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MATHEMATICAL MODEL FOR THE CALCULATION OF RESISTANCE TO HEAT TRANSMISSION AT THE CROSS-FLOW OF GAS IN TUNNEL OVENS FOR THE PRODUCTION OF CONSTRUCTION CERAMICS

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Abstract. *The paper presents a mathematical model for the calculation of resistance to heat transmission at the cross-flow of gas. The obtained mathematical expressions show that the resistance to heat transfer at the cross-flow of gas is determined by the following influential values: the method of setting processed elements, the setting height, the available cross passages, elements density, the heating/cooling rate and the characteristic temperature in the voids existing between set elements.*

Tunnel ovens represent complex plants in which heat energy and mass are transferred in physical and chemical processing of materials. Fig. 1 shows the simplest process developing in a tunnel oven for the production of construction ceramics. The upper part of the Figure shows the flow of dry clay materials, then the flow of air and fuel heating gases, while the bottom part of the Figure shows the distribution of temperatures of both media flowing in the opposite direction.

The cross flow of gas in tunnel ovens occurs as a consequence of local temperature differences between the edge gaps and the center of the setting. So, for example, in the cooling zone, set elements leaving the heating zone are warmer that the air flowing in the opposite direction through the edge gaps and passages provided, with the air in voids in the heap being warmer than that in the edge gaps and passages. Due to this, a difference in density occurs between the air inside the voids in the heap and the air of the axial flow current.

On the other hand, this difference in densities causes a difference in pressure, due to

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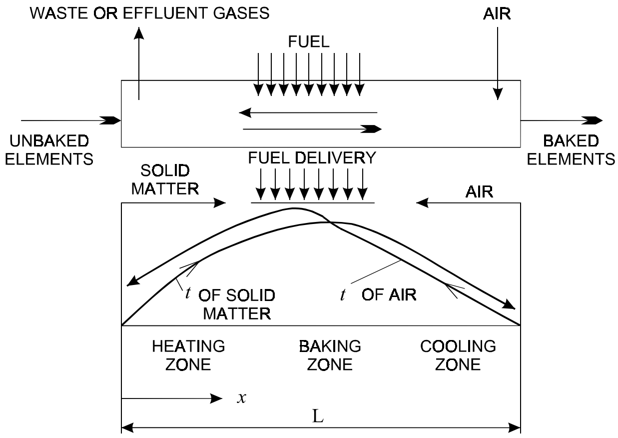


Fig. 1. Schematic presentation of current flows
and temperature distribution in the tunnel oven

which the relatively hot air from the voids in the heap flows upward, whereas simultaneously relatively colder air from the edge gaps and passages flows in the lower part of the heap (Fig. 2). In compliance with this, and based on other temperature relations prevailing in the heating zone, a cross flow occurs with the direction of flow opposite to that shown in Fig. 2.

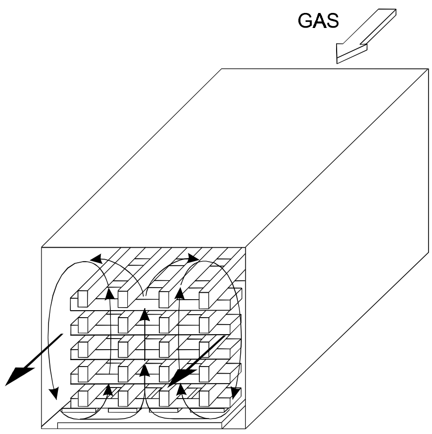


Fig. 2. Schematic illustration of a tunnel oven with the gas flowing axially through the setting and the edge gaps and the cross-flow through the setting

Fig. 3 shows schematically cross sections in the tunnel oven heating and cooling zones with current flows and temperature distribution. It is noted that temperature over the oven cross section is constant in principle. Deviations occur only in the edge areas where there is direct contact with the axial flow of gas. Measurements taken on tunnel ovens confirm this distribution of temperature [7].

