# TRACK STIFFNESS AND THE VERTICAL TRACK GEOMETRY DETERIORATION MODELING\*

## *UDC 625.083.4*

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Abstract. Railway track stiffness is a basic parameter of track design which influences the bearing capacity, the dynamic behavior of passing vehicles, track geometry quality and the life of track components. In this paper track stiffness is treated from the point of view of the vertical track geometry deterioration modeling. In most of the models, the vertical geometry deterioration is considered to be a function of the number of loading cycles and/or a function of the magnitude of the loading. Deterioration should also be a function of the properties of the track superstructure and substructure, so, the track stiffness should be an indispensable part of those models. The aim of this study is to analyze the vertical track stiffness and its influence on geometry deterioration, and to make a comparative analysis between deterioration models that directly or indirectly include the effect of the vertical track stiffness.

Key Words: Ballasted Track, Track Stiffness, Geometry Deterioration, Prediction Models, Track Maintenance

#### 1. INTRODUCTION

Under the impact of dynamic traffic loading, the geometry of ballasted railway tracks inevitably deteriorates. Optimizing track maintenance to minimize costs is a complex task and the vertical geometry deterioration prediction is the main part of the optimization process. In order to develop a mathematical model for the track geometry deterioration prediction, the phenomena have to be understood and described in an engineering way.

Track stiffness is a significant parameter from the aspect of designing, construction and maintenance of the railway superstructure and substructure. This parameter represents the basis for calculating stresses in the elements of track and track foundation. During the

Received September 05, 2012

<sup>\*</sup> Acknowledgements. This work is supported by the Ministry of Education and Science of the Republic of Serbia through the research projects "Research of technical-technological, staff and organizational capacity of Serbian Railways, from the viewpoint of current and future European Union requirements" (No. 36012) and "Reconstruction and revitalization of railway infrastructure in accordance with regional development" (No. 680-00-140/2012-09/10).

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track service life, track stiffness considerably influences the dynamic behavior of passing vehicles, the life of track components and the track geometry deterioration.

The objective of this paper is to analyze the vertical track stiffness and its influence on the geometry deterioration. A comparative analysis is performed between deterioration models that directly or indirectly include the effect of the vertical track stiffness.

#### 2. ANALYSIS OF TRACK STIFFNESS

Track stiffness (D) presents the proportion between vertical load (Q) and track deflection (y) in a given moment (t):

$$D(t) = \frac{Q(t)}{y(t)} \tag{1}$$

Traditionally, the track behavior analysis under vertical load is based on the beam on an elastic foundation (BOEF) model and the Winkler's Hypothesis. This approach assumes that the behavior of the superstructure and substructure elements is linear, that there is proportionality between the load and the deflection, i.e. the track stiffness has a constant value.

The modern approach to the stiffness definition includes both inelastic and nonlinear behavior of the superstructure and substructure elements, the existence of the difference between the stiffness under static and under dynamic load, and the unevenness of the stiffness along the track.

#### 2.1. Nonlinearity of the load-deflection dependence

In reality, the elements of the superstructure and substructure behave neither linearly nor completely elastically [1-4]. This can be explained by using the example of ballast behavior under the real conditions. In the majority of cases, the sleepers' leaning on the ballast is not ideal. There are voids beneath the sleepers which cause great deflections at small load intensity. Moreover, at great load intensities, nonlinearity and track stiffness increases are a consequence of the ballast and substructure layers compaction. Load distribution through ballast is done through contact surfaces between ballast stones. As the load value increases, the stone deformations lead to an increase of these contact surfaces and thus the ballast stiffness increases.

The absence of linearity of the load-deflection connection actually means that there is no unique value of the track stiffness. Fig. 1 shows the procedure for determining linearized stiffness as one of the possible procedures for determining numerical stiffness values in the calculations.

Linearization of the nonlinear load-deflection diagram is performed in the proper load range, which can be the range of dynamic load on the section the stiffness is being determined for. Since there is a difference between the real and linearized stiffness (depending on the load value, the real stiffness can be lower or higher than the linearized one), it is necessary to keep in mind the error, which is the consequence of linearization, at the application of the calculation data in calculation models [5].

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Fig. 1 Linearized track stiffness for the corresponding load range [5]

#### 2.2. Dynamic track stiffness

The term track stiffness, presented in the previous section, is referring to static stiffness, i.e. stiffness under static load. However, it is necessary to consider the stiffness under dynamic load as well.

Except for the load value, track stiffness also depends on excitation frequency (f) and thus a frequency related definition of stiffness is necessary. The term receptance or dynamic flexibility ( $\alpha$ ) is introduced. It actually presents inverse dynamic stiffness and it is measured on the track under dynamic load:

$$\alpha(f) = \frac{y(f)}{Q(f)} \tag{2}$$

Fig. 2 shows dependence of receptance on excitation frequency, according to [6]. Dynamical stiffness increases with the increase in frequency.



