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# LYAPUNOV EXPONENTS AND STOCHASTIC STABILITY OF COUPLED LINEAR SYSTEMS SUBJECTED TO WIDE - BAND CORRELATED RANDOM PROCESSES

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**Abstract.** *The almost - sure asymptotic stability of a class of two degrees of freedom linear systems subjected to parametric wide - band correlated random processes of small intensity are investigated. By combined use of the method of stochastic averaging and well - known procedure due to Khas'minskii, asymptotic expressions for the largest Lyapunov exponent for various values of the system parameters are obtained. As an application, the example of the flexural - torsional instability of closed thin - walled beam acted upon by a stochastically fluctuating of axial loads and ends moments at the central cross - section at the beam is discussed.*

**Key words:** *Lyapunov exponents, stochastic stability, coupled linear systems, flexural - torsional instability*

## 1. INTRODUCTION

In recent years dynamic stability of elastic systems subjected to stochastically fluctuating loads has been explored in the papers of many authors. The stochastic moment stability of such systems was examined previously in the paper given by Ariaratnam and Srikantaiah [1] using the method of stochastic averaging and the technique of Khas'minskii [5,6]. For a similar dynamic system the conditions for the uniform stochastic stability of the trivial solution of the corresponding system of Itô - equations have been determined by using the Lyapunov functional in the paper presented by Tylikowski [7]. In the paper Ariaratnam and Tam [2] the stochastic stability of the solution of the linear oscillatory system with one degree of freedom has been analyzed while subjected to three correlated random processes. Explicit asymptotic expressions for the largest Lyapunov exponent for various values of the system parameters are obtained by using a combination

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of the method of stochastic averaging and a well - known procedure due to Khas'minskii, from which the asymptotic stability boundaries are determined in the papers Ariaratnam et. all. [2,3]. In this paper the stability of coupled flexural - torsional instability of a closed thin - walled beam under the combined action of axial loads and equal ends moments, which are correlated stationary stochastic wide - band processes of small intensity and of small correlation time thus being different from the study in the paper given by Jochi and Suryanarayan [4] in which these parameters are deterministic, is considered. Conditions for stochastic almost - sure asymptotic stability are obtained by a method for the exact evaluation of the Lyapunov exponent of linear systems described by stochastic differential equations of Itô type that was given by Khas'minskii [9].

## 2. FORMULATION

Consider a class of the oscillatory systems described by stochastic differential equations of the form

$$\ddot{q}_i + \omega_i^2 q_i + 2\sum_{j=1}^{n} \beta_{ij}\dot{q}_j + \omega_i\left[\xi_1(t)\sum_{j=1}^{n} k_{ij}q_j + \xi_2(t)\sum_{j=1}^{n} c_{ij}q_j\right] = 0\,,\quad i = 1,2,..n \tag{1}$$

where the $q_i$ are generalized coordinates, $\beta_{ij}$ are damping constants, $\omega_i$ are natural frequencies, $k_{ij}$ and $c_{ij}$ are constants. The excitations are represented by $\xi_1(t)$ and $\xi_2(t)$ which are taken to be an ergodic stochastic process with zero mean value and a sufficiently small correlation time. Equation (1) describe exactly the parametrically excited motion of certain non-gyroscopic, discrete, linear elastic systems with $n$ degrees-of-freedom about the equilibrium configuration $q_i(t)$=0. They also describe approximately the motion of certain continuous elastic structures whose equations of motion have been discretized by some suitable techniques such as Rayleigh - Ritz, Galerkin, finite differences or finite elements. In equation (1), the generalized coordinates $q_i(t)$ and velocities $\dot{q}_i(t)$ are transformed to polar coordinates by means of the relations,

$$q_i = a_i\cos\theta_i\,,\ \dot{q}_i = -a_i\omega_i\sin\theta_i\,,\ \theta_i = \omega_i t + \theta_i\,,\quad i = 1,2,..n \tag{2}$$