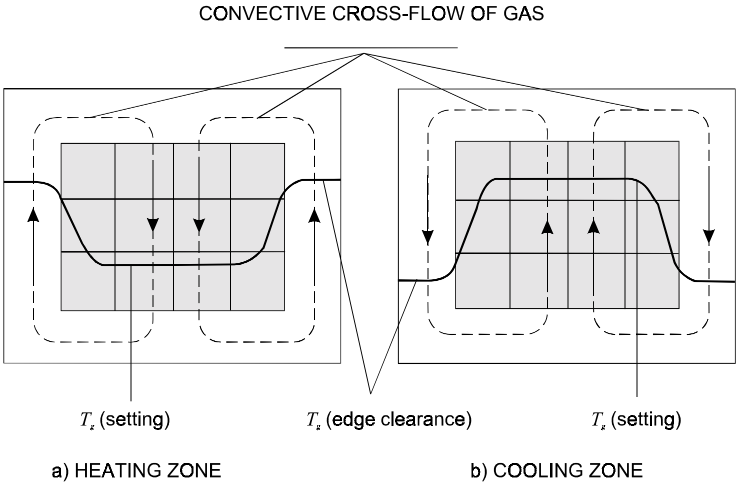


Fig. 3. Schematic presentation of tunnel oven cross sections with current flows and temperature distribution

The updraft force resulting from the difference in density is opposed to by the resistance to the flow. It is assumed that all the resistances to flow in relation to the resistance to the flow through the setting arrangement are negligible [6]. In compliance with this, the following dependence applies:

$$g \cdot h_s \cdot \Delta\rho = \Psi_z \cdot \rho_z \cdot \frac{w^2}{2}\,, \tag{1}$$

where:

g - gravity,

h_s - setting height,

$\Delta\rho$ - difference between the mean density of fluids in the edge gaps and passages and the mean density in the voids in the setting,

Ψ_z - coefficient of resistance to vertical flow through the setting,

ρ_z - cross-flow fluid density,

w_z - cross-flow fluid velocity.

Taking into account the connection between the coefficient of resistance Ψ_z and the coefficient of pressure fall ξ_z [2], defined on the basis of the isothermal non-compressible flow of ideal gases in a pipe – where the height of the setting h_s is taken for the length of the flow channel, and for the diameter – hydraulic diameter d_h – we have

$$g \cdot h_s \cdot \Delta\rho = \xi_z \cdot \frac{h_s}{d_h} \cdot \rho_z \cdot w_z^2\,, \tag{2}$$

wherefrom:

$$w_z^2 = \frac{g \cdot d_h \cdot \Delta\rho}{\xi_z \cdot \rho_z} \tag{3}$$

From the above equation, it can be concluded that the height of the setting does not affect the velocity of cross-flow, and hence the intensity of heat exchange.

Using the equation for the ideal gas state, the relation $\Delta\rho/\rho$ can be expressed in the following form:

$$\frac{\Delta\rho}{\rho} = \left| \frac{\overline{T}_g - \overline{T}_z}{\overline{T}_g} \right| , \tag{4}$$

where: $\overline{T}_g$ – the mean temperature of gas in the axial current; $\overline{T}_z$ – characteristic temperature of gas in the setting voids.

The mass cross-flow of fluid $\dot{m}_z$ over the unit length of the oven is determined from the equation of continuity:

$$\dot{m}_z = w_z \cdot \rho_z \cdot A_z \ . \tag{5}$$

With the S. Ergun's [8] coefficient of the setting looseness introduced, the characteristic (free) area for the vertical current flow, which, reduced to the unit length l_e, is:

$$A_z = \varepsilon\, A_{os} \ , \tag{6}$$

where: A_{os} – the area of the setting base over the unit length.

From equations (3), (4) and (5) it follows that:

$$\dot{m}_z = \rho_z A_z \left[\frac{g d_h \left(\overline{T}_g - \overline{T}_z \right)}{\xi_z \overline{T}_g} \right]^{1/2} . \tag{7}$$

For the setting design shown in Fig. 4 and the geometric relations b/a and c/b, the coefficient of pressure fall ξ_z for the longitudinal (axial) flow through the setting, according to the reference literature [5], is given by the expression:

$$\xi_z = \left(\frac{b}{a} \right)^{0.5} \left(\frac{c}{b} \right)^{0.45} \left[\frac{160}{R_e} + 0.48 \left(\frac{c}{b} \right)^{0.2} \right] . \tag{8}$$

Numerous tests have confirmed that equation (8) can be applied with sufficient accuracy for vertical flows through the setting [5]. It is worth also to mention that between the heat exchange and the exchange of impulses in the setting – with the assumptions made – there is a full analogy.

