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CONVECTIVE HEAT AND MASS TRANSFER UNDER THE CONDITIONS OF HYDRODYNAMIC STABILIZATION OF THE FLOW

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Abstract. *In this paper, the solution for convective heat and mass transfer in the part of hydrodynamic stabilization of the flow through the channel formed of two parallel plates, is given. The solution is given for the boundary conditions of the first kind .The similarity method between this problem and corresponding potential flow is applied, in order to obtain the solution.*

1. INTRODUCTION

Heat and mass transfer in hydrodynamic initial part of the channel between parallel plates, in principle, is similar to the same process in potential flow along isothermal or isoconcentrational flat plate.

In general case, the development of laminar boundary layer from the entering edge of the channel, is occurring, independently of external flow turbulence. When the certain thickness of laminar boundary layer has been achieved, the transition from laminar to turbulent flow will occur for so-called critical Re-number (Re_{xk}).

In order to determine critical Re-number, the mean velocity and turbulence intensity profiles were measured along the channel having the following dimensions 25x550x1000mm.

2. DESCRIPTION OF PHENOMENON

By the determination of the velocity and turbulence intensity profiles in the above mentioned channel, it was established that the fully developed flow is achieved for $Re_{xk}=1\cdot10^5$ (see chapter 5),where $Re_{xk}=(U_\delta\cdot x_k)/\nu$ (see Fig. 1)

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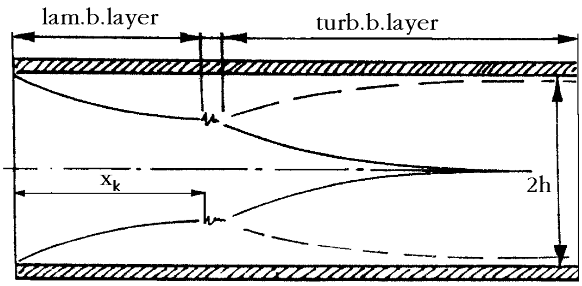

Fig. 1. Development of boundary layers in flow between two parallel plates

Since this value of Re-number is needed in relations for Nu-number, thus the determination of its critical value is of great importance.

Laminar fluid flow. In the case of the potential flow along isothermal flat plate, Blasius has determined the local Nusselt number as:

$$Nu_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{0,33} \tag{1}$$

According to the similarity of flow between parallel plates with flow along single plate for viscous flow in inlet part of channel, the local Nu-number can be expressed as:

$$Nu_x = K_1\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{o,33} \tag{2}$$

The constant K_1 will be determined later.
The expression (2) can be written in the following form:

$$\frac{\alpha \cdot x}{\lambda} = K_1\,\mathrm{Pr}_x^{1/3}\sqrt{\mathrm{Re}_x} = K_1\,\mathrm{Pr}^{1/3}\sqrt{\frac{u_\delta x}{\nu}} \tag{3}$$

The local thickness of laminar boundary layer [3] in the developing region can be determined as:

$$\delta = 2h\sqrt{1-\sqrt{1-\frac{2K^2}{\mathrm{Re}}\frac{x}{2h}}}\,,\ 2K^2 = 19,35 \tag{4}$$

and the maximal velocity in boundary layer [3]:

$$u_\delta = \left[1+\frac{\delta}{h}\left(1-\frac{1}{2}\frac{\delta}{h}\right)\right]V \tag{5}$$

If the velocity u_δ, from the eq.(5) is introduced into eq.(3) and using the definition of Nu and Re numbers, the eq.(3) get the following form:

$$\frac{Nu}{2}\frac{x}{2h} = K_1\,\mathrm{Pr}^{1/3}\sqrt{\left[1+\frac{\delta}{h}-\frac{1}{2}\left(\frac{\delta}{h}\right)^2\right]\frac{\mathrm{Re}}{2}\frac{x}{2h}} \tag{6}$$