Then one obtains the equations of motion in terms of $a_i(t)$ and $\theta_i(t)$ :

$$\dot{a}_i = -\frac{2}{\omega_i}\sum_{j=1}^{n}\beta_{ij}a_j\omega_j\sin\theta_j\sin\theta_i + \sum_{j=1}^{n}\left[\xi_1(t)k_{ij} + \xi_2(t)c_{ij}\right]a_j\cos\theta_j\sin\theta_i \tag{3a}$$

$$\dot{\theta}_i = -\frac{2}{a_i\omega_i}\sum_{j=1}^{n}\beta_{ij}a_j\omega_j\sin\theta_j\sin\theta_i + \frac{1}{a_i}\sum_{j=1}^{n}\left[\xi_1(t)k_{ij} + \xi_2(t)c_{ij}\right]a_j\cos\theta_j\cos\theta_i \tag{3b}$$

It is supposed that the damping constants and the stochastic perturbation are small, i.e. $\beta_{ij}=O(\varepsilon)$, $S_1=O(\varepsilon)$, $S_{12}=O(\varepsilon)$, $S_{21}=O(\varepsilon)$, $S_2=O(\varepsilon)$, $\Psi_1=O(\varepsilon)$, $\Psi_{12}=O(\varepsilon)$, $\Psi_{21}=O(\varepsilon)$, $\Psi_2=O(\varepsilon)$, $0<|\varepsilon|<<1$, where $S_1(\omega)$, $S_{12}(\omega)$, $S_{21}(\omega)$, $S_2(\omega)$, $\Psi_1(\omega)$, $\Psi_{12}(\omega)$, $\Psi_{21}(\omega)$, $\Psi_2(\omega)$ denote the cosine and sine power spectral densities of the stochastic processes $\xi_1(t)$ and $\xi_2(t)$, defined by

$$S_j(\omega)+i\psi_j(\omega)=2\int_0^\infty E\left[\xi_j(t)\xi_j(t+\tau)\right]e^{i\omega\tau}d\tau\,,\ \ j=1,2,12,21.$$

$E[\cdot]$ denoting the expectation operation. Under these conditions, the method of stochastic averaging Khas'minskii [5] may be applied to equations (3a, 3b) to yield the following Itô equations for the averaged amplitudes $a_i$ and phase angle $\theta_i$, whose solutions provide a uniformly valid first approximation,

$$da_i = m_{a_i}dt+\sum_{j=1}^{n}\sigma_{ij}dW_j\ , \tag{4a}$$

$$d\theta_i = m_{\theta_i}dt+\sum_{j=1}^{n}\mu_{ij}dB_j\,, \quad i=1,2,...n, \tag{4b}$$

where $W_j(t)$, $B_j(t)$, $j$ = 1,2,...$n$ are mutually independent unit Wiener processes and the drift coefficients $m_{a_i}$, $m_{\theta_i}$ and the diffusion coefficients $\sigma_{ij}$, $\mu_{ij}$ are given by:

$$\begin{aligned}m_{a_i} &= -\beta_{ii}a_i+\frac{3a_i}{16}\{k_{ii}^2S_1(2\omega_i)+k_{ii}c_{ii}[S_{12}(2\omega_i)+S_{21}(2\omega_i)]+c_{ii}^2S_2(2\omega_i)\}+\\ &+\sum_{\substack{j=1\\ j\neq i}}^{n}\{\frac{a_i}{8}[k_{ij}k_{ji}S_{1ij}^- +k_{ij}c_{ji}S_{12ij}^- +k_{ji}c_{ij}S_{21ij}^- +c_{ij}c_{ji}S_{2ij}^-]+\\ &+\frac{a_j^2}{16a_i}[k_{ij}^2S_{1ij}^+ +k_{ij}c_{ij}(S_{12ij}^+ +S_{21ij}^+)+c_{ij}^2S_{21ij}^+]\},\end{aligned}$$