For the flow through the setting, the Reynolds's criterion of similarity is defined in the following way:

$$R_e = \frac{w_z d_h}{\nu} \ , \tag{9}$$

where: ν – kinematics viscosity of fluid in the cross-flow, w_z – the characteristic velocity of the cross-flow fluid, d_h – hydraulic diameter, defined in the following way [8]:

$$d_h = 4 \frac{V_z}{A_{so}} \ , \tag{10}$$

where V_z – is the free volume of the setting, and A_{so} – is the sum of free outer gaps within the setting (effective heat exchange area).

The heat flow over the unit length of the oven effected by the cross-flow of gas is equal to the difference of the main axial current fluid state enthalpy and the state enthalpy of the fluid within the setting voids:

$$\dot{Q} = \dot{m}_z c_{pm} \left(\overline{T}_g - \overline{T}_z \right) \ . \tag{11}$$

The heat flow transmitted by the cross-flow of gas will cause a corresponding increase of the enthalpy of products, which, for the unit length l_e, is determined by the following expression:

$$\dot{H} = \dot{m}_s \cdot c_s \cdot \frac{d\overline{T}_s}{dx} \cdot l_e \ . \tag{12}$$

Upon introducing the rate of products heating defined as:

$$\dot{T}_s = \frac{d\overline{T}_s}{d\tau} \ , \tag{13}$$

equation (12) can be expressed in the following form:

$$\dot{H} = \dot{m}_s \cdot c_s \cdot \dot{T}_s \cdot \frac{d\tau}{dx} \cdot l_e \ . \tag{14}$$

Taking into account that the speed of products motion through the oven is constant, it follows that:

$$\dot{H} = \dot{m}_s \cdot c_s \cdot \dot{T}_s \cdot \frac{\tau}{L} \cdot l_e \ , \tag{15}$$

where: τ – baking time, L – total length of the oven.

On the other hand, using the equation of state for the ideal gas and normal conditions, equation (7) obtains the following form:

$$\dot{m}_z = \rho_n \frac{T_n}{\overline{T}_z} A_z \left[\frac{g d_h}{\xi_z} \frac{\overline{T}_g - \overline{T}_z}{\overline{T}_g} \right] \ , \tag{16}$$

the substitution of which in equation (11), with subsequent equalization of the right-hand sides of equations (11) and (15) and their sorting results in the following expression:

$$\frac{\overline{T}_g - \overline{T}_z}{\left(\overline{T}_g\right)^{1/3}} = \left[\left(\frac{c_s}{\rho_n \cdot c_{pm}} \cdot \dot{T}_s \cdot \dot{m}_s \cdot \tau \cdot \frac{l_e}{L \cdot A_z} \cdot \frac{\overline{T}_z}{T_n} \right)^2 \frac{\xi_z}{g \cdot d_h} \right]^{1/3} \ . \tag{17}$$

In order to simplify the expression for determining the temperature differential $\overline{T}_g - \overline{T}_z$, the notion of the stacking coefficient is introduced and it represents the ratio between the sum of the setting bases areas and the total area of the tunnel oven base:

$$\varphi = \frac{A_{os,uk}}{L \cdot B} \ . \tag{18}$$

So, the characteristic cross section for vertical flow over the unit length l_e can be expressed as:

$$A_z = \varepsilon \cdot A_{os,uk} \frac{l_e}{L} \ , \tag{19}$$

or, by using expression (18), as:

$$A_z = \varepsilon \cdot \varphi \cdot l_e \cdot B \ . \tag{20}$$

If the above expression (20) is included in equation (17), the following is obtained:

$$\frac{\overline{T}_g - \overline{T}_z}{\left(\overline{T}_g\right)^{1/3}} = \left[\left(\frac{c_s}{\rho_n \cdot c_{pm}} \cdot \dot{T}_s \cdot \dot{m}_s \cdot \frac{\tau}{L \cdot B \cdot \varphi} \cdot \frac{\overline{T}_z}{T_n}\right)^2 \frac{\xi_z}{\varepsilon^2 \cdot g \cdot d_h}\right]^{1/3} \ . \tag{21}$$