By determination of the ratio δ/h from the eq.(4) and by its introduction into eq. (6), Nu-number can be expressed as:

$$Nu = 2K_1 \Pr^{1/3} \sqrt{\frac{2h}{x}\left[\frac{1}{2} + \sqrt{1-\sqrt{1-\frac{2K^2}{\mathrm{Re}}\frac{x}{2h}}}\left(1-\sqrt{1-\sqrt{1-\frac{2K^2}{\mathrm{Re}}\frac{x}{2h}}}\right)\right]\mathrm{Re}} \tag{7}$$

The following notation can be introduced:

$$X = \sqrt{1-\sqrt{1-\frac{2K^2}{\mathrm{Re}}\frac{x}{2h}}} \quad , \tag{8}$$

and the eq.(7) obtains the following form:

$$Nu_x = 2K_1 \Pr^{1/3} \sqrt{\frac{2h}{x}\left[\frac{1}{2} + X(1-X)\right]\mathrm{Re}} \tag{9}$$

The constant K_1 is determined from the known Nu-number for the fully developed flow:

$$\lim_{x \to x_k} Nu_x = 7{,}54 \tag{10}$$

Using the expression for the developing entry length

$$\frac{2h}{x} = \frac{1}{0{,}0225}\mathrm{Re}$$

one can obtain:

$$K_1 \Pr^{1/3} = 0{,}6545$$

and finally one can get the local Nusselt number:

$$Nu_x = 1{,}309\sqrt{\frac{2h}{x}\left[\frac{1}{2} + X(1-X)\right]\mathrm{Re}} \tag{11}$$

The expression for the Shervood-number can be obtained by analogous way.

Turbulent fluid flow. Using the assumption that the heat and mass transfer in turbulent fluid flow is similar to the heat and mass transfer along isothermal, (i.e. isoconcentrational) flat plate in potential fluid flow, and observing the case $0{,}5 \leq \Pr \leq 10$ and $0{,}5 \leq \mathrm{Sc} \leq 10$, one can, with sufficient accuracy, take that the thickness of hydrodynamic, heat and diffusion boundary layers are equal or proportional.

From the solution of the problem of heat transfer from isothermal flat plate in potential fluid flow [1]:

$$t_\delta - t_0 = -\frac{q_0''}{C_p}\frac{1}{\sqrt{\frac{\tau_0}{\rho}}}\left[5\Pr + 5\ln(5\Pr + 1) + \frac{u_\gamma}{\sqrt{\frac{\tau_0}{\rho}}} - 14\right], \tag{12}$$

since the flow in the entry region of channel is similar to the flow along single flat plate, for the considered case one can write:

$$\frac{\delta}{x} = k\,\mathrm{Re}_x^{-0,2} \tag{13}$$

In the considered case, there are the collinear entering of fluid into channel and high level of turbulence of incident fluid flow. Under these conditions, it can be taken that the developing length depends only on Re-number of flow in channel. Let find the functional dependence.

Substituting $\delta = h$, $x = x_k$ in eq.(13) one can obtain:

$$\left(\frac{x_k}{h}\right)^{-0,8} = K\left(\frac{u_c h}{\upsilon}\right)^{-0,2}$$

Taking into account the ratio V/u_c= 0,875 in velocity distribution according to the "power law" with power coefficient n = 1/7, and to the Re number definition (Re=$V \cdot 4h/\nu$) the above eq. will be:

$$\frac{x_k}{2h} = k_1\,\mathrm{Re}^{0,25}$$

The value of k_1 = 2,5 is obtained during the experimental procedure shown in this paper (see further), and the above reduces to the form:

$$\frac{x_k}{2h} = 2{,}5\,\mathrm{Re}^{0,25} \tag{14}$$

Finally for the developing length $\delta = h$, $x = h_k$, and the eq.(13) becomes:

$$\frac{2h}{x_k} = 2k\,\mathrm{Re}_{x_k}^{-0,2} = 2k\left(\frac{\mathrm{Re}}{2}\frac{x_k}{2h}\right)^{-0,2} = 2k\left(\frac{\mathrm{Re}}{2}2{,}5\,\mathrm{Re}^{0,25}\right)^{-0,2} = 2\cdot 0{,}956k\,\mathrm{Re}^{-0,25} \tag{15}$$