Fig. 2 Dependence of receptance on excitation frequency [6]

### 2.3. Spatially varying track stiffness

Spatially varying track stiffness is one of the basic causes of differential track settlement, which is of primary influence on the track geometry deterioration [5]. The basic causes for the emergence of the spatially varying track stiffness are the change of the superstructure and substructure characteristics along the line, variable ballast thickness, variable blanket layer thickness, characteristics of the material that the embankment is made of, moisture content and geological characteristics of the subsoil.

Uneven track stiffness along the track is the usual problem which has been explored within numerous research projects [7]. Numeric analysis that López Pita and Fonesco performed at the Technical University of Catalonia [8] clearly point that if two adjacent sections with considerably different track stiffness are being considered, the stress level on the ballast can be between 30 and 50% higher than the level corresponding to the hypothesis which assumes constant stiffness along the track. This additional stress accelerates the ballast deterioration process. That is why it is necessary to set the new acceptance criteria, which also considers track stiffness homogeneity together with the requirements related to the quality of the track geometry.

#### 3. ANALYSIS OF THE VERTICAL TRACK GEOMETRY DETERIORATION MODELS

Due to complexity of the degradation mechanism, deterioration of vertical geometry of ballasted track is very difficult to present in a mathematical model. Through the years a lot of research has been carried out and a lot of different models have been formulated. In those models, the track geometry deterioration is mostly considered as a function of the number of loading cycles and/or a function of the loading magnitude [5].

An overview of the track geometry deterioration models that are directly or indirectly influenced by track stiffness is given in Table 1.

According to the exponential model, the factor of relative settlement, which is directly proportional to the interval between successive track maintenance, depends on the ballast pressure. Ballast pressure, among other things, is the function of track stiffness. Based on the analysis of this model, conducted in [5], it can be concluded that increasing the track stiffness adversely affects the settlement, and the deterioration of the vertical track geometry.

According to the German DSM model, settlement of the track depends on the sleeper force. As the sleeper forces are lower in the sections with lower track stiffness, according to this model, increased stiffness causes an increase in deterioration of the vertical track geometry.

In the Satoh's model, the settlement depends on coefficient  $\beta_{Sh}$  that is directly proportional to the sleeper pressure and ballast acceleration. Sleeper pressure and ballast acceleration are both a function of the track stiffness and their values increase with increasing stiffness. Similar to this model, in the Hoshino's and Sugiyama's models the vertical track geometry deterioration is directly proportional to structure factor *J*, which is influenced by sleeper pressure, and ballast acceleration. And in the fourth Japanese model, the Sato's model, the ballast settlement is directly proportional to the sleeper-ballast contact pressure.

The French model of the vertical track geometry deterioration, the Guérin's one, expresses the intensity of the settlement as a function of the maximum elastic deflection during the loading cycle. Increasing deflection leads to an increase in the intensity of settlement. Therefore, according to this model, the stiffness reduction causes an increase in track settlement. It is similar to the South African model, the Fröhling's one, where the track settlement is directly proportional to the measured track stiffness.

Thus, according to the exponential, the German and the Japanese vertical track geometry deterioration models, an increase of track stiffness adversely affects the deterioration. On the other hand, according to the French and South African models, an increase in stiffness leads to reduction in track settlement, i.e. it has a positive effect on the deterioration.