$$\begin{aligned}m_{\theta_i} &= \frac{1}{8}\{k_{ii}^2\psi_1(2\omega_i)+k_{ii}c_{ii}[\psi_{12}(2\omega_i)+\psi_{21}(2\omega_i)]+c_{ii}^2\psi_2(2\omega_i)\}+\\ &+\sum_{\substack{j=1\\ j\neq i}}^{n}\{\frac{1}{8}[k_{ij}k_{ji}\psi_{1ij}^- +k_{ij}c_{ji}\psi_{12ij}^- +k_{ji}c_{ij}\psi_{21ij}^- +c_{ij}c_{ji}\psi_{2ij}^-],\end{aligned}$$

$$\begin{aligned}\left[\sigma\sigma^T\right]_{ii} &= \frac{a_i^2}{8}\{k_{ii}^2S_1(2\omega_i)+k_{ii}c_{ii}[S_{12}(2\omega_i)+S_{21}(2\omega_i)]+c_{ii}^2S_2(2\omega_i)\}+\\ &+\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{a_j^2}{8}[k_{ij}^2S_{1ij}^- +k_{ij}c_{ij}S_{12ij}^-(S_{12ij}^+ +S_{21ij}^+)+c_{ij}^2S_{21ij}^+],\end{aligned}$$

$$\left[\sigma\sigma^T\right]_{ij} = \frac{a_ia_j}{8}[k_{ij}k_{ji}S_{1ij}^- +k_{ij}c_{ji}S_{12ij}^- +k_{ji}c_{ij}S_{2ij}^- +c_{ij}c_{ji}S_{2ij}^-],\ (i\neq j)$$

$$\begin{aligned}\left[\mu\mu^T\right]_{ii} &= \frac{1}{8}\{k_{ii}^2[2S_1(0)+S_1(2\omega_i)]+k_{ii}c_{ii}[2S_{12}(0)+2S_{21}(0)+S_{12}(2\omega_i)+S_{21}(2\omega_i)]+\\ &+c_{ii}^2[2S_2(0)+S_2(2\omega_i)]\}+\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{a_j^2}{8a_i^2}[k_{ij}^2S_{1ij}^+ +k_{ij}c_{ij}(S_{12ij}^+ +S_{21ij}^+)+c_{ij}^2S_{21ij}^+]\},\end{aligned}$$

$$\left[\mu\mu^T\right]_{ij} = \frac{1}{4}[k_{ii}k_{jj}S_1(0) + k_{ii}c_{jj}S_{12}(0) + c_{ii}k_{jj}S_{21}(0) + c_{ii}c_{jj}S_2(0)]$$
$$+\frac{1}{8}[k_{ij}k_{ji}S_{1ij}^{+} + k_{ij}c_{ji}S_{12ij}^{+} + k_{ji}c_{ij}S_{21ij}^{+} + c_{ij}c_{ji}S_{2ij}^{+}],\ (i \neq j)$$

In the above expressions, the functions $S^+$, $S^-$, $\psi^+$, $\psi^-$ are defined by

$$S_{kij}^{\pm} = S_k(\omega_i + \omega_j) \pm S_k(\omega_i - \omega_j)\ ,\ \psi_{kij}^{\pm} = \psi_k(\omega_i + \omega_j) \pm \psi_k(\omega_i - \omega_j)\ ,\ \begin{matrix} k=1,2,12,21 \\ i,j = 1,2,..n \end{matrix}.$$

The $n$ degrees-of-freedom system given by equation (1) is difficult to study in its general form. Hence, the discussion from now on will be restricted to two degrees-of-freedom system described by the equations:

$$\ddot{q}_1 + 2\beta_{11}\dot{q}_1 + 2\beta_{12}\dot{q}_2 + \omega_1^2 q_1 + \omega_1(k_{11}q_1 + k_{12}q_2)\xi_1(t) + \omega_1(c_{11}q_1 + c_{12}q_2)\xi_2(t) = 0, \tag{5a}$$

$$\ddot{q}_2 + 2\beta_{21}\dot{q}_1 + 2\beta_{22}\dot{q}_2 + \omega_2^2 q_2 + \omega_2(k_{21}q_1 + k_{22}q_2)\xi_1(t) + \omega_2(c_{21}q_1 + c_{22}q_2)\xi_2(t) = 0. \tag{5b}$$

