

## BRIDGE MEASURING CIRCUITS IN THE STRAIN GAUGE SENSOR CONFIGURATION

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**Abstract.** *In engineering sciences student education, electrical measurement of mechanical quantities can often be found. It is also widely used in engineering applications force measurements, particularly in machine tools. Load sensors, which are frequently used in mechanical engineering research, use strain gauges, bonded to a metal beam, to measure bidirectional force. The applied force induces extremely small changes in the resistance of strain gauges. In order to measure strain with a bonded resistance strain gauge, they must be connected to an electrical bridge measuring circuit. Strain gauge transducers usually employ four strain gauge elements, electrically connected to form a Wheatstone bridge circuit. The aim of this paper is to present the strain gauge conjunction with various measuring circuits, especially in Wheatstone bridge configuration. Also, the model for student education in the field of electrical measurement will be presented.*

**Key Words:** *Measurements, Strain Gauge Sensor, Measuring Circuits, Wheatstone Bridge, Student Education*

### 1. INTRODUCTION

The study of material properties under the working conditions is very important in engineering research since the mechanical properties of materials are influenced by stress application. Stress in a material cannot be measured directly. Therefore, in the stress analysis, the measured strain is combined with the other properties of the material so as to calculate the stress for a given load. In order to understand the behavior of used materials, it is very important to know the stress, as well as the strain, and their influence on designed machine part [1]. Based on various mechanical, optical, acoustic and electrical phenomena, there are numerous methods for measuring strain or deformation.

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An extensometer, developed in the 1800s, like the other initial devices for strain measurement, was mechanical in nature. This mechanical device is applied in materials testing, as well as the scientific research, to measure changes in the length of an object [2]. There are several optical methods for strain measurement [3]. Some of them use the interference fringes [4], photostress [5] or photoelasticity [6]. Defined as a sensor, whose resistance varies with applied force, the most frequent method of measuring strain, involving the mechanical engineering and electrical engineering, is a strain gauge (SG) [7]. It converts force, pressure, tension, weight, etc., into a change in electrical resistance. But, measuring a very small resistance change in a strain gauge ( $m\Omega$ ) with an ohmmeter is usually difficult. Therefore, the Wheatstone bridge, with strain gauges in one or more of the four arms of the bridge, is widely used in practice [8].

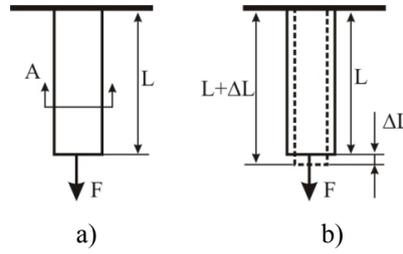
The history of SG manufacturing is very long. This device is applied in many fields, from the material testing applications, to numerous areas in experimental measurement technology. The main area of the strain gauges application is an experimental stress analysis [7], such as its use in civil engineering. Considering the fact that the distribution of strain on the tension zone is not uniform, the load-testing of highway bridges is nowadays a very important procedure. What is described here is the design and testing of a simple and accurate transducer for strain measurement of one particular bridge. [9].

There are different types of SG, different in construction and technical specification, applied in medicine, such as models for determining force and load curves in dental and oral medicine [10], used to evaluate stresses in implant-supported prostheses, in vivo and in vitro, under static and/or dynamic loads [11]. Force platforms are widely used in biomechanical estimation of human movement, especially in walk analysis [12]. Within the last decade, much experimentation on skeletal muscle has been undertaken, and one of the developed methods was based on a novel capacitive strain gauge (CSG) [13]. All the above-mentioned applications speak in favor of a wide use of strain gauges.

In this paper we discuss applying a strain gauge, together with a Wheatstone bridge, to measure (i.e. calculate) force or load. In addition to several electrical circuits, used in strain measurements, the principles of the Wheatstone bridge circuit and the strain gauge are explained. The model designed for laboratory use, with a description of the experiment used in student education, has been described.