On the other hand, the total volume of the setting that passes through the oven within time τ is defined by the expression:

$$V_{s,uk} = (1-\varepsilon) h_s \cdot L \cdot B \cdot \varphi \ . \tag{22}$$

Therefore, based on expression (22), the mass flow can be expressed as:

$$\dot{m}_s = \rho_s (1-\varepsilon) h_s \frac{L \cdot B \cdot \varphi}{\tau} \ , \tag{23}$$

wherefrom:

$$\dot{m}_s \frac{\tau}{L \cdot B \cdot \varphi} = (1-\varepsilon) \rho_s h_s \ . \tag{24}$$

By including this expression in equation (22), we get:

$$\frac{\overline{T}_g - \overline{T}_z}{\left(\overline{T}_g\right)^{1/3}} = \left[\left(\frac{\rho_s \cdot c_s}{\rho_n \cdot c_{pm}} \cdot \dot{T}_s \cdot \frac{\overline{T}_z}{T_n}\right)^2 \frac{h_s}{g} \cdot \xi_z \frac{h_s}{d_h} \left(\frac{1-\varepsilon}{\varepsilon}\right)^2\right]^{1/3} \ . \tag{25}$$

In order to simplify further the formula for determining the temperature differential $\overline{T}_g - \overline{T}_z$, the non-dimensional coefficient of through-flow defined by the below given expression is introduced:

$$\omega = \xi_z \frac{h_s}{d_h} \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 \ . \tag{26}$$

By substituting the expression (26) in equation (25), the following is obtained:

$$\frac{\overline{T}_g - \overline{T}_z}{\left(\overline{T}_g\right)^{1/3}} = \left[\left(\frac{\rho_s \cdot c_s}{\rho_n \cdot c_{pm}} \cdot \dot{T}_s \cdot \frac{\overline{T}_z}{T_n}\right)^2 \frac{h_s}{g} \cdot \omega\right]^{1/3} \ . \tag{27}$$

As shown by equation (27), the temperature differential $\overline{T}_g - \overline{T}_z$ is directly proportional to the cube root from:

– the square of the baked products density ρ_s,

– the square of the heating rate $\dot{T}_s$,

– the square of $\overline{T}_z$,
– the height of the setting h_s,
– the through-flow coefficient.

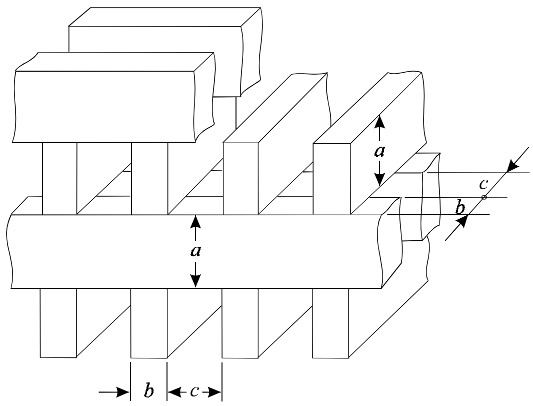


Fig. 4. Setting geometry

The through-flow coefficient ω, defined by equation (26), is determined with the help of equation (6), (8) and (10).

