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ON EFFICIENCY OF A SINGLE-LAYER SHELL ELEMENT FOR COMPOSITE LAMINATED STRUCTURES

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Abstract. *An increasing need for lightweight structures requires the development of new materials that are supposed to bring various benefits over the conventional structural materials. The fiber-reinforced composite laminates are among the most advanced structural materials nowadays. Effective simulation of behavior of the structures involving such a material system is the ingredient that provides for their easier, faster and less expensive design. For this purpose, the authors use a recently developed single-layer shell element. The paper investigates the influence of the order of numerical integration on the effectiveness and accuracy of the developed numerical tool. A uniformly reduced integration technique is compared to full order integration for a set of examples involving fiber-reinforced laminated shell structures.*

Key Words: *Composite Laminates, Thin-walled Structures, Shell Element, Reduced Integration*

1. INTRODUCTION

A great number of applications, like space structures, transport applications, etc., call for high specific mechanical properties, i.e. the ratio between mechanical properties and mass density. Such material properties would offer great possibilities for achieving improvements in structural behavior, both static and dynamic. A new generation of advanced lightweight structural materials has been developed to provide for the required properties. The fiber-reinforced laminates not only bring high strength and stiffness to mass ratio but they can also be tailored by adjusting the nature and proportions of their constituents, orientation of fibers, sequence and thickness of layers, in order to meet very specific requirements of various structures.

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The new emerged structural material is characterized by more complex material properties and behavior compared to classical structural materials. The possibility of setting different material configurations calls for adequate numerical tools so that the behavior of various design solutions can be efficiently simulated in order to check their adequacy. In this manner, numerical simulations reduce the dependence on extensive and sometimes even prohibitively expensive testing and enable easier and faster design developments.

Regarding this objective, an important aspect is the accuracy and efficiency of developed numerical tools, both crucially affecting the quality of performed numerical simulations. Structures made of composite laminates belong to the group of thin-walled structures. Application of solid elements for the analysis of thin-walled structures is numerically very time-consuming, especially in cases of large-scale applications of composite laminates and, therefore, the attention has turned to shell finite elements. A recently developed shell finite element [2] that supports modeling of structures made of such a material system is used in this paper.

Accurate and robust shell finite elements have been and are still a challenging task for researchers [3]. The key issue behind the difficulty is denoted as locking effects. Many researchers have dedicated their work to identify the problems and offer adequate, efficient and general solutions. Though one of the simplest solutions and even characterized as a "variational crime" [3] reduced integration technique is widely utilized as a solution that very efficiently alleviates locking phenomena with shell elements. This paper demonstrates the reduced integration technique and its peculiarities in combination with the shell element for composite laminated structures.

2. Fiber-reinforced Laminates – Properties and Modeling Requirements

Generally speaking, composite materials are formed by combining two or more already existing materials with different properties. The aim is to form a material with unique new properties, which are actually a combination of advantageous properties of the constituents. In the case of fiber-reinforced composites, the reinforcements are in the form of thin fibers (carbon, glass, aramid, etc.) placed in a matrix. The fibers may be of different length and shape, but quite commonly they are long and unidirectional. The most common architecture of fiber–reinforced composite materials is a laminate, which consists of a number of layers with different orientations of fibers and certain sequence (Fig. 1). Anisotropy of fiber-reinforced composites can be systematically utilized for local stiffness optimization, so that the design process can already begin on the material level.

