FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol. 11, N° 1, 2013, pp. 13 - 18

CREEP TRANSITION STRESSES OF ORTHOTROPIC THICK-WALLED CYLINDER UNDER COMBINED AXIAL LOAD UNDER INTERNAL PRESSURE

UDC 624.072.4

Pankaj Thakur

Department of Mathematics, Indus International University Bathu, India

Abstract. *Effect of anisotropy has been illustrated graphically. It is found that the anisotropy of the material has little effect on radial stress but the effect is more prominent on the axial and tangential stresses for higher ratio of anisotropic constants along the radius. The results indicate the maximum stress is not always on the surface.*

Key Words: *Creep, Cylinder, Load, Pressure, Stresses*

1. INTRODUCTION

A thick walled circular cylinder is widely and commonly used either as a pressure vessel intended for storage in industrial gases or as a media transportation of high pressurized fluids. Creep of the thick-walled cylinder under internal pressure has been discussed by many authors [1-10, 12, 13, 15]. At high temperature, its thick walled tubes subjected to internal pressure and axial load exhibit continuously increasing deformation with directions. The thick walled cylinder creep under internal pressure alone may be found in literature for isotropic [6-9] and orthotropic [10-11] materials. At high temperature, the thick-walled tubes under internal pressure are usually placed in vertical position. Axial load is thus exerted on the tube not only by internal pressure but also by its own weight. When axial load is exerted on the tube in addition to the internal pressure, the stress distribution is altered. Solutions become more complicated. It must be noted that in general, the solution cannot be obtained by combining the effects of separate loads. The problem of the thick walled cylinder creep under combined axial load and internal pressure has been discussed by Finnie [12], Schwicker and Sidebottom [13] in the isotropic creep the-

---

Received February 21, 2012
* **Acknowledgements**. The author are grateful to the referee for his critical comments, which led to a significant improvement of the paper.
Corresponding author: Pankaj Thakur
Department of Mathematics, Indus International University Bathu • Una, Himachal Pradesh-174301, India
E-mail: pankaj_thakur15@yahoo.co.in

ory. Rimrott [6] analyzed the above problem by taking into consideration large strain. This author made the following assumption:

1) The material is incompressible *i.e.*

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0. \tag{1}$$

2) Axes of anisotropy coincide with the principal axes of the cylinder.
3) The creep deformation is infinitesimal and the pressure is assumed to be applied slowly and held constant during loading.
4) This problem is assumed to be that of generalized plane strain so that

$$\dot{\varepsilon}_z = K. \tag{2}$$

In this paper, the distribution of stress and creep rate have been obtained with the help of digital computer by considering the constitutive equations of the anisotropic theory given by Bhatanagar and Gupta [11].

2. Solution and Discussion of the Problem

Consider a uniform thick walled cylinder of internal and external radii a and b respectively, subjected to internal pressure p and axial load L. Co-ordinates r, θ and z are taken in radial, circumferential and axial directions of the cylinder respectively. The compatibility equation is:

$$r\frac{d\dot{\varepsilon}_\theta}{dr} = \dot{\varepsilon}_r - \dot{\varepsilon}_\theta . \tag{3}$$

Combining equation (3) with equation (1) and integrating, one gets:

$$\dot{\varepsilon}_\theta = \frac{C'}{r^2} - \frac{K}{2},\ \dot{\varepsilon}_r = -\frac{C'}{r^2} - \frac{K}{2},\ \dot{\varepsilon}_z = K. \tag{4}$$

where C' is the constant of integration.

Fundamental constitutive equations for anisotropic theory are given by Bhatnagar and Gupta [9] as follow:

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_i}{2\sigma_i}[(G+H)\sigma_r - H\sigma_\theta - G\sigma_z]\ \dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_i}{2\sigma_i}[(H+F)\sigma_\theta - F\sigma_z - H\sigma_r],$$

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_i}{2\sigma_i}[(F+G)\sigma_z - G\sigma_r - F\sigma_\theta]. \tag{5}$$

Stress invariant σ_i is defined as:

$$\sigma_i = \frac{1}{\sqrt{2}}[F(\sigma_\theta - \sigma_z)^2 + G(\sigma_z - \sigma_r)^2 + H(\sigma_r - \sigma_\theta)^2]^{\frac{1}{2}}. \tag{6}$$

where F, G, H are constants of the material and $\dot{\varepsilon}_i$ is the strain rate invariant.