## 2 THEORY

**Strain and stress definition:** The results of an external force application to a stationary object are stress and strain. If we consider a wire fixed to the top (Fig. 1), with original cross-sectional area of the wire ( $A$ ), and original wire length ( $L$ ), due to applied force ( $F$ ), the wire stretches vertically. Stress  $\sigma$  (called an axial stress) is defined as the material's internal resistance.



**Fig. 1** The wire before (a) and after force application (b)

$$\sigma = \frac{F}{A} \tag{1}$$

With the applied force, the wire length increases ( $\Delta L$ ), the cross-sectional area decreases (Fig. 1), and the deformation that occurs, defines axial strain  $\epsilon$ :

$$\epsilon = \frac{\Delta L}{L} \tag{2}$$

There are tensile strain (elongation) with sign (+), or compressive strain (contraction) with sign (-). Compressed in one direction, a material usually tends to expand in the other two directions, perpendicular to the direction of compression. This phenomenon, called the Poisson effect, is characterized by the Poisson ratio (for most metals around 0.3).

The relationship between stress and strain, for elastic materials, can be described by Hooke's law:

$$\sigma = E \cdot \epsilon \tag{3}$$

where  $E$  is Young's modulus, also called the modulus of elasticity. We will consider only the elastic stress region [1].

The same wire (Fig. 1) of length  $L$ , cross-section area  $A$  and resistivity  $\rho$  of the wire material has electrical resistance  $R$ :

$$R = \frac{\rho \cdot L}{A} \tag{4}$$

The fact that the resistance of a conductor changes when the conductor is exposed to strain is essential for the strain gauges application. With strain, the electrical resistance of wire will change, and the wire (Fig. 1.b), according to the equation (4), has higher resistance because it is longer and thinner. In real application, the strain gauge resistance will change not only due to deformation of the surface to which the sensor is attached, but also because of the influence of some other parameters, such as temperature or material properties [14].

The relationship between strain and resistance variation is almost linear, and the constant of proportionality is known as the "sensitivity factor":

$$G = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon} \quad \text{or} \quad \frac{\Delta R}{R} = G \cdot \varepsilon \quad (5)$$

where:  $G$  – sensitivity factor,  $R$  – initial resistance,  $\Delta R$  – change in resistance,  $L$  – initial length,  $\Delta L$  – change in length. A typical value for  $G$ , known as the ‘gauge factor’, is around 2 for commercially available strain gauges [1]. Since in practice resistance change  $\Delta R$  is very small, and ratio  $\Delta R/R$  for typical  $120\Omega$  strain gauges can be between  $10^{-3}$  and  $10^{-6}$  [1], it is very difficult to use ohmmeter for direct, precise measurement of such a small resistance change. Therefore, there is a need to design an electronic circuit to measure the change in resistance, preferably to the resistance itself. The most widely used electric circuit, with the ability to accurately detect small changes in resistance, is the Wheatstone bridge.

#### Bridges used for strain measurements

Beside the Wheatstone bridge, there are some other electric circuits, the measuring bridges, with the ability to detect very low resistance.

**Kelvin double bridge or Thomson bridge or Four-wire Ohm circuit.** This bridge is a modification of the Wheatstone bridge circuit, used to measure resistance in the range from  $1\Omega$  to  $0.00001\Omega$ . With this bridge, it is possible to overcome the limitations of the Wheatstone bridge in determining very low values of resistance, taking into account the effect of contact resistances, and the resistances of the wires, used to connect the elements in the electric circuit, which is usually neglected. The operation of the Kelvin bridge is similar to the Wheatstone, except for the presence of additional resistors and the configuration of the bridge are arranged to essentially reduce measurement errors. The change in absolute value of gauge resistance to compute strain can be measured very accurately, using a four-wire ohms measurement technique with high digital multimeter (Fig. 2).