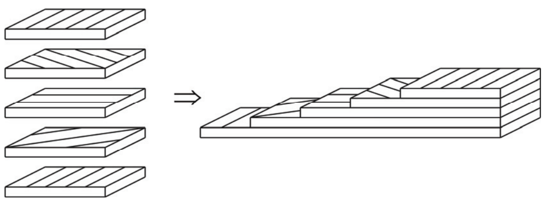

Fig. 1 Fiber-reinforced composite laminate

In modeling, heterogeneity and anisotropy of fiber-reinforced composites are handled by idealizations. Heterogeneity is treated by determining homogenized material properties based on a representative volume element, which results in transversely isotropic material properties of a single layer, requiring five material constants to characterize the material elastic behavior. Effective modeling of thin-walled structures made of fiber-reinforced laminates is, furthermore, driven by the recognition that the nature of their general behavior allows condensation of the complex 3D-field to the essential ingredients of the structural response described by a 2D approach [2]. The basic assumption of such a theory consists in expressing the displacement components of a point lying on the thickness line as a power series of the thickness coordinate with coefficients which are functions of the corresponding reference surface point position in the global coordinate system. A distinction can be made between layer-wise theories, which use layer-wise approximations for the displacements, and single-layer theories that treat the laminate as one single layer. Among single-layer theories, the first-order theories are the workhorse for simulation of thin-walled structures within the FEM, due to a very good compromise between the achievable accuracy and the required numerical effort. Two major first-order theories are used, namely, the Kirchhoff-Love and the Mindlin-Reissner ones. The essential difference between these two is the kinematical assumption related to the rotation of the thickness direction line, which further determines whether the transverse shear strains and stresses are included (Mindlin-Reissner) or not (Kirchhoff-Love) in a formulation. The Mindlin-Reissner kinematical assumptions are more frequent in FEM since they require only C^0-continuity from the FE shape functions. Upon reducing the material low in accordance with the assumptions of zero normal stress in the thickness direction and integration of material properties across the thickness based on the Mindlin-Reissner kinematics, one obtains the laminate constitutive equation according to the First-order Shear Deformation Theory (FSDT) as:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [F] \end{bmatrix} \begin{Bmatrix} \{\varepsilon_m\} \\ \{\kappa_f\} \\ \{\varepsilon_s\} \end{Bmatrix} \tag{1}$$

where {N}, {M} and {Q} are the in-plane forces, bending/torsional moments and transverse shear resultants, respectively, [A], [D], [B] and [F] are the extensional stiffness, bending stiffness, bending-extensional coupling stiffness and transverse shear stiffness matrices, respectively, and finally, $\{\varepsilon_m\}$, $\{\kappa_f\}$ and $\{\varepsilon_s\}$ are the membrane, flexural/torsional and transverse shear strains, respectively.

2. Properties of the Developed Shell Element and its Numerical Integration

Only the most important features of the developed shell element for composite laminates are to be given here, since a thorough description of the element formulation is already available in [2].

The developed element is an isoparametric 9-node element (full biquadratic shape functions). It is obtained by a “degeneration process,” in which the 3D field is condensed to 2D one by implementing the Mindlin-Reissner kinematical assumptions. In this manner, a shell element is obtained that is applicable to thin-walled structures with a wide range of thickness and curvature. The element formulation involves three different coordinate systems – global, local and natural coordinate system (Fig. 2). The local coordinate system is a

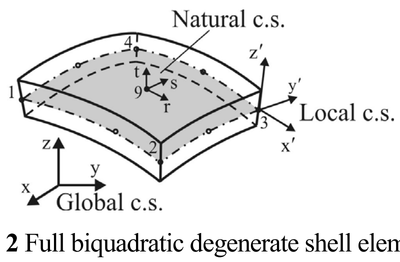

Fig. 2 Full biquadratic degenerate shell element

curvilinear coordinate system with two axes lying in the plane tangential to the structure reference surface, while the third axis is oriented along the thickness direction. This coordinate system is very important for the description of element

geometry and kinematics, but also the material properties, i.e. the ABD matrix (Eq. (1)), are given with respect to this coordinate system.

The integration of the element vectors and matrices in the thickness direction is performed analytically, while the in-plane integration is performed by means of numerical integration procedures. Due to its efficiency, the Gauss quadrature is the most frequently used numerical integration procedure within the FEM. It allows for exact integral evaluation for polynomials of degree *(2n-1)* with *n* evaluations of the function. For the element at hand, the 3×3 Gauss rule is required for the exact integration. However, the class of degenerate shell elements is generally susceptible to shear and membrane locking problems. Locking phenomena represent an intrinsic problem of the finite elements which, in their application, encounter constrained field problems. For instance, thin-walled structures with high slenderness are known to deform in such a manner that transverse shear strains are not involved in the deformational behavior. Hence, in this type of deformation, the transverse shear strain and, therefore, stress fields are constrained. Similarly, pure bending occurs without membrane strains. The incapability of an element to correctly represent such deformational states implies the presence of parasitic strains and stresses, whereas they are not a part of the considered physical regimes. In the aforementioned cases, the effects are denoted as shear and membrane locking, respectively. As a consequence, simulations yield stiffer structural behavior compared to the actual behavior.