Steady-state creep rate $\dot{\varepsilon}_i$ is a function of stress σ_i only. Assuming Norton's law of creep, we can write:

$$\dot{\varepsilon}_i = \beta\sigma_i^n, \tag{7}$$

where β constant measure. Substituting in equation (5) the value of $\dot{\varepsilon}_r$ and $\dot{\varepsilon}_\theta$ from equation (4) and solving for σ_z, we obtain:

$$\sigma_z = \frac{\left[(F+2H)\frac{K}{2}+\frac{C'}{r^2}F\right]\sigma_\theta+\left[\frac{C'}{r^2}G-(G+2H)\frac{K}{2}\right]\sigma_r}{\left[\frac{C'}{r^2}(F+G)+(F-G)\frac{K}{2}\right]}. \tag{8}$$

Substituting $\dot{\varepsilon}_\theta$ from equation (4) and σ_z from equation (8), in equation (5), one gets:

$$\sigma_\theta-\sigma_r = \left[\frac{C'}{r^2}(F+G)+(F-G)\frac{K}{2}\right]\frac{1}{HF+FG+GH}\left(\frac{2\sigma_i}{\dot{\varepsilon}_i}\right). \tag{9}$$

After substituting the value of stress invariants σ_i and strain rate invariants $\dot{\varepsilon}_i$ from equation (6) and (7) in equation (9) the resulting equation becomes:

$$\sigma_\theta-\sigma_r = \frac{1}{\beta^{\frac{1}{n}}}\left[\frac{2}{HF+FG+GH}\right]^{\frac{1+n}{2n}}\left[\frac{C'}{r^2}(F+G)+(F-G)\frac{K}{2}\right]\begin{bmatrix}\frac{C'^2}{r^4}(F+G)\\+\frac{K^4}{4}(F+G+4H)\\+\frac{2C'}{r^2}\frac{K}{2}(F-G)\end{bmatrix}^{\frac{1-n}{2n}}. \tag{10}$$

The equation of equilibrium are given by:

$$r\frac{d\sigma_r}{dr} = \sigma_\theta-\sigma_r. \tag{11}$$

Integrating equation (11) after substituting value of $(\sigma_\theta-\sigma_r)$ from equation (10) and inserting boundary condition $\sigma_r = 0$ at $r = b$ leads to:

$$\sigma_r = \frac{1}{\beta^{\frac{1}{n}}}\left[\frac{2}{HF+FG+GH}\right]^{\frac{1+n}{2n}}\int_b^r\begin{bmatrix}\frac{C'}{r^2}(F+G)\\+(F-G)\frac{K}{2}\end{bmatrix}\begin{bmatrix}\frac{C'}{r^4}(F+G)\\+\frac{K^2}{4}(F+G+4H)\\+\frac{C'}{r^2}K(F-G)\end{bmatrix}^{\frac{1-n}{2n}}\frac{dr}{r}, \tag{12}$$

The tangential stress is given by:

$$\sigma_\theta = \sigma_r + r\frac{d\sigma_r}{dr}. \tag{13}$$

The axial stress balances the sum of external axial load L and internal pressure $\pi a^2 p$ *i.e.*

$$2\pi\int_a^b \sigma_z r dr = \pi a^2 p + L. \tag{14}$$

Let us define the dimensionless quantities as follows: $T_{rr} = \sigma_r / p, T_{\theta\theta} = \sigma_\theta / p,$

$$D = \left[\frac{2}{FG + GH + HF}\right] \cdot \frac{1}{p\beta^{\frac{1}{n}}} C^{\frac{1}{n}}, \quad T_{zz} = \sigma_z / p, \quad R = r / b, C = C' / b^2 \quad K = \frac{2}{\sqrt{3}} \beta C \text{ and}$$

$$R_0 = a / b. \tag{15}$$

Equation (12) in non dimensional becomes:

$$T_{rr} = D \int_1^a \left[\frac{1}{a^4}(F+G) + \beta^2 \frac{(F+G+4H)}{3} + \frac{2\beta}{\sqrt{3}a^2}(F-G)\right]^{\frac{1-n}{2n}} \cdot \left[\frac{1}{a^2}(F+G) + \frac{\beta}{\sqrt{3}}(F-G)\right] \frac{da}{a}, \tag{16}$$

with $T_{rr} = -1$ at $a = R$. Also

$$T_{\theta\theta} = T_{rr} + D\left[\frac{1}{a^4}(F+G) + \beta^2 \frac{(F+G+4H)}{3} + \frac{2\beta}{\sqrt{3}a^2}(F-G)\right]^{\frac{1-n}{2n}} \cdot \left[\frac{F+G}{a^2} + \frac{F-G}{\sqrt{3}}\beta\right]. \tag{17}$$

and the axial stress becomes:

$$T_{zz} = \frac{\left[(F+2H)\frac{\beta}{\sqrt{3}} + \frac{F}{a^2}\right] T_{\theta\theta} + \left[\frac{G}{a^2} - (G+2H)\frac{\beta}{\sqrt{3}}\right] T_{rr}}{\left[\frac{F+G}{a^2} + \frac{F-G}{\sqrt{3}}\beta\right]}. \tag{18}$$