The coefficient of looseness ε, figuring in the expression for ω, in compliance with expression (6) and Fig. 4, will be:

$$\varepsilon = \frac{c}{c+b}, \tag{28}$$

whereas the hydraulic diameter, reduced to a single cell of the setting, will be:

$$d_h = 2\frac{a \cdot c}{a+(1-\varepsilon)c}, \tag{29}$$

with:

$$\omega = 0.5\left(\frac{b}{a}\right)^{0.5}\left(\frac{c}{b}\right)^{-1.55}\left[\frac{160}{R_e}+0.48\left(\frac{c}{b}\right)^{0.2}\right]\left[\frac{h_s}{c}+\frac{h_s}{a}\left(1-\frac{c}{b}\right)^{-1}\right]. \tag{30}$$

For the JNF hollow brick, the size of which is a = 0.120m and b =0.065m, we would have:

$$\frac{\omega}{h_s} = \left(\frac{0.8511}{R_e c^{1.55}}+\frac{0.0044}{c^{1.35}}\right)\left(\frac{1}{c}+\frac{0.5417}{c+0.065}\right). \tag{31}$$

From equation (31), it may be concluded that the quotient ω/h_s depends on the R_e number and distance between bricks.

Fig 5 shows the dependence of quotient ω/h_s from Reynolds's number R_e and the distance between bricks c. For this calculation, using Fig. 5 or equation (31), it will be appropriate to use R_e =400.

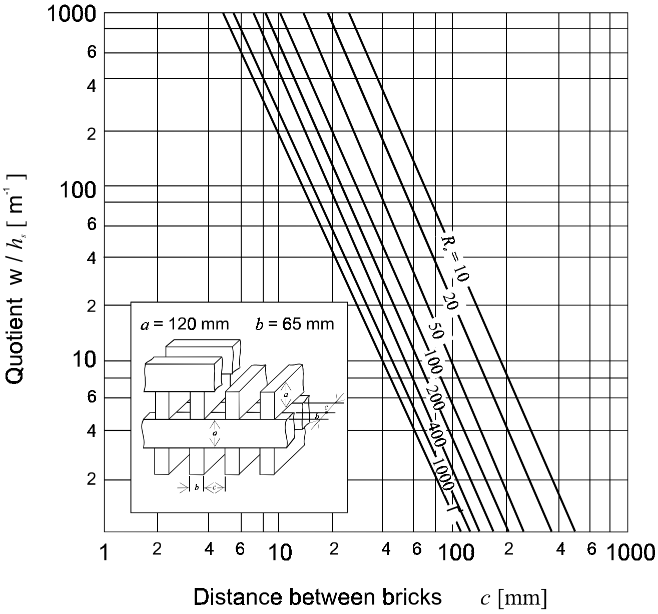


Fig. 5. Dependence of ω/h_s quotients from the distance between bricks c and R_e-number for JNF hollow bricks setting

By introducing the values of ρ_n, T_n and g into equation (27) and sorting the latter, we shall obtain:

$$\frac{\overline{T}_g-\overline{T}_z}{\left[\left(\overline{T}_g-\overline{T}_z\right)+\overline{T}_z\right]^{1/3}}=4\cdot10^{-5}\left[\left(\rho_s\cdot\dot{T}_s\cdot\overline{T}_s\right)^2 h_s\cdot\omega\right]^{1/3} . \qquad (32)$$

If the following units are taken for ρ (=) kg/m^3, $\dot{T}_s$ (=) K/h, $\overline{T}_z$ (=) K and h_s (=) m, then the right-hand side of the above equation is expressed in K$^{2/3}$.

Fig. 6 provides a diagram from which the temperature differential $\overline{T}g-\overline{T}z$ can be determined from equation (32) and the mean temperature of gas in the setting voids. Wherein the mean temperature of gas $\overline{T}g$ is substituted by the expression $\left(\overline{T}g-\overline{T}z\right)+\overline{T}z$. Taking into account that differences in temperature occurring in actual ovens are always below 1000K, the upper part of the curve set can be excluded from consideration.