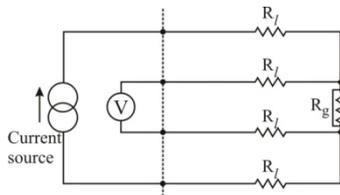


Fig. 2 Four wire Ohm circuit

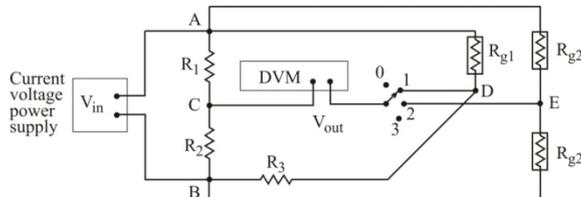


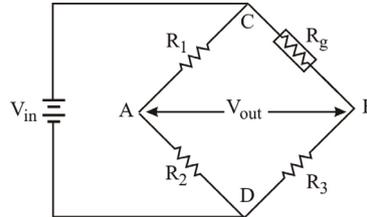
Fig. 3 Schematic view of Chevrone bridge

**The Chevrone bridge.** For strain measurements on rotating machines, the Chevrone bridge (Fig. 3) can be applied.

These two bridges are described in more details in the literature [14].

**The Wheatstone bridge.** The Wheatstone bridge (WB) (Fig. 4) is one of the most sensitive and precise methods, with the ability to measure small changes in resistance, which makes it frequently used nowadays for strain measurements [8,14]. In a simple Wheatstone bridge circuit, input voltage ( $V_{in}$ ) is applied across the bridge with four resistors, while in the middle of the bridge (between points A and B) output voltage ( $V_{out}$ ) is measured. When  $R_g$  is the resistance of the strain gauge and  $R_1$ ,  $R_2$  and  $R_3$  are the

resistors in the bridge, this bridge configuration is called a quarter-bridge (Fig. 4). In general, if we define  $n$  as the number of active gauges in the WB,  $n=1$  for a quarter bridge,  $n=2$  for a half bridge and  $n=4$  for a full bridge.



**Fig. 4** The Wheatstone bridge circuit

Considering the strain gauge,  $R_g$ , as the only variable resistor, due to a change in strain on the surface of the sample to which it is attached [14], after the circuit analysis, the relation between the output and the input voltage is given by the equation:

$$V_{out} = V_{in} \left( \frac{R_3}{R_g + R_3} - \frac{R_2}{R_2 + R_1} \right) \quad (6)$$

When  $R_g/R_3=R_1/R_2$ , output voltage is  $V_{out}=0$  and the bridge is called balanced. When the gauge change their resistance to  $R_1+\Delta R_1$ ,  $R_2+\Delta R_2$ ,  $R_3+\Delta R_3$ ,  $R_4+\Delta R_4$ , the relation between input and output voltage of four-gauge system, frequently applied to strain-gauge transducers, is as follows:

$$V_{out} = V_{in} \frac{1}{4} \left( \frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \quad (7)$$

This result, according to equation (5), indicates that the output voltage only depends on the gauge factor and strain.

For a quarter and a half bridge the relations for  $V_{out}$  are described in details in the literature [15].

In the classic Wheatstone bridge configuration, three out of four resistors are with fixed resistance value, while one is variable. The variable resistor is the sensing element (transducer), and it allows its resistance to change due to a change in an external factor, such as stress, pressure, or temperature [15].

The aim of this work is the development and use of the model for force measuring bridge, on the basis of resistance elements. The model we have made should be suitable for the laboratory applications, especially in the measuring of mechanical quantities by electrical means.

### 3. MATERIALS AND METHODS

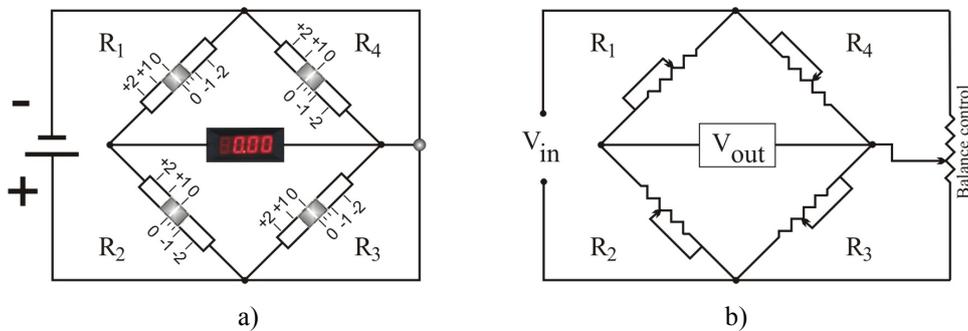
The Wheatstone bridge model described in this paper consists of four variable resistance elements ( $R_1$  to  $R_4$ ), representing a strain gauges. Strain gauges are resistant

