setting of bricks. This method can be used to calculate the coefficient of temperature diffusiveness of other non-homogenous and homogenous materials also.*

The task of determining the temperature field is among the essential tasks of the analytical theory of heat conduction. The temperature field equation describing the distribution of temperature in space and time contains a physical parameter of substance:

$$a = \frac{\lambda}{\rho \cdot c} \tag{1}$$

named as the heat conduction coefficient and having as the unit $m^2/s$. If the heat conduction coefficient $\lambda$ characterizes the capability of a body to conduct heat, then $a$ as the coefficient of temperature diffusiveness represents the measure of thermo-technical properties of bodies. The rate of temperature change in any point of the body will be higher if the value of the coefficient $a$ is higher. Gases and liquids possess considerable heat inertia, and their coefficients of temperature diffusiveness are low. Metals, on the other hand, do not have a high heat inertia since their coefficients $a$ have high values.

In a large number of cases, the coefficient of temperature diffusiveness is determined experimentally, and for homogenous materials that are most frequently used in industry is given in the form of tables. For the cases of very unhomogeneous materials (bricks setting in a tunnel oven), it is more appropriate to refer to the effective coefficient of temperature diffusiveness.

With a staggered type of setting, heat is transmitted both by the mechanism of conduction and by convection and radiation through the voids filled up with gas, so that the calculation of the non-stationary temperature field is practically almost impossible, with questionable accuracy of results. An attempt has been made, therefore, to describe

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the non-stationary temperature field by means of the effective coefficient of temperature difussivity covering all of the present mechanisms of heat exchange, and the problem of calculating the non-stationary temperature field within the brick setting has been reduced essentially to solving the differential equation of heat conduction.

The method of determining the effective coefficient of temperature diffusivity presented in this paper is analogous to that applied in determining the effective heat conduction coefficient of insulation and construction materials. On the other hand, the notion of the heat conduction coefficient has been used by other authors [1-5] for the distribution of temperature in granulated materials.

In this example of determining the effective coefficient of temperature diffusiveness of a staggered type brick setting, limit conditions close to practice have been used. Besides, the procedure is to some extent independent of limit conditions.

It may be stated that this method is generally applicable, independent of materials, and that it can be applied to homogenous materials also.

With this procedure, differential quotients approximate to difference quotients, so that finite local differences are used in determining temperature derivatives. It is assumed that inside a homogenous, impermeable solid body, the differential equation of heat conduction must be satisfied in any point.

The differential equation of non-stationary heat conduction is:

$$\frac{\partial T}{\partial \tau} = a \cdot \Delta T = a \cdot \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \tag{2}$$

where: $T$ - thermodynamic temperature; $\Delta$ - Laplas's operator; and $a$ - the coefficient of temperature diffusiveness.

According to differential equation (1) for a, the following applies:

$$a = \frac{\frac{\partial T}{\partial \tau}}{\Delta T} . \tag{3}$$

The derivative temperature by time can be determined per the well known procedure of finite differences, where it is recommended to operate with the central difference, because then we would have the square convergence with respect to time steps, which means that the approximation of temperature derivatives by time at a predetermined value of steps is comparatively more accurate. Other differences for $\Delta T$ are also comparatively simple to determine, if temperature values on coordinate axes are known, i.e. when thermocouples of all of the three coordinate directions are set on a straight line.

Difficulties arise, however, when the thermocouples are more or less randomly distributed in the body, which cannot be avoided in practical measurements on settings in tunnel ovens.

This proposed procedure for determining the effective coefficient of temperature diffusiveness has been conceived also for randomly distributed thermocouples. In experimental work, thermocouples should be arranged in a single line, with a larger number of thermocouples installed in order to allow the consideration of more complex cases.

The problem of determining the second derivative of temperature by coordinates with randomly distributed thermocouples has not yet been solved in a satisfactory way. The finite difference procedure is used for that purpose, which provides the same accuracy as the procedure of finite differences for special arrangement of thermocouples. With a larger number of thermocouples, it is possible to have a more accurate approximation of Laplas's operator by means of the finite differences method.

In carrying out the difference procedure for Laplas's operator, the equation of a three-dimensional temperature field is developed into Taylor's series:

$$\begin{aligned} &T(x_o+i_n, y_o+j_n, z_o+k_n)-T_0(x_o, y_o, z_o)= \\ &=i_n\cdot\frac{\partial T}{\partial x}+j_n\cdot\frac{\partial T}{\partial y}+k_n\cdot\frac{\partial T}{\partial z}+i_n\cdot j_n\cdot\frac{\partial^2 T}{\partial x\partial y}+i_n\cdot k_n\cdot\frac{\partial^2 T}{\partial x\partial z}+{}_n\cdot k_n\cdot\frac{\partial^2 T}{\partial y\partial z}+, \\ &+\frac{i_n^2}{2}\cdot\frac{\partial^2 T}{\partial x^2}+\frac{j_n^2}{2}\cdot\frac{\partial^2 T}{\partial y^2}+\frac{k_n^2}{2}\cdot\frac{\partial^2 T}{\partial z^2}\Big|_{x_o, y_o, z_o}+\ldots \end{aligned} \tag{4}$$

where $n$ is the number of thermocouples, while $i$, $j$ and $k$ are local differences in relation to the reference thermocouple $0(x_o, y_o, z_o)$ for which the effective coefficient of temperature diffusiveness is determined. The development into the series goes up to the second member, so that linear convergence may be expected for the difference presentation of Laplas's operator.