The results derived for two degrees-of-freedom system may be generalized to $n$ degrees-of freedom system under certain conditions on the spectral density distribution of the parametric excitations, and for the case $k_{12}=k_{21}=0$, $c_{11}=c_{22}=\beta_{12}=\beta_{21}=0$, $\beta_{11}=\beta_1$, $\beta_{22}=\beta_2$ (See Section Application). For the two degrees-of-freedom system, the amplitude equations of (4a) become

$$da_1 = m_1 dt + \sigma_{11}dW_1 + \sigma_{12}dW_2,\ \ da_2 = m_2 dt + \sigma_{21}dW_1 + \sigma_{22}dW_2, \tag{6}$$

where

$$m_1 = \left[-\beta_1 + \frac{3k_{11}^2}{16}S_1(2\omega_1) + \frac{c_{12}c_{21}}{8}S_2^{-}\right]a_1 + \frac{c_{12}^2 a_2^2}{16a_1}S_2^{+}$$

$$m_2 = \left[-\beta_2 + \frac{3k_{22}^2}{16}S_1(2\omega_2) + \frac{c_{12}c_{21}}{8}S_2^{-}\right]a_2 + \frac{c_{21}^2 a_1^2}{16a_2}S_2^{+}$$

$$\left[\sigma\sigma^T\right]_{11} = \frac{1}{8}k_{11}^2 S_1(2\omega_1)a_1^2 + \frac{1}{8}c_{12}^2 S_2^{+} a_2^2,\quad \left[\sigma\sigma^T\right]_{12} = \frac{1}{8}c_{12}c_{21}S_2^{-}a_1 a_2,$$

$$\left[\sigma\sigma^T\right]_{12} = \frac{1}{8}c_{12}c_{21}S_2^{-}a_1 a_2,\quad \left[\sigma\sigma^T\right]_{22} = \frac{1}{8}k_{22}^2 S_1(2\omega_2)a_2^2 + \frac{1}{8}c_{21}^2 S_2^{+} a_1^2,$$

$$S_k^{\pm} = S_k(\omega_1 + \omega_2) \pm S_k(\omega_1 - \omega_2),\quad k = 1,2,12,21$$

It may be noted that the averaged amplitude vector ($a_1$, $a_2$) is a two dimensional diffusion processes and that the coefficients of the right side terms of equations (6) are homogeneous in $a_1$, $a_2$ of degree one. Hence the procedure of Khas'minskii [5] may be employed to derive an expression for the largest Lyapunov exponent of the amplitude process. In order to this a further logarithmic polar transformation is applied:

$$\rho = \frac{1}{2}\log(a_1^2 + a_2^2),\quad \phi = \tan^{-1}\left(\frac{a_2}{a_1}\right),\ \ 0 \le \phi \le \frac{\pi}{2},$$

and by the use of Itô's differential rule or otherwise, the following pair of Itô equations, $\rho$, $\phi$ are obtained:

$$d\rho = Q(\phi)dt + \Sigma(\phi)dW, \quad \mathrm{d}\phi = \Phi(\phi)\mathrm{dt} + \psi(\phi)\mathrm{dW}, \tag{7}$$

where $W(t)$ is a Wiener process of unit intensity and

$$Q(\phi) = \lambda_1 \cos^2\phi + \lambda_2 \sin^2\phi + \frac{1}{8}c_{12}c_{21}S_2^- + \psi^2(\phi), \tag{8a}$$