measuring elements, bounded (i.e. glued) to the elastic measuring element, which is loaded by force. Force application causes the deformation of the elastic element, as well as the simultaneous change of strain gauges dimensions. The deformation of the strain gauges leads to an increase or decrease in their resistance, compared to nominal. This characteristic of strain gauges, in our example, is provided by variable potentiometers (resistors).

### The electrical connection

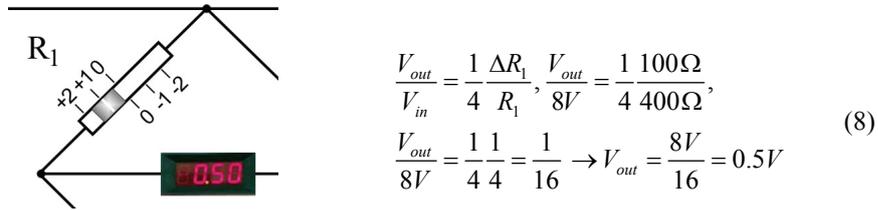
In order to build the model, linear slider resistors with maximum resistance of  $1\text{k}\Omega$ , power of  $0.3\text{ W}$  and the highest allowed input current of  $0.35\text{ A}$ . have been used. On the model, shown in Fig. 5a, the initial position of the slider of the potentiometer on the scale is marked with zero (0). It is a position corresponding to unloaded state, when the resistance of strain gauges is equal to their nominal value. In this example, all the four resistors have in initial position resistance of  $400\ \Omega$ . Depending on the direction of the external loads (forces), there may be either the contraction or the elongation of the strain gauges, which respectively leads to a decrease or increase of their resistance. Marks on the scale, next to resistors, indicate increase (+1 and +2) or decrease (-1 and -2) of resistance. It is chosen that each mark on the scale corresponds to an equal change in resistance of  $\Delta R = 100\ \Omega$ .

The resistors are connected in a Wheatstone bridge. Bridge input voltage ( $V_{in}$ ) of  $8\text{ V}$  is provided by a standard  $220\text{ V}$  adapter to  $12/24\text{ V}$ , using voltage stabilizer 7808CT and 7805CT. The output voltage is measured with a digital voltmeter. In order to facilitate the balancing of the bridge in unloaded (initial) state, parallel with resistors  $R_3$  and  $R_4$ , another variable resistor is connected. Its maximum resistance is  $10\text{ k}\Omega$  and its fine adjustment is possible by 15-turn gear drive. Block diagram of the system is shown in Fig. 5b.



**Fig. 5** Model of the Wheatstone bridge and the corresponding block diagram

From equation (Eq. 6), in the case that all the elements in the Wheatstone bridge have the same resistance, it follows that the value of the output voltage is equal to zero ( $V_{out} = 0$ ). A change in the resistance, for example,  $R_1$ , produces an imbalance of the bridge, that is, the appearance of the output voltage. The result of bringing resistors  $R_1$  in a position +1 on the scale, is the increase of its resistance to  $\Delta R_1 = 100\ \Omega$ , and the appearance of the corresponding output voltage (Fig. 6).



**Fig. 6** Increase of resistance  $R_1$  and the appearance of the corresponding output voltage

On the other hand, reducing the value of resistance  $R_1$  (for example position  $-1$  on scale) will lead to the appearance of output voltage  $V_{out} = -0.5V$ .