By combining eq.(14) and eq.(15) one can obtain the constant k = 0,2092. On this way, eq.(13) becomes:

$$\frac{\delta}{x} = 0{,}2092\,\mathrm{Re}_x^{-0,2} \tag{16}$$

The shear stress, in boundary layer is defined by [1]:

$$\tau_0 = 0{,}0228\rho u_\delta^2\left(\delta u_\delta / \upsilon\right)^{-1/4} \tag{17}$$

To calculate this shear stress, the approximation of constant maximal flow velocity in boundary layer, which is u_δ = 1,14V, is introduced. It means that this velocity is equal to the fluid velocity on axis of channel in the case of developed flow.

It is proved, by calculation [3], that the values of velocity u_δ at the transition of laminar to turbulent boundary layer are:

Re	12000	16000	20000	50000
u_δ	1,19 V	1,15 V	1,125 V	1,11 V

It can be seen that the value of maximal flow velocity in boundary layer given in upper table, is changed up to. $1{,}14V$ in developed flow.

The differences are not so great, so the previous assertion is proved.

According to the eq.(17) it is obtained that:

$$\frac{\tau_0}{\rho} = 0{,}0228 \cdot u_\delta^2 \cdot \left(\frac{\delta \cdot u_\delta}{\nu}\right)^{-0{,}25} = 0{,}0228 \cdot (1{,}14 \cdot V)^2 \cdot \left(\frac{\delta \cdot 1{,}14 \cdot V}{\nu}\right)^{-0{,}25} = \\ = 0{,}028676 \cdot V^2 \cdot \left(\frac{\delta \cdot V}{\nu}\right)^{-0{,}25} \tag{18}$$

The definition of thickness δ is one of the greatest problem. Because of that, it is eliminated as:

$$\frac{\delta}{x} = 0{,}2092 \cdot \left(\frac{u_\delta \cdot x}{\nu}\right)^{-0{,}2} = 0{,}2092 \cdot \left(\frac{1{,}14 \cdot V \cdot x \cdot 4 \cdot h}{\nu \cdot 4 \cdot h}\right)^{-0{,}2} = \\ = 0{,}23409 \cdot \mathrm{Re}^{-0{,}2} \cdot \left(\frac{x}{2 \cdot h}\right)^{-0{,}2}$$

Then :

$$\delta = 0{,}23409 \cdot \mathrm{Re}^{-0{,}2} \cdot \left(\frac{x}{2 \cdot h}\right)^{-0{,}2} \cdot \frac{x}{2 \cdot h} = 0{,}46818 \cdot h \cdot \mathrm{Re}^{-0{,}2} \cdot \left(\frac{x}{2 \cdot h}\right)^{0{,}8} \tag{19}$$

so:

$$\frac{\delta_V}{\upsilon} = \frac{0{,}46818 \cdot 4 \cdot h \cdot V}{4 \cdot \nu} \cdot \mathrm{Re}^{-0{,}2} \cdot \left(\frac{x}{2 \cdot h}\right)^{0{,}8} = 0{,}1175 \cdot \mathrm{Re}^{0{,}8} \cdot \left(\frac{x}{2 \cdot h}\right)^{0{,}8} \tag{20}$$

By substitution eq.(18) in eq.(20):

$$\frac{\tau_0}{\rho} = 0{,}049025 \cdot V^2 \cdot \mathrm{Re}^{-0{,}2} \cdot \left(\frac{x}{2 \cdot h}\right)^{-0{,}2} \tag{21}$$

To solve the eq.(12),it is needed to know the difference t_δ - t_0, i.e. the temperature t_δ at the border of boundary layer. According to the previous analogy for flow velocity, one can consider that the temperature on the same border is equal to the temperature in axial plane of channel in developed flow, i.e.