The iterative procedure is applied to determine the value of Reynolds's number, which was taken as known for the graphical solution of equation (31). For the first step of iteration, it was assumed that $R_e = 400$, because for the cross-flow this value lies in the middle of the usual range of Reynolds's numbers. After the ratio ω/h_s has been found, then, based on the selected temperature of gas in the setting voids, the iterative procedure is applied using equation (32) to obtain the temperature differential $\overline{T}g-\overline{T}z$. In this procedure, temperature $\overline{T}z$ changes gradually beginning from $\overline{T}g=\overline{T}z$ until both sides of

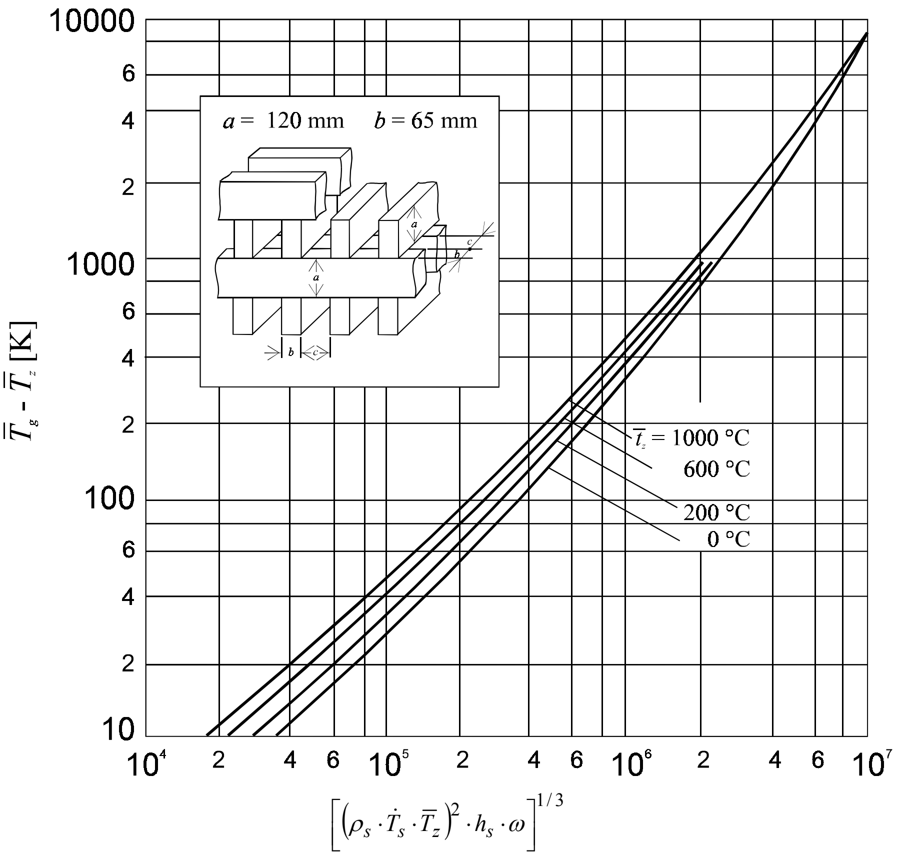


Fig. 6. Diagram for calculation of $\overline{T}g - \overline{T}z$ temperature difference

equation (32) become equal. The second iteration series develops within the iteration band for determining the value of Reynolds's number. In order to set up the expression for Reynolds's number, it is necessary first to determine the rate of cross-flow, which, with equations (7) and (5) taken into account, can be written in the following form:

$$w_z = \left[\frac{gd_h}{\xi_z}\frac{\overline{T}_g - \overline{T}_z}{\overline{T}_g}\right]^{1/2} . \tag{33}$$

Using equations (26) and (28), the above expression receives the form:

$$w_z = \frac{b}{c}\left[g\frac{h_s}{\omega}\frac{\overline{T}_g - \overline{T}_z}{\overline{T}_g}\right]^{1/2} , \tag{34}$$

where h_s/ω is the reciprocal of the previously determined quotient ω/h_s, shown in Fig. 5.

Then, based on the known value of Reynolds's number (9), with equations (28), (29) and (34) applied:

$$R_e = \frac{1}{\nu}\frac{2ab}{a+\dfrac{bc}{b+c}}\left[g\frac{h_s}{\omega}\frac{\overline{T}_g-\overline{T}_z}{\overline{T}_g}\right]^{1/2} . \tag{35}$$

The following expression applies to JNF hollow bricks having the following dimensions: a = 0,120m and b =0,065m:

$$R_e = \frac{1}{\nu}\frac{0.065+c}{0.16+3.783c}\cdot\frac{1}{\left(\dfrac{\omega}{h_s}\right)^{1/2}}\cdot\frac{1}{\left(\dfrac{\overline{T}_z}{\overline{T}_g-\overline{T}_z}+1\right)^{1/2}} , \tag{36}$$

with units for ν (=) m^2/s, c (=) m, h_s (=) m and $\overline{T}$ (=) K.