Stress distribution equation (16) is solved by Simpson's Rule for values of a. Constant D in equation (16) is found by taking $T_{rr} = -1$ at $a = R$ and then, C is determined from equation (15).Thus, stresses T_{rr}, $T_{\theta\theta}$ and T_{zz} are determined for various values of a from equations (16), (17) and (18) for known values of n and β. Curves have been drawn in Fig. 1 for stress distribution along radius for different values of G/F, H/F and $n = 5$. The effect of anisotropy is noticeable more on the axial and tangential stresses. The stress where the variation along the radius is prominent for higher ratios of anisotropic constants. The results indicate that maximum stress is not always on the surface.

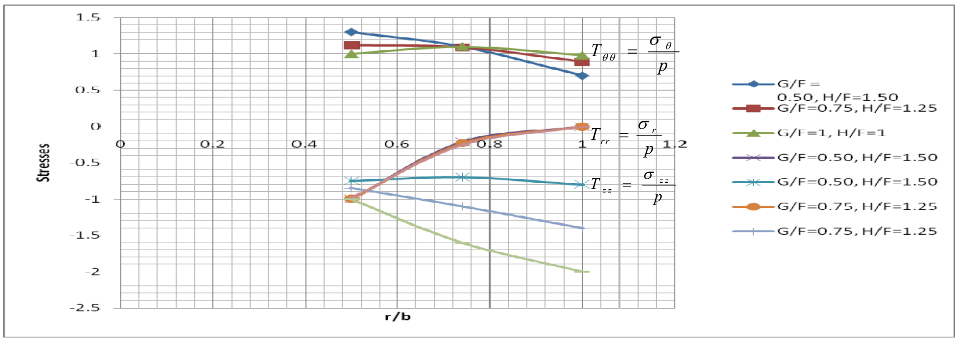

Fig. 1 Stress distribution along the radius of thick walled cylinder under combined axial Load and internal pressure.

3. PARTICULAR CASE

Case 1 For vanishing anisotropy, $F = G = H = 1$ and Equations (16), (17) and (18) becomes:

$$T_{rr} = \left(\frac{2}{\sqrt{3}}\right)^{\frac{1+n}{n}} \frac{1}{p}\left[\frac{C}{\beta}\right]^{\frac{1}{n}} \int_1^a \left[\frac{1}{a^4}+\beta^2\right]^{\frac{1-n}{2n}} \frac{da}{a^3}, \tag{19}$$

$$T_{\theta\theta} = T_{rr} + \left(\frac{2}{\sqrt{3}}\right)^{\frac{1+n}{2n}} \frac{1}{p}\left[\frac{C}{\beta}\right]^{\frac{1}{n}} \left[\frac{1}{a^4}+\beta^2\right]^{\frac{1-n}{2n}} .\frac{1}{a^2}, \tag{20}$$

$$T_{zz} = \left[\frac{\sqrt{3}}{2} - \beta a^2 + \frac{1}{2}\right] T_{\theta\theta} + \left[\frac{1}{2} - \frac{\sqrt{3}}{2} - \beta a^2\right] T_{rr}. \tag{21}$$

Equations (19) and (20) are same as given by Finnie [7] for isotropic case.

Case 2 Plane strain under internal pressure: If $\beta = 0$ and $L = 0$, the problem reduces to the thick walled orthotropic cylinder internal pressure alone for the case of plane strain. On substituting values β and L in equations (16), (17), (18) and solving with boundary condition $T_{rr} = -1$ at $a = R$, one gets:

$$T_{rr} = \frac{1}{b^{-\frac{2}{n}} - a^{-\frac{2}{n}}} \left[r^{-\frac{2}{n}} - b^{-\frac{2}{n}} \right], \tag{22}$$

$$T_{\theta\theta} = \frac{1}{b^{-\frac{2}{n}} - a^{-\frac{2}{n}}} \left[\left(1 - \frac{2}{n}\right) r^{-\frac{2}{n}} - b^{-\frac{2}{n}} \right], \tag{23}$$

and

$$T_{zz} = \frac{F T_{\theta\theta} + G T_{rr}}{F+G}. \tag{24}$$

Equations (22) – (24) are same as given by Pooja Kumari [15] and Bhatanagar and Gupta [11] in case of plain strain.