**Gauge factor**

Strain gauges are used in situations where we want to determine strain ( $\epsilon$ ) on element loaded by force. The corresponding value of deformation leads to changes in the resistance of strain gages. In order to define the strain by the resistance change, the sensitivity factor (gage factor  $G$ ) of the strain gauge material must be determined.

In industrial measuring strain gages, the sensitivity factor is predetermined and known. For example, for the strain gauges made of constantan, the gauge factor is  $G = 2$ . However, in our model, the value of  $G$  is unknown and must be calculated.

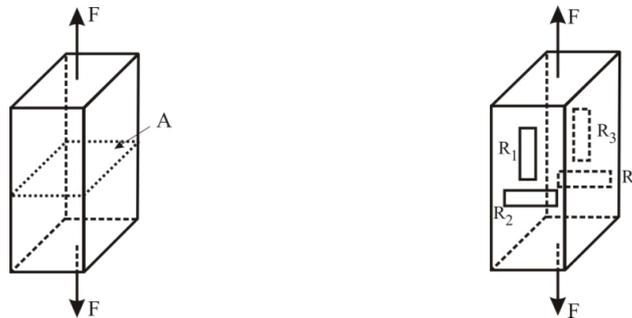
On the model, one mark on the scale corresponds to the change of resistance  $\Delta R=100\Omega$ . Suppose that the change is caused by strain  $\epsilon = 0.1$ . Taking the previous assumption into account, the gauge factor can be determined by the calculation below:

$$\frac{\Delta R}{R} = G \cdot \epsilon, \quad \frac{100\Omega}{400\Omega} = G \cdot 0.1, \quad \frac{1}{4} = G \frac{1}{10} \rightarrow G = \frac{10}{4} = 2.5$$

**Use of the model in the case of tensile forces**

Under the influence of the mechanical forces on some element, the resistance that opposes the effect of external load, occurs in the material. This resistance is called stress. The stress values cannot be measured, but can be calculated from the strain. The procedure of using measured deformation to determine the stress in the material, that is, the force acting on the element, will be presented in the example of uniaxial stress state.

Consider a rod loaded with tensile forces (Fig. 7).



**Fig. 7** Rod loaded with tensile forces **Fig. 8.** Bridge configuration for uniaxial tensile load

Tensile force  $F$  leads to elongation of the rod in the direction of the load. At the same time, normal stress appears in rod material ( $\sigma$ ). This stress is proportional to the tensile force and the cross-sectional area:

$$\sigma = \frac{F}{A} \quad (10)$$

where:  $F$  – tensile force and  $A$  – cross sectional area

To measure the value of tensile strain, and reject possible bending strain, four active strain gages are bounded on the surface of the rod (two in the direction, and two perpendicular to the direction of the force). They are electrically connected to form a full Wheatstone bridge, as shown in Fig. 8.

From the equation 7, according to equation (5) it follows:

$$V_{out} = \frac{G}{4}(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) \cdot V_{in} \quad (11)$$

For configuration displayed in Fig. 8, where two strain gauges ( $R_1$  and  $R_3$ ) are elongated, and the other two ( $R_2$  and  $R_4$ ) compressed, there follow deformation values:

$$\varepsilon_1 = \varepsilon_3 = \varepsilon, \quad \varepsilon_2 = \varepsilon_4 = -\mu \cdot \varepsilon \quad (12)$$

where:  $\varepsilon$  – local deformation (strain) in the direction of the force,  $\mu$  – coefficient of lateral contraction (Poisson ratio); for steel  $\mu = 0.3$ .

Substituting the above values, equation (11) takes the form:

$$\frac{V_{out}}{V_{in}} = \frac{G}{4}[\varepsilon - (-\mu \cdot \varepsilon) + \varepsilon - (-\mu \cdot \varepsilon)] = \frac{G}{4}2.6 \cdot \varepsilon \quad (13)$$

The above equation allows calculation of strain value based on both known and measured electrical quantities:

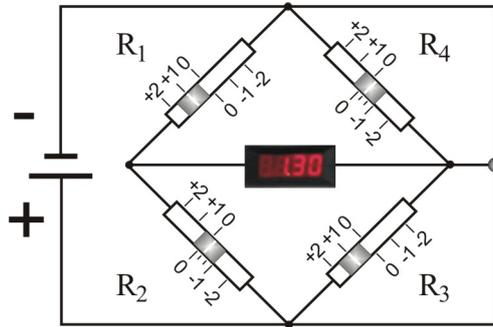
$$\varepsilon = \frac{V_{out}}{V_{in}} \frac{4}{G} \frac{1}{2.6} \quad (14)$$

Using strain-stress relationship (Eq. 3), the stress value can be determined, i.e. force acting on the rod. For a known value of the strain, material type ( $E$  – Young's modulus) and cross-sectional area, force can be calculated by the formula:

$$\frac{F}{A} = E \cdot \varepsilon \rightarrow F = A \cdot E \cdot \varepsilon = A \cdot E \cdot \frac{V_{out}}{V_{in}} \frac{4}{G} \frac{1}{2.6} \quad (15)$$

Assume, for example, that an unknown force ( $F$ ) causes the rod elongation of  $\varepsilon = 0.1$ . Under the load and according to the Wheatstone bridge configuration in Fig. 8, the following layout is set on the model: resistances  $R_1$  and  $R_3$  are higher (position +1), while the value of resistance  $R_2$  and  $R_4$  are smaller (position - 0.3), compared to nominal (unloaded condition).

In the case of such given load condition, bridge output ( $V_{out}$ ) value can be read (Fig. 9).



**Fig. 9** The value of the output voltage for given load condition

Substituting the values in equation (15) for  $A = a^2 = (10 \text{ mm})^2 = 100 \text{ mm}^2$ ,  $E = 2,1 \cdot 10^5 \text{ N/mm}^2$ ,  $G = 2,5$ ,  $V_{in} = 8 \text{ V}$ ,  $V_{out} = 1,3 \text{ V}$ , it follows:

$$F = 100 \text{ mm}^2 \cdot 2,1 \cdot 10^5 \frac{\text{N}}{\text{mm}^2} \cdot \frac{1,3 \text{ V}}{8 \text{ V}} \cdot \frac{4}{2,5} \cdot \frac{1}{2,6}, \quad F = 2,1 \cdot 10^4 \text{ N} = 21 \text{ kN}$$

By specifying the type of loading and layout of active strain gauges, this example helps us to determine the value of mechanical force.

#### 4. CONCLUSION

Strain gauge measurement techniques touch upon two specialist areas: mechanical engineering and electrical engineering. For many materials tests, it is particularly important to know the stress which occurs in the material. It allows us to draw important conclusions about the structural stability, regardless of its previous shape and size.

The objectives of this work are to gain practical experience with resistance strain-measurements techniques and to learn about the Wheatstone bridge, as well as how it is used in strain measurements. The idea behind this is to help students understand strain and stress better. For this purpose, a very simple model able to measure electrical parameters in the electric circuit with the acceptable accuracy is made in order to calculate the non-electrical quantities such as force. The presented model is a learning tool, applied in the laboratory for measurements.

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## MERNI MOSTOVI KOD SENZORA NA BAZI MERNIH TRAKA

*Električno merenje mehaničkih veličina je često u podučavanju studenata inženjerstva. Ono se široko primenjuje kod merenja sile, posebno pri radu alatnih mašina. Senzori sile koriste merne trake povezane na elastični merni element za merenje spoljašnjeg opterećenja - sile. Primenjena sila proizvodi male promene u otpornosti mernih traka. U cilju merenja deformacije otporne merne trake, moraju biti vezane u električno kolo. Pretvarači na bazi mernih traka obično koriste četiri merne trake vezane u električno kolo pri čemu formiraju Vitstonov most. Cilj ovog rada je da opiše vezu između mernih traka u različitim mernim kolima, a posebno konfiguraciju Vitstonovog mosta. Takođe će biti predstavljen model u obrazovanju studenata u oblasti merenja mehaničkih veličina električnim putem.*

Ključne reči: *merenje, merne trake, merna kola, Vitstonov most, obrazovanje*