$$t_0 - t_\delta = 1{,}125 \cdot (t_0 - t_m) \tag{22}$$

Now, the eq.(12) can be solved,

$$t_\delta - t_0 = -\left(\frac{q_{0x}}{\rho \cdot C_p}\right) \cdot \frac{1}{\sqrt{\frac{\tau_0}{\rho}}} \cdot \left[\frac{u_\delta}{\sqrt{\frac{\tau_0}{\rho}}} - B\right] \tag{23}$$

where the constant B is :

$$B = 5 \cdot \mathrm{Pr} + 5 \cdot \ln(\mathrm{Pr} + 1) - 14 \tag{24}$$

If one introduces the convective heat transfer:

$$q_{0x} = \alpha_x \cdot (t_0 - tm_x) \tag{25}$$

or, when t_δ is equal t_c, then $tm_x = tm$, and eq.(25) can be expressed as:

$$q_{0x} = \alpha_x \cdot (t_0 - tm) \tag{26}$$

By substituting eq.(22) and eq.(21) in eq.(23), it follows:

$$\frac{\alpha_x}{\rho \cdot C_p} = \frac{1{,}125 \cdot \tau_0/\rho}{\frac{u_\delta}{\sqrt{\tau_0/\rho}} - B} \tag{27}$$

If τ_0/ρ evaluated from eq.(27), is substituted in eq.(21), keeping in mind that u_δ = 1,14 V and using the connexion of Stanton and Nusselt number $Nu_x = St_x Re\, Pr$, it is obtained the final expression for dimensionless coefficient of local heat transfer at entry part of channel formed by two parallel plates:

$$Nu = \frac{0{,}249 \cdot \mathrm{Re}^{0{,}9} \cdot \mathrm{Pr} \cdot (x/2 \cdot h)^{-0{,}1}}{5{,}15 \cdot \mathrm{Re}^{0{,}1} \cdot (x/2 \cdot h)^{0{,}1} - B} \tag{28}$$

In the case of flat phase separation surface (for example: mass transfer from immovable liquid into gas flow), the Shervood number can be obtained on analogous way by substituting in eq.(24) from (28) the Pr number by the value for Sc-number.

In the another case i.e. convective mass transfer from moving liquid film into gas flow, the Shervood number is expressed in modified form. The pseudolaminar structure of the liquid film surface, disturbs the structure of laminar sublayer in the boundary layer, because of the increasing of liquid film velocity and the waving on the liquid surface.

Applying the semiempirical theory of the turbulent mass transfer, the Shervood number [2] is determined:

$$Sh = \frac{0{,}249 \cdot \mathrm{Re}^{-0{,}9} \cdot \left(\frac{x}{2 \cdot h}\right)^{-0{,}1} \cdot Sc}{5{,}15 \cdot \mathrm{Re}^{-0{,}1} \cdot \left(\frac{x}{2 \cdot h}\right)^{0{,}1} + B_1} \tag{29}$$

Where B_1 is defined as:

$$B_1 = \frac{y_1^+ + 5 \cdot \ln\left(\frac{y_b^+}{5} - 1 + \frac{1}{Sc}\right)}{\left[\frac{y_1^+}{5} - 1\right] + \frac{1}{Sc}} - 14 \tag{30}$$

The dimensionless distance y_b^+ of the film surface from the border between the buffer layer and turbulent core, is expressed:

$$y_b^+ = e^{1{,}8 + \ln y_1^+} \tag{31}$$

and the dimensionless thickness of laminar sublayer y_1^+, is defined as:

$$y_1^+ = 5 - f\left(\mathrm{Re}, \mathrm{Re}_1\right) \tag{32}$$

The form of function f is experimentally defined. For the case of air cooling of water film, the eq.(32) becomes:

$$y_1^+ = 5 - 0{,}018 \cdot \left(\mathrm{Re}_1 - 160\right) \cdot \mathrm{Re}^{-0{,}1} \tag{33}$$

with:

$$160 \le \mathrm{Re}_1 \le 500, 4000 \le \mathrm{Re} \le 25000$$

where Re_1 - Reynolds number of water film.
For $\mathrm{Re}_1 < 160$ the intensity of mass transfer is not dependent on water film velocity.

Finally, one can conclude that the heat and mass transfer at the entry part of channel is defined only by geometrical characteristics of channel, thermo-physical properties of fluid and flow velocity.

3. Fully developed velocity profile

This flow can be taken as the particular case of fluid flow in initial part of channel. Then exists: $\delta = h$.