4. CONCLUSION

Material anisotropy is found to have little effect on radial stress but the effect is more prominent on the axial and tangential stresses for higher ratio of anisotropic constants along the radius.

Nomenclature

a, b – Inner and outer radius of the cylinder
r, θ, z – cylindrical polar co-ordinate
σ_r, σ_θ, σ_z – stresses components
n – measure
p – pressure
L – Load

C' – constant of integration

F, G, H – constant of the materials

$\dot{\varepsilon}_i$ – strain rate invariant

Greek letters

$R = r / b$; $R_0 = a/b$ – Radii ratio

T_{rr} – Radial stress component (σ_r / p)

$T_{\theta\theta}$ – Circumferential stress component (σ_θ / p)

T_{zz} – Circumferential stress component (σ_z / Y).

REFERENCES

1. Gupta, S.K., Dharmani, R.L., 1979, Creep transition in thick walled cylinder under internal Pressure, Z.A.M.M., 59(10) pp. 517- 521.
2. Gupta, S.K. 'Elastic-plastic and creep Transition of Thick-Walled Cylinder under Uniform Pressure', Proc. of Int. Symp. on Non-linear Mech. Kharagpur, 1980, pp. 169-175.
3. Thakur Pankaj, Gaurav Sharma,2009, Creep transition stresses in thick walled rotating cylinder by finitesimal deformation under steady state temperature, International Journal of Mechanics and Solids, 4(1) pp. 39-44.
4. Thakur Pankaj,2012,*Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature*, Thermal Science, DOI Reference: 10.2298/TSCI110315080P.
5. Thakur Pankaj, 2012, *Steady thermal stress and strain rates in a circular cylinder with non- homogeneous compressibility subjected to thermal load*, Thermal Science, DOI Reference: 10.2298/TSCI110318079P.
6. Rimrott, F.P.J.,1959, Creep of thick-walled tubes under internal pressure considering large strains, J. Appl. Mech., 81(series E) pp. 271-276.
7. King, R.H. and Mackie, W. W.,1967, *Creep of thick-walled cylinder*, J. Basic Engr., 87(series D) pp. 877-882.
8. Taira, S., Koterazawa, R. and Ohtani, R.,1965, *Creep of Thick-walled cylinder under internal pressure at elevated temperature*, Proc. 8th Japan Congress on Testing Materials, J. Japan Soc. Mat. Sci., 83 pp. 53-60.
9. Weir, C.D.,1957, The creep of thick-walled cylinder under internal pressure, J. Appl. Mech., 24(2) pp. 464-468.
10. Pai, D.H.,1967, Steady state creep analysis of thick-walled orthotropic cylinders, Int. J. Mech. Sci., 9 (6) pp. 335-339.
11. Bhatnagar, N.S. and Gupta S. K.,1969, *Analysis of thick-walled orthotropic cylinder in the theory of creep*, J. Phys. Soc. Japan, 27(6) pp. 1655-1662.
12. Finnie, I.,1960, Steady-state creep of a thick-walled cylinder under combined axial load and internal pressure, Trans. ASME, 82(D) pp.689-694.
13. Schweiker, J.W. and Sidebottom, O.M.,1965, Creep of thick-walled cylinders subjected to internal pressure and axial load, Expt. Mech., 82(1) pp.435-440.
14. Bhatnagar, N.S. and Gupta R.P.,1966, On the constitutive equations of the orthotropic theory of creep, *Journal of the Physical Society of Japan, 2*1(4) pp. 1003-1007.
15. Kumari Pooja, 2006, *Ph.D Thesis*, Elastic-Plastic and Creep Problems in Transversely isotropic materials, Department of Mathematics, H.P.U. Shimla, pp. 80-81.

PUZEĆI TRANZICIONI NAPONI ORTOTROPSKOG DEBELOZIDNOG CILINDRA POD KOMBINOVANIM AKSIJALNIM OPTEREĆENJEM POD UNUTRAŠNJIM PRITISKOM

Dejstvo anizotropije grafički je ilustrovano. Utvrđeno je da anizotropija materijala ima malo uticaja na radijalni napon a da je uticaj mnogo izraženiji na aksijalnim i tangencijalnim naponima za više koeficijente anizotropksih konstanti duž radijusa. Rezultati ukazuju da maksimalni napon nije uvek na površini.

Ključne reči: *puženje, cilindar, opterećenje, pritisak, naponi*