The value of Reynolds's number is calculated, based on the known temperature differential $\overline{T}_g-\overline{T}_z$ (equation 32), through the expression under (36). The iterative process is carried on until the Reynolds's number value entered into equation (31) coincides with the value obtained from equation (36).

If the quotient ω/h_s and the temperature differential $\overline{T}_g-\overline{T}_z$ (determined from Figs. 5 and 6) are entered in equation (36), then Reynolds's number is obtained in function of the distance between bricks c and temperature $\overline{T}_z$. Fig. 7 provides the graphical presentation of this dependence, including the dependence of kinematics viscosity from temperature.

Based on equations (11) and (15), the right-hand sides of which are made equal, the following expression is obtained for the mass cross-flow of gas:

$$\dot{m}_z = \dot{m}_s\frac{c_s}{c_{mp}}\dot{T}_s\frac{\tau}{L}l_e\frac{1}{\overline{T}_g-\overline{T}_z} . \tag{37}$$

By rearranging the expression (37) with the help of equation (24), the following is obtained:

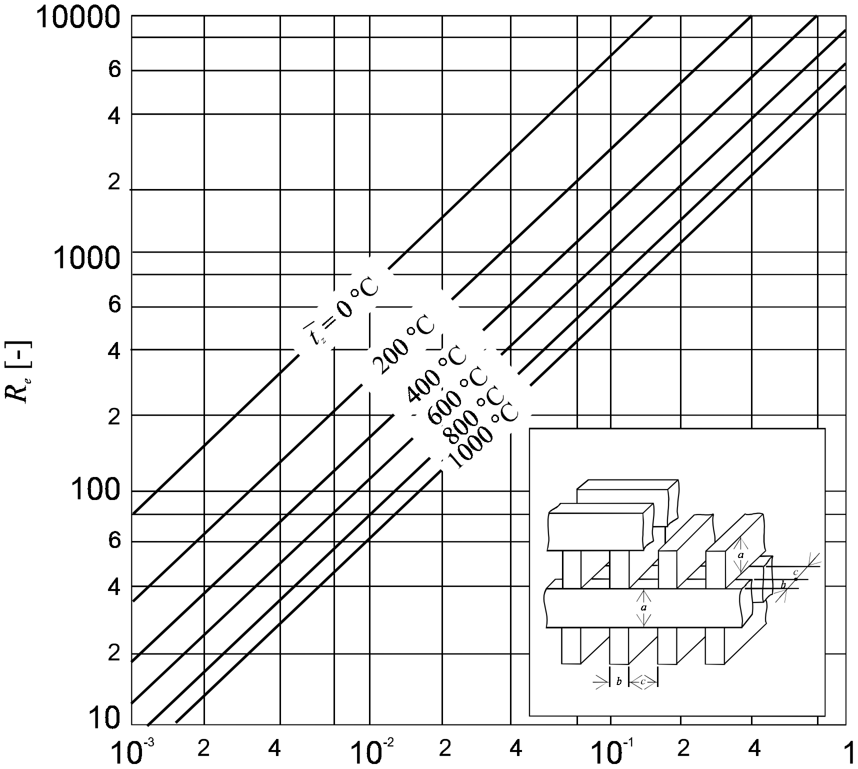
$$\dot{m}_z = \rho_s\frac{c_s}{c_{mp}}(1-\varepsilon)\varphi B h_s l_e\frac{\dot{T}_s}{\overline{T}_g-\overline{T}_z} . \tag{38}$$

On the other hand, the total current contact area within the unit length of the tunnel oven l_e is given by the following expression:

$$A_{so} = 2l_e\frac{h_s b_s}{a(b+c)}\left[a+c(1-\varepsilon)\right] . \tag{39}$$