$$\begin{aligned}\Phi(\phi) = -\frac{1}{2}(\lambda_1 - \lambda_2)\sin 2\phi + \frac{1}{32}\left[4(\lambda_1 + \lambda_2 + \beta_1 + \beta_2) - c_{12}c_{21}S_2^-\right]\sin 4\phi + \\ +(c_{12}^2 \sin^4\phi + c_{21}^2\cos^4\phi)\frac{S_2^+}{8}\cos 2\phi,\end{aligned} \tag{8b}$$

$$\begin{aligned}\Sigma^2(\phi) = (\lambda_1 + \beta_1)\cos^4\phi + (\lambda_2 + \beta_2)\sin^4\phi + \\ + \frac{1}{32}\left[(c_{12}^2 + c_{21}^2)S_2^+ + 2c_{12}c_{21}S_2^-\right]\sin^2 2\phi,\end{aligned} \tag{8c}$$

$$\begin{aligned}\psi(\phi) = \frac{1}{16}\{[4(\lambda_1 + \lambda_2 + \beta_1 + \beta_2) - c_{12}c_{21}S_2^-]\sin^2 2\phi + \\ + 2(c_{12}^2 \sin^4\phi + c_{21}^2\cos^4\varphi)S_2^+\}.\end{aligned} \tag{8d}$$

The constants $\lambda_1$, $\lambda_2$ are defined by

$$\lambda_1 = -\beta_1 + \frac{1}{8}k_{11}^2 S_1(2\omega_1), \quad \lambda_2 = -\beta_2 + \frac{1}{8}k_{22}^2 S_1(2\omega_2). \tag{9}$$

From the second of equations (7), it is clear that the $\phi$ - process is itself a diffusion on the first quadrant of the unit circle. If $\psi(\phi)$ vanishes in $0 \le \phi \le \pi/2$, the diffusion process is singular, otherwise it is non-singular. In this paper we shall analyze only a non-singular case which is possible only if the following relation is valid

$$\lambda_1 + \lambda_2 + \beta_1 + \beta_2 > \frac{1}{4}c_{12}c_{21}S_2^-.$$

## 3. Non-singular case

Since $\psi^2(\phi)$ does not vanish in $0 \le \phi \le \pi/2$, the diffusion is non-singular, the density $\mu(\phi)$ of the invariant measure being governed by the Fokker-Planck equation

$$\frac{1}{2}\left[\psi^2(\phi)\mu(\phi)\right]'' - \left[\psi(\phi)\mu(\phi)\right]' = 0, \tag{10}$$

where a prime denotes differentiation with respect to $\phi$. The invariant density $\mu(\phi)$ which satisfies the Fokker-Planck equation is given, see [6,7],

$$\mu(\phi) = \frac{C}{\psi^2(\phi)U(\phi)}. \tag{11}$$

Where $C$ is integration constant and

$$U(\phi)=\exp\left\{-2\int^{\phi}\left[\Phi(t)\psi^{-2}(t)\right]dt\right\}=\frac{1}{\sin 2\phi}\exp\left[8(\lambda_1-\lambda_2)\int^{\sin^2\phi}\frac{dx}{ax^2+bx+c}dx\right]. \tag{12}$$

the constants $a$, $b$ and $c$ being given by

$$a=(c_{12}^2+c_{21}^2)S_2^+ + 2c_{12}c_{21}S_2^- - k_{11}^2 S_1(2\omega_1)-k_{22}^2 S_1(2\omega_2),$$

$$b=k_{11}^2 S_1(2\omega_1)+k_{22}^2 S_1(2\omega_2)-2c_{12}c_{21}S_2^- - 2c_{21}^2 S_2^+,$$

$$c=c_{21}^2 S_2^+.$$

The form at the integral in equation (l0) depends on the sign of the discriminant $\Delta$, where

$$\Delta=[k_{11}^2 S_1(2\omega_1)+k_{22}^2 S_1(2\omega_2)-2c_{12}c_{21}S_2^-]^2-4c_{12}^2c_{21}^2(S_2^+)^2.$$