Model name and equation	Explanation
Exponential model	$\gamma$ – factor of relative settlement
$\gamma = \frac{N_{ref}}{N_p} = \left[\frac{\sigma_p}{\sigma_{ref}}\right]^w$	$\sigma_{ref}$ - ballast pressure from reference number of loading cycles $\sigma_p$ - ballast pressure $N_{ref}$ - reference number of loading cycles
	$N_p$ – number of loading cycles
	w – exponent*
$\frac{\text{DSM model}}{\overline{\alpha}} = \overline{\alpha} (1 + K + 1 + \overline{N})$	$\overline{S}_N$ – track settlement after $\overline{N}$ loading cycles
$S_N = S_1 (1 + K_H \ln N)$	$\overline{S}_1$ – initial settlement (a function of the sleeper force)
	$K_H$ – coefficient*
Satoh's model	S – settlement
$S = \gamma_{sh} \cdot (1 - e^{-\alpha_{sh}N}) + \beta_{sh}N$	N-number of loading cycles
	$\alpha_{Sh}, \gamma_{Sh}$ – coefficients*
	$\beta_{Sh}$ – coefficient proportional to sleeper pressure
Hoshino's model	$\Delta$ – coefficient of track deterioration
	$L_H$ – load factor
$\Delta = L_H \cdot J \cdot Z$	J – structure factor (influenced by sleeper pressure, ballast acceleration,
	and track stiffness)
0	Z – state factor
Sugiyama's model	S – average growth of track irregularities in section
2 0.21	T- passed tonnage
$\overline{S} = 2.09 \cdot 10^{-3} \cdot T^{0.31} \cdot$	V – average running speed
$_{V}0.98$ $_{I}1.10$ $_{P}0.21$ $_{V}$ 0.26	J- structure factor
$V \cdot J \cdot K \cdot K p$	R – influence factor (jointed rail /CWR)
$(\overline{s})$	$K_p$ – influence factor for subgrade
$P_{ir} = 31.7 \log \left  \frac{3}{4} \right  + 31.6$	$P_{ir}$ – percentage of errors exceeding 3 mm
$\begin{pmatrix} A_T \end{pmatrix}$	$A_T$ – rate of tamping per year
Sato's model	$S_1, S_2$ – ballast settlement
$\int a_s (p_b - p_{b,gr})^w, \ p_b > p_{b,gr}$	$a_s, \alpha_s - \text{coefficients}^*$
$S_1 = \begin{cases} 0 & n \leq n \end{cases}$	$p_b$ – sleeper-ballast contact pressure
$(, P_b - P_{b,gr})$	$p_{b,gr}$ – threshold limit value of sleeper-ballast contact pressure
$S_2 = \alpha_s p_b^w$	$w - exponent^*$
Guérin's model	S – settlement
$dS_{\beta_c}$	N – number of loading cycles
$\frac{dN}{dN} = \alpha_G y^{r_G}$	y – maximum elastic deflection during the loading cycle
	$\alpha_G, \beta_G$ – material parameters*
Fröhling's model	$S_{Ni}$ – track settlement
$\begin{bmatrix} K_{F1} + K_{F2} \end{bmatrix}$	$D_{2mi}$ – measured track stiffness at a particular sleeper i
$S_{Ni} = \left  \left( \frac{D_{2mi}}{K} \right) - \left  \frac{Q_{tot}}{Q_{ref}} \right  \log N \right $	$K_{Fl}, K_{F2}, K_{F3}$ – settlement constants*
	$Q_{tot}$ – prevailing wheel load
$\left[ \left[ \left( \mathbf{\Lambda}_{F3} \right) \right]^{2} \right]$	$Q_{ref}$ - reference wheel load
	w = exponent*
	w – exponent*

Table 1 Overview of the track geometry deterioration models [5]

\* - coefficients whose values depend on local conditions and are determined empirically

It is obvious that a unified and consistent view on the impact of track stiffness on the vertical track geometry deterioration does not exist, which leads to the necessity of finding the optimum track stiffness.

#### 4. CONCLUSIONS

The vertical track geometry deterioration is a function of the number of loading cycles, the magnitude of loading and the properties of track superstructure and substructure. Therefore, the track stiffness should be an indispensable part of the track geometry deterioration models.

Taking into account the vertical track stiffness may seem straightforward, but the situation is not so simple. The stiffness definition must include inelastic and nonlinear behavior of the superstructure and substructure elements, as well as the existence of the difference between the stiffness under static and under dynamic load. Moreover, there is a problem with unevenness of the stiffness along the track. In the end, even if the right value of stiffness is chosen for a deterioration model, the relationship between vertical geometry deterioration and stiffness is complex.

Too low stiffness value would cause track settlement, with a considerable stress increase in the rails. Too high value would increase dynamic load and thus accelerate track deterioration. The necessity to find the optimum track stiffness is obvious.

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# KRUTOST ŠINSKE PODLOGE I MODELIRANJE PROPADANJA VERTIKALNE GEOMETRIJE KOLOSEKA

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Krutost šinske podloge je osnovni parametar konstrukcije gornjeg i donjeg stroja železničke pruge koji utiče na nosivost, dinamičko ponašanje vozila, kvalitet geometrije koloseka i vek trajanja elemenata konstrukcije. U radu se krutost šinske podloge razmatra sa aspekta modeliranja propadanja vertikalne geometrije koloseka. U većini modela, propadanje vertikalne geometrije koloseka je funkcija broja ciklusa opterećenja i/ili funkcija intenziteta opterećenja. Pored toga, propadanje je funkcija karakteristika konstrukcije gornjeg i donjeg stroja, tako da krutost šinske podloge treba da bude neizostavni deo tih modela. Cilj istraživanja je analiza krutosti šinske podloge i njenog uticaja na propadanje geometrije, kao i komparativna analiza modela propadanja koji na direktan ili indirektan način uključuju uticaj vertikalne krutosti.

Ključne reči: kolosek u zastoru, krutost šinske podloge, propadanje geometrije, modeli predviđanja, održavanje koloseka