Based on the developed equations (38) and (39) and the expression $R_m = A_{so}/(\dot{m}_z\cdot c_{pm})$ from the referenced literature [1], and by their combining, the following expression for the thermal resistance to heat transmission by the cross-flow of gas is obtained:

$$R_m = 2\frac{b_s}{\varphi\cdot B}\cdot\frac{1}{\rho_s\cdot c_s}\cdot\left(\frac{\varepsilon}{a}+\frac{1}{b}\right)\cdot\frac{\overline{T}_g-\overline{T}_z}{\dot{T}_s} . \tag{40}$$



$$\frac{0.065+c}{0.16+3.783c}\cdot\frac{1}{\left(\dfrac{\omega}{h_s}\right)^{1/2}}\cdot\frac{1}{\left(\dfrac{\overline{T}_z}{\overline{T}_g-\overline{T}_z}+1\right)^{1/2}}$$

Fig. 7. Reynolds's number for cross-flow through the JNF hollow brick setting

In order to simplify further the expression for determining the thermal resistance R_m, similar to the introduction of the setting coefficient φ non-dimensional coefficient of longitudinal utilization of the tunnel oven is introduced, and it is defined by:

$$\varphi_L = \frac{l_{s,uk}}{L} \quad , \tag{41}$$

where $l_{s,uk}$ is the total length of all settings (the total length of the tunnel oven less the lateral passages for fuel burning).

Thus, based on expression (18) and (41), the following relation is obtained:

$$\varphi = \frac{b_s}{B}\cdot\varphi_L \quad , \tag{42}$$

the introduction of which in equation (40) gives:

$$R_m = 2\frac{1}{\varphi_L}\cdot\frac{1}{\rho_S\cdot c_S}\cdot\left(\frac{1}{b}+\frac{\varepsilon}{a}\right)\cdot\frac{\overline{T}_g-\overline{T}_z}{\dot{T}_s} \quad . \tag{43}$$

For the JNF brick setting, equation (43) gets the following form:

$$R_m = \frac{1}{\varphi_L}\cdot\frac{1}{\rho_S}\cdot\frac{8.19+194.27c}{0.065+c}\cdot\frac{\overline{T}_g-\overline{T}_z}{\dot{T}_s} \tag{44}$$

where the following units apply: ρ_s (=) kg/m; c (=) m; $\overline{T}_g$ (=) K, $\overline{T}_z$ (=) K and $\dot{T}_s$ (=) K/h.

From equation (44), it may be concluded that the thermal resistance to heat transmission by the cross-flow of gas is determined by the following influential values:

– the method and geometry of setting a, b, c,
– the height of setting h_s,
– the participation of lateral passages $(1-\varphi_L)$,
– the density of elements ρ_s,
– the rate of heating/cooling $\dot{T}_s$ and
– temperature $\overline{T}_z$.

Having in mind the importance of the cross-flow of gas for heat transmission in the existing baking tunnel ovens, it is necessary to pay particular attention to the geometry of the setting. Increasing the coefficient of looseness of the setting reduces resistance to the cross-flow of gas, and this for different values of distance c may result in a ratio of even 1:5.

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MATEMATIČKI MODEL ZA PRORAČUN OTPORA PRENOSU TOPLOTE POPREČNIM STRUJANJEM GASA U TUNELSKIM PEĆIMA ZA PROIZVODNJU GRAĐEVINSKE KERAMIKE

Kemal Dž. Tahirbegović, Dimitrije K. Voronjec, Nenad V. Radojković

U radu je prikazan matematički model za proračun otpora prenosu toplote poprečnim strujanjem gasa. Dobijeni matematički izrazi pokazuju da je otpor prenosu toplote poprečnim strujanjem gasa određen sledećim uticajnim veličinama: načinom slaganja i geometrijom sloga, visinom sloga, učešćem poprečnih pasaža, gustinom elemenata, brzinom zagrevanja odnosno hlađenja i karakterističnom temperaturom u međuprostoru rešetke sloga.