On the basis of eq.(17) and keeping in mind that in this case $\delta = h$ and $u_\delta = u_c = 1{,}14V$ one can obtain:

$$\sqrt{\tau_0/\rho} = 0{,}20137 \cdot V \cdot \mathrm{Re}^{-0{,}125} \tag{34}$$

Taking into account that the temperature at external border of boundary layer is $t_\delta = t_0$ eq.(12) becomes:

$$t_c - t_0 = -\left(\frac{q_0}{\rho \cdot C_p}\right) \cdot \frac{1}{\sqrt{\frac{\tau_0}{\rho}}} \cdot \left[\frac{u_c}{\sqrt{\frac{\tau_0}{\rho}}} - B\right] \tag{35}$$

By using t_0 - t_c = 1,125 (t_0 - t_m), as the connexion between the temperature in axial plane of channel and mean fluid temperature, introducing convective heat transfer: $q_0=\alpha(t_0 - t_m)$, and using the relation between Stanton and Nusselt number, one can obtain:

$$Nu = \frac{0{,}227 \cdot \mathrm{Re}^{0{,}875} \cdot \mathrm{Pr}}{5{,}66 \cdot \mathrm{Re}^{0{,}125} - B} \tag{36}$$

when the ratio $x_k/2h$ = 2,5 $\mathrm{Re}^{0{,}25}$ is substituted in eq.(28), the eq.(36) is obtained. The eq.(36) is obtained, practically, without any simplification.

On the analogous way, for the case of the mass transfer from the moving liquid surface into gas flow, one can obtain:

$$Sh = \frac{0{,}227 \cdot \mathrm{Re}^{0{,}875} \cdot Sc}{5{,}66 \cdot \mathrm{Re}^{0{,}125} + B_1} \tag{37}$$

4. THE GENERAL CASE OF HEAT AND MASS TRANSFER

In the initial part of channel, at the entrance of it, laminar boundary layer is developed the first, independently of the bulk fluid flow turbulence. When the thickness of the boundary layer achieves the given value, it becomes turbulent. That transition appears as very short transient part. Since this length is small, the quantity of exchanged heat and mass can be neglected. Then this process takes part as in laminar and turbulent boundary layer.

As the transient criterion can be used the following equation [3]:

$$\frac{x_k}{2\cdot h}\cdot\left[1+2\sqrt{1-\sqrt{1-\frac{2\cdot k^2}{\mathrm{Re}}\cdot\frac{x_k}{2\cdot h}}}\cdot\left(1-2\sqrt{1-\sqrt{1-\frac{2\cdot k^2}{\mathrm{Re}}\cdot\frac{x_k}{2\cdot h}}}\right)\right]=\frac{2\cdot 10^5}{\mathrm{Re}} \qquad (39)$$

where $2k^2$ = 19,35.

The numerical values of Shervood number are obtained on the basis of the eq.(11) for the laminar boundary layer, and from the eq.(29) for the turbulent boundary layer in the part of hydrodynamic development, and from the eq.(37) for the developed turbulent flow. These results are shown on Fig.2. for the case of air cooling the water and Re=20000.

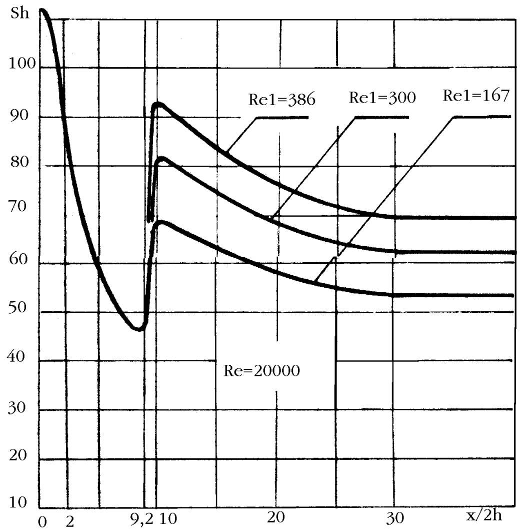

Fig.2. Variation of Sh - number along the channel for different Re_1 - numbers

5. EXPERIMENTAL PROCEDURE

The experimental verification of obtained analytical results is divided into two parts:

In the first part, the length of hydrodynamic stabilization was investigated. Experiments were done in vertical air channel, with dimensions 25x550x1000mm. The channel was mounted on the suction side of the fan, as it is shown in Fig. 3.