Equation (3) may be written for each installed thermocouple, and by using ten thermocouples a system of equations with nine unknowns is obtained. In these, the local temperature derivatives figure as unknowns, and the temperature difference in relation to the central thermocouple as non-homogeneity.

This system of equations can be presented very conveniently by a matrix system:

$$\begin{pmatrix} & \vdots & \\ i_n\cdot j_n\cdot k_n\cdot\frac{i_n^2}{2}\cdot\frac{j_n^2}{2}\cdot\frac{k_n^2}{2}\cdot i_n\cdot j_n\cdots \\ & \vdots & \end{pmatrix}\cdot\begin{pmatrix} \frac{\partial T}{\partial x} \\ \vdots \\ \frac{\partial^2 T}{\partial x^2} \\ \vdots \\ \frac{\partial^2 T}{\partial x\partial y} \end{pmatrix}=\begin{pmatrix} \vdots \\ T(x_o+i_n, y_o+j_n, z_o+k_n)-T(x_o, y_o, z_o) \\ \vdots \end{pmatrix} \tag{5}$$

$$C\cdot D=T \tag{6}$$

Thus, the matrix coefficient $C$ should be multiplied with vector $D$ containing the unknown temperature derivatives in order to obtain the vector of temperature differences.

The solution of this system of equations is obtained by multiplying vector $T$ from the left-hand side with the inverse matrix of coefficient $C$. Solving of this system does not represent a problem today since such programs are quite common in computer software.

From the structure of this system, it may be noted that ten thermocouples should be installed here in order to get a straightforward solution. In addition to this, it is possible also to get the straightforward solution with seven thermocouples arranged in a line. The system of equations in this case would have the block-diagonal structure. Analogous are also the relations with eight and nine thermocouples.

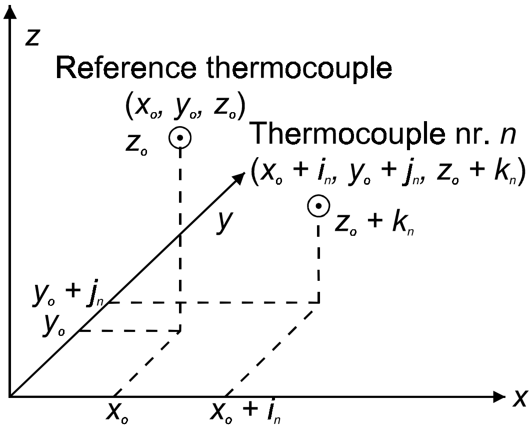


Fig. 1 - Thermocouples arrangement scheme

By adding the corresponding vector elements, it is possible to derive their Laplas's operator from vector *D*. However, it is possible also to determine the temperature gradient, the components of which are already contained in vector *D*, which is of decisive importance with non-stationary problems of heat tension. In addition to this, there occur mixed derivatives which indicate in principle a change in the temperature gradient in certain direction. By means of these mixed derivatives, it is possible to determine exactly the point with the largest gradient, which frequently corresponds to the point with the largest heat tension load. If more than ten thermocouples are installed in a brick setting, then the system of equations is excessively determined. Then it is possible to take additional information by means of known procedures – as is, for example, the method of the least squares of errors – in order to increase the statistical safety of results. On the other hand, by using more equations, higher derivatives of the temperature field may be eliminated, and thereby achieve a better convergence of the procedure.

It is important for this procedure that in the investigated temperature area a uniform change in temperature takes place, i.e. that the sign of the temperature change rate is the same at all measurement points. This procedure can be applied appropriately especially when uniform limit conditions are given, if possible. For this reason, application is limited for this procedure to the bricks heating and cooling stages. In addition to this, the most favorable results can be expected at linear heating/cooling processes.

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# JEDNA OD MOGUĆNOSTI ODREĐIVANJA EFEKTIVNOG KOEFICIJENTA TEMPERATURNE PROVODLJIVOSTI

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*U radu je prikazana metoda proračuna efektivnog koeficijenta temperaturne provodljivosti rešetkastog sloga opeka. Ova se metoda može primeniti za proračun koeficijenta temperaturne provodljivosti i drugih nehomogenih, kao i homogenih materijala.*