The invariant density $\mu(\phi)$ is of the form,

$$\mu(\phi)=C\frac{\sin 2\phi}{\psi^2(\phi)}\exp\left[-\frac{16(\lambda_2-\lambda_1)}{\Delta^{1/2}}\tanh^{-1}\left(\frac{b+2a\sin^2\phi}{\Delta^{1/2}}\right)\right],\quad \Delta>0, \tag{13a}$$

where $C$ is determined from the normalization condition and is found to be,

$$C=\frac{\lambda_1-\lambda_2}{2}\operatorname{csc h}\left[-\frac{16(\lambda_2-\lambda_1)\alpha}{\Delta^{1/2}}\right]\exp\left\{\frac{16(\lambda_2-\lambda_1)}{\Delta^{1/2}}\left[\tanh^{-1}\left(\frac{b+2a}{\Delta^{1/2}}\right)+\frac{\alpha}{2}\right]\right\},$$

where $\alpha$ is given by

$$\tanh\alpha=\left[1-\frac{2c_{12}c_{21}S_2^+}{k_{11}^2 S_1(2\omega_1)+k_{22}^2 S_1(2\omega_2)-2c_{12}c_{21}S_2^-}\right]^{1/2}.$$

For $\Delta<0$ these expressions are

$$\mu(\phi)=C\frac{\sin 2\phi}{\psi^2(\phi)}\exp\left[-\frac{16(\lambda_2-\lambda_1)}{(-\Delta)^{1/2}}\tan^{-1}\left(\frac{b+2a\sin^2\phi}{(-\Delta)^{1/2}}\right)\right],\quad \Delta<0, \tag{13b}$$

$$C=\frac{\lambda_1-\lambda_2}{2}\operatorname{csc h}\left[-\frac{16(\lambda_2-\lambda_1)\alpha}{(-\Delta)^{1/2}}\right]\exp\left\{\frac{16(\lambda_2-\lambda_1)}{(-\Delta)^{1/2}}\left[\tanh^{-1}\left(\frac{b+2a}{(-\Delta)^{1/2}}\right)+\frac{\alpha}{2}\right]\right\},$$

$$\tan\alpha=\left[1-\frac{2c_{12}c_{21}S_2^+}{k_{11}^2 S_1(2\omega_1)+k_{22}^2 S_1(2\omega_2)-2c_{12}c_{21}S_2^-}\right]^{1/2},$$

and for $\Delta=0$,

$$\mu(\phi)=C\frac{\sin 2\phi}{\psi^2(\phi)}\exp\left[-\frac{16(\lambda_2-\lambda_1)}{b+2a\sin^2\phi}\right],\quad \Delta=0, \tag{13c}$$

$$C=\frac{\lambda_1-\lambda_2}{2}\operatorname{csch}\left[\frac{4(\lambda_2-\lambda_1)\alpha}{\left|c_{12}c_{21}S_2^+\right|}\right]\exp\left\{\frac{16(\lambda_2-\lambda_1)}{b+2a}-\frac{4(\lambda_2-\lambda_1)}{\left|c_{12}c_{21}S_2^+\right|}\right\}.$$

Employing Khas'minskii [6] formulation (See also Ariaratnam et. al. [2,3]) the largest Lyapunov exponent of system (6) is given by

$$\lambda=E[Q(\phi)]=\int_0^{\pi/2}Q(\phi)\mu(\phi)d\phi\,. \tag{14}$$

Substituting from equation (8) and (13) in equation (14) and performing the indicated integration yields the following expression for the Lyapunov exponent:

$$\lambda=\frac{1}{2}\left\{(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)\coth\left[\frac{8(\lambda_1-\lambda_2)\alpha}{\Delta^{1/2}}\right]\right\}+\frac{c_{12}c_{21}S_2^-}{8},\quad \Delta>0\ , \tag{15a}$$