On the basis of numerous measurements of mean velocities and turbulence intensities (10 equidistant positions in longitudinal direction). The critical value of Re-number was obtained and shown in part 2.

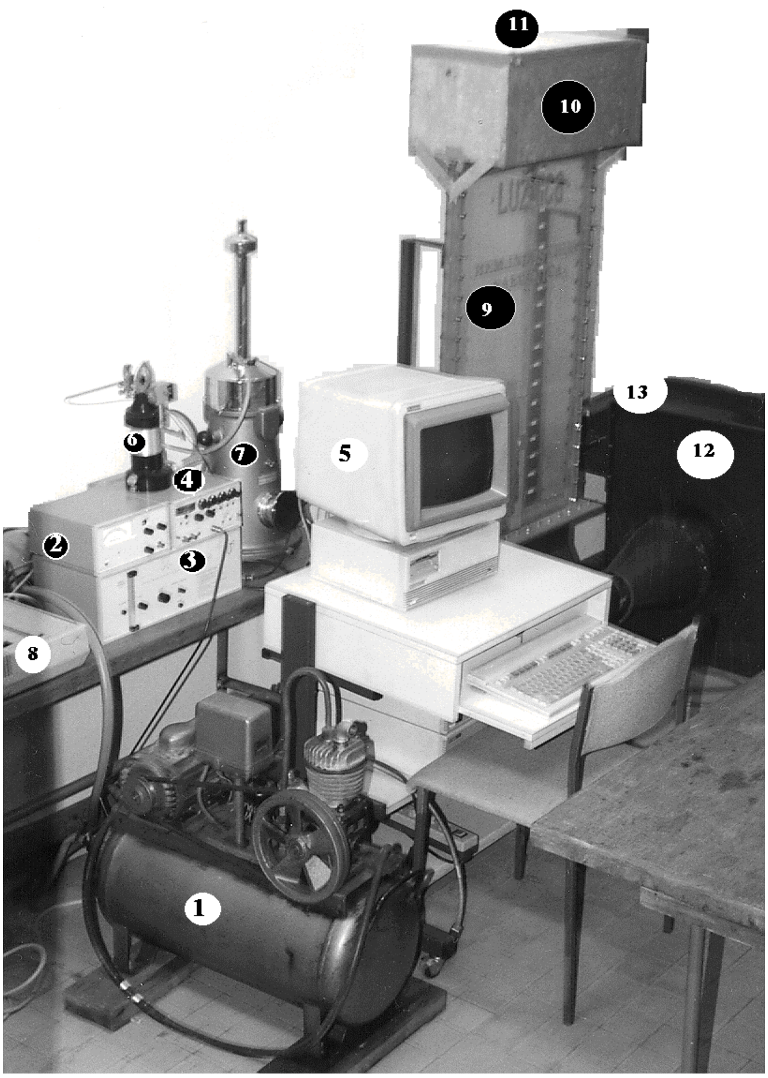

Fig. 3. Test facility

1-COMPRESSOR, 2-PRESSURE CONVERTER DISA 55D46, 3-PRESSURE CONTROL UNIT DISA 55D44, 4-MAIN UNIT DISA 55M01, 5-AQUIZITION SYSTEM HEWLET PACKARD 9133, 6-NOZZLE UNIT DISA 55D45, 7-BETZ'S MICROMANOMETER, 8-PRINTER, 9-TEST SECTION - CHANNEL, 10-CONFUSER, 11-FILTER, 12-FAN IMPELLER, 13-REGULATION VALVE

In figures 4 and 5, the characteristic diagrams, for axial distance x=600 mm from inlet, have been shown.

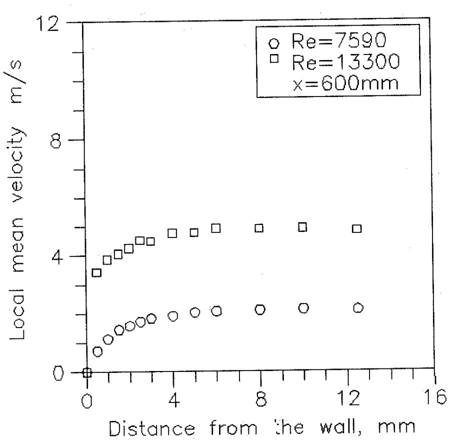

Fig. 4. Variation of local mean velocity versus distance from the wall

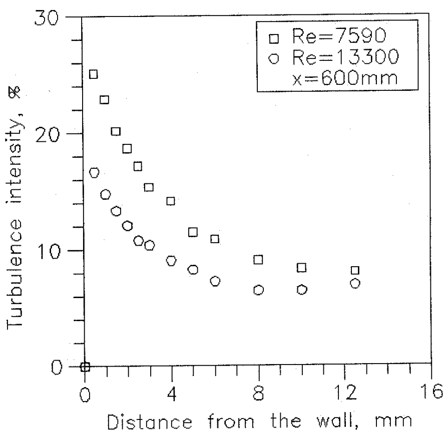

Fig. 5. Variation of turbulence intensity versus distance from the wall

The experimental investigation of mass transfer was done on the film type cooling tower, 700x700x74000mm. The dimensions of plates in packing were 700x770x1600 mm and the channel, formed of two neighboring plates, was wide 35mm.

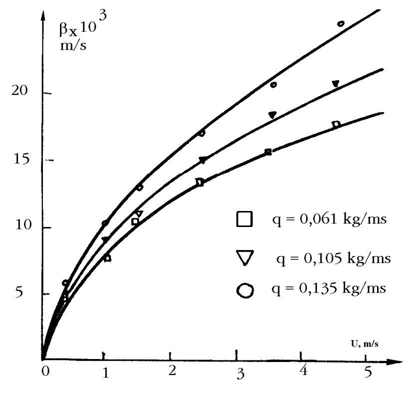

Fig. 6. Variation of mass transfer coefficient versus distance from the wall

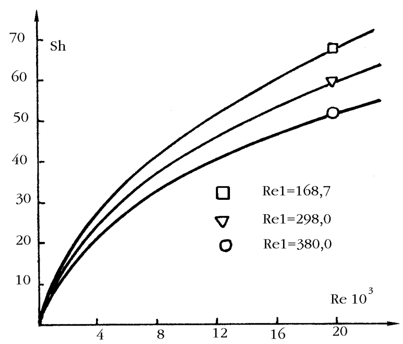

Fig. 7. Variation of Sh – number versus distance from the wall

The graphical representation of results for mass transfer coefficient is shown in Fig.6 and for Shervood number in Fig.7.

In order to compare analytical and experimental results, by numerical integration of curves on Fig.2 the average values for Sh criterion was obtained. These values are compared with experimental values shown on Fig.7. The agreement between them are very good (less then 5%).

Finally, instead of conclusion, it can be stated that it is possible, and for practical application acceptable, to use the theoretical solution of convective heat and mass transfer for both the part of developed flow and the part of flow hydrodynamic stabilization, in calculation characteristic quantities in cooling towers.

NON-STANDARD SYMBOLS

V - mean velocity of fluid
δ - local thickness of boundary layer
h - half of distance between plates

INDEXES

m - average
k - critical
o - wall
c - center

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KONVEKTIVNI TRANSPORT TOPLOTE I MASE U USLOVIMA HIDRODINAMIČKE STABILIZACIJE STRUJANJA

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U ovom radu dato je rešenje problema konvektivnog transporta toplote i mase u delu hidrodinamičke stabilizacije u kanalu izmedju dve paralelne ploče. Rešenje je dobijeno za granične uslove prve vrste. Za dobijanje ovog rešenja korišćen je metod sličnosti izmedju razmatranog problema i odgovarajućeg potencijalnog strujanja.