$$\lambda=\frac{1}{2}\left\{(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)\coth\left[\frac{8(\lambda_1-\lambda_2)\alpha}{(-\Delta)^{1/2}}\right]\right\}+\frac{c_{12}c_{21}S_2^-}{8},\quad \Delta<0, \tag{15b}$$

$$\lambda=\frac{1}{2}\left\{(\lambda_1+\lambda_2)+(\lambda_1-\lambda_2)\coth\left[\frac{4(\lambda_1-\lambda_2)\alpha}{\left|c_{12}c_{21}S_2^+\right|}\right]\right\}+\frac{c_{12}c_{21}S_2^-}{8},\quad \Delta=0\ . \tag{15c}$$

The system is asymptotically stable with probability one (w. p. 1) if $\lambda$ is negative and unstable w. p. 1 if $\lambda$ is positive.

## 4. APPLICATION

As an application let us determine the stochastic stability of a thin-walled beam of the length $L$ loaded by a random axial force $P(t)$ and by equal random moments $M(t)$ at the beam ends, as shown in Fig. 1 is considered. The partial differential equations of such elastic systems are given in the paper written by Joshi and Suryanarayan [4] for the deterministic case of loading,

$$-EI_x\upsilon''''+P(t)\upsilon''+M(t)\theta''-\rho A\ddot{\upsilon}-\beta\dot{\upsilon}=0\ , \tag{16a}$$

$$\left(GJ+\frac{P(t)I_0}{A}\right)\theta''+M(t)\upsilon''-\rho I_0\ddot{\theta}-\beta\dot{\theta}=0\ . \tag{16b}$$

Here $EI_x$ and $GJ$ the Euler bending and St. Venant torsional stiffness; $P(t)$ and $M(t)$ are the random loads; $\beta$, $\gamma$ is the viscous damping coefficients; $\rho$ is the mass density at the material; $A$ is the area of cross-section; $I_0$ is the polar moment of inertia $\upsilon$ and $\theta$ are the infinitesimal flexural and torsional displacements of the cross-section. The primes and dots denote partial differentiation with respect to $z$ and $t$, respectively. For the case of simply supported ends, the following boundary conditions must be satisfied: without warping and without possibility of rotation,

$$\upsilon(0,t)=\upsilon(L,t)=\upsilon''(0,t)=\upsilon''(L,t)=0\,,$$
$$\theta(0,t)=\theta(L,t)=0\ .$$

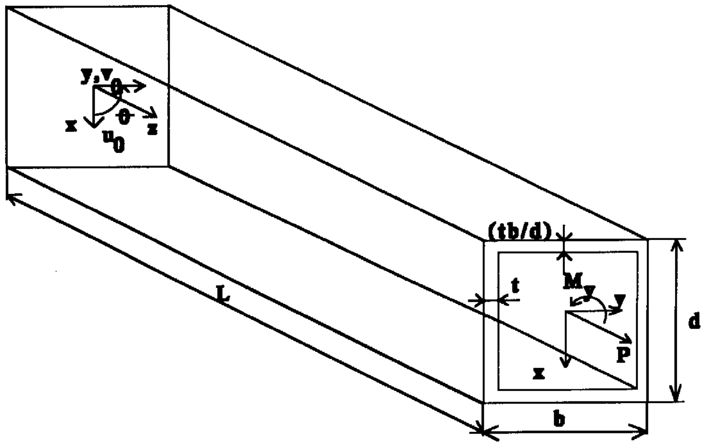


Fig. 1. A typical thin-walled beam of warples cross-section

Considering the fundamental mode, the above boundary conditions are satisfied by taking,

$$\upsilon = q_1(t)\sin\left(\frac{\pi z}{L}\right),\ \ \upsilon = q_2(t)\sin\left(\frac{pz}{L}\right)\,,$$

which, when substituted in the equations motion, yield,

$$\ddot{q}_1 + 2\beta_1\dot{q}_1 + \omega_1^2 q_1 + \omega_1[\omega_2\zeta_1(t)q_1 + i_0^2\omega_2\zeta_2(t)q_2] = 0\,, \tag{17a}$$

$$\ddot{q}_2 + 2\beta_2\dot{q}_2 + \omega_2^2 q_2 + \omega_2[\omega_1\zeta_1(t)q_2 + \omega_2\zeta_2(t)q_1] = 0\,, \tag{17b}$$

where

$$\omega_1^2 = \frac{EI_x\pi^4}{\rho AL^4}, \omega_2^2 = \frac{GJ\pi^2}{\rho I_0 L^2},\ \beta_1 = \frac{\beta}{2\rho A},\ \beta_2 = \frac{\gamma}{2\rho I_0}, = 0\,,$$
$$i_0^2 = \frac{I_0}{A},\ \zeta_1(t) = \frac{Li_0}{\pi}\frac{P(t)}{(EI_xGJ)^{1/2}},\ \zeta_2(t) = \frac{L}{\pi i_0}\frac{M(t)}{(EI_xGJ)^{1/2}}\ .$$

Comparing equations (17) with (5), $k_{11}=\omega_2$, $k_{22}=\omega_1$, $c_{12} = i_0^2\omega_2$, $c_{21}=\omega_1$, so that substituting these values in equation (15) leads to the boundary of almost-sure stability, obtained by setting $\lambda = 0$, is given by the equation

$$\left[-8\beta_1 + k_{11}S_1(2\omega_1) + c_{12}c_{21}S_2^-\right]\exp\left[\frac{8\beta_2 + k_{11}S_1(2\omega_1)}{4p}\right]$$
$$= \left[-8\beta_2 + k_{22}S_1(2\omega_2) + c_{12}c_{21}S_2^-\right]\exp\left[\frac{8\beta_1 + k_{22}S_1(2\omega_2)}{4p}\right], \tag{18}$$

where

$$p = \frac{\Lambda^{1/2}}{8\alpha}, \ \tanh\alpha = \left[1 - \left(\frac{2c_{12}c_{21}S_2^+}{k_{11}^2 S_1(2\omega_1) + k_{22}^2 S_1(2\omega_2) - 2c_{12}c_{21}S_2^-}\right)^2\right]^{1/2}, \ \Delta > 0,$$

$$p = \frac{(-\Lambda)^{1/2}}{8\alpha}, \ \tan\alpha = \left[\left(\frac{2c_{12}c_{21}S_2^+}{k_{11}^2 S_1(2\omega_1) + k_{22}^2 S_1(2\omega_2) - 2c_{12}c_{21}S_2^-}\right)^2 - 1\right]^{1/2}, \ \Delta < 0,$$

$$p = \left|c_{12}c_{21}S_2^+\right|, \ \Delta = 0.$$

## 5. CONCLUSIONS

A method of calculating the Lyapunov exponent at a class two degrees-of-freedom systems subjected combined random parametric excitations of two correlated stationary stochastic wide-band processes has been presented. Explicit expressions for the largest Lyapunov exponent, valid in the first approximation for non-singular case, have been obtained and applied to an example in the stochastic stability of coupled flexural-torsional oscillations of closed thin-walled beam under combined action of axial loads and equal ends moments. The amount of damping necessary to ensure stability has been found to depend only on those values of excitation spectral density of axial loads near twice the eigenfrequency and the sum and difference of the eigenfrequencies of excitation of spectral density of the ends moments.

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# EKSPONENTI LJAPUNOVA I STOHASTIČKA STABILNOST SPREGNUTIH LINEARNIH SISTEMA POD DEJSTVOM POVEZANIH SLUČAJNIH PROCESA ŠIROKOG SPEKTRA

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*U ovom radu istraživana je skoro sigurna asimptotska stabilnost jedne klase linearnih sistema sa dva stepena slobode pod dejstvom parametarskih široko pojasnih korelisanih slučajnih procesa malog intenziteta.*