The problem itself has been recognized a couple of decades ago, but it did take time to realize the actual reasons for it. In order to resolve the problem in a variationally consistent manner, Prathap [3] has proposed the paradigms of field-consistency and variational correctness. He has successfully demonstrated his postulates on simpler elements such as beam or plate elements. However, regarding the class of degenerate shell elements, his final conclusion is that the complexity of multiple mappings from one coordinate system to another inhibits tracking of inconsistent constrained fields and carrying out their reconstitution in order to obtain consistent fields. Hence, simpler techniques are to be used to avoid locking phenomena with this type of element, which, however, do not guarantee the variational correctness. One of the simplest and very effective techniques is the uniformly reduced integration.

It should be emphasized that, due to the full biquadratic shape functions, the developed shell element does not lock severely but suffers a sub-optimal convergence. The convergence behavior can be significantly improved by uniformly reduced integration, in this case the 2×2 rule, since it alleviates the locking effects. The developed element is coded so that the user is given the choice between the two integration rules. Further, the convergence analysis will be carried out for a set of simple composite laminated shell structures, whereby both integration rules are used.

3. Composite Laminated Shells – Aspect of Numerical Integration

The following examples consider composite laminated shell structures constituted of two types of layers – a composite (T300/976 graphite/epoxy) layer and an isotropic layer. The properties of the layers are summarized in Table 1. The properties are given with respect to the principal material directions for the composite layer and the material direction is, of course, irrelevant for the isotropic layer. The sequence and thickness of layers will be defined separately in the considered examples. The structures will be discretized with several FE meshes, with each succeeding mesh being finer, and the convergence of the results with full and reduced integration will be analyzed.

Table 1 Material properties of layers with respect to the principal material directions

Material properties	Isotropic layer	T300/976 graphite/epoxy
	Elastic properties	
E_{11} (GPa)	63	150
E_{22} (GPa)	63	9
ν_{12}	0.3	0.3
G_{12} (GPa)	24.2	7.1

3.1. Clamped composite shell

The first example considers a cylindrical composite shell with in-plane dimensions $a \times b$ = 254×254 mm (Fig. 3). It is a rather shallow shell with radius R=10×a (Fig. 3). The laminate consists of 8 layers with the stacking sequence given as $[ISOL/30_2/0]_s$, where ISOL stands for isotropic layer (Table 1). As a structure reference direction used to define the orientation of composite layers, the x-axis is taken. The thickness of each composite layer is 0.138 mm and of each isotropic layer 0.254 mm.

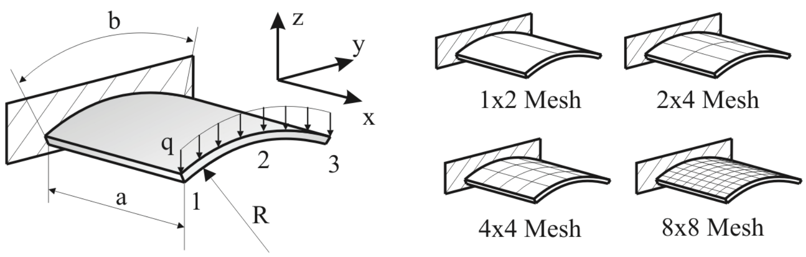

Fig. 3 Clamped composite shell with 4 FE meshes

The structure is subjected to a uniformly distributed transverse force over the free edge of shell q=2×10^{-2} N/mm (Fig. 3, left). It should be noted that the stacking sequence of the laminate is unbalanced, thus leading to coupling in structural behavior not typical for classical structural materials. Hence, the acting load causes both bending and twisting of the shell. In order to characterize this behavior properly, the transverse deflection is observed at five points of the free edge – the two end-points, the mid-point, denoted in Figs. 3 as 1, 3 and 2, respectively, and the points at mid-distance between points 1 and 2, and 2 and 3.

The considered FE meshes are depicted on the right-hand side of Fig. 3, while the diagram in Fig. 4 represents the obtained results in the form of normalized displacements at the aforementioned five points, with normalization done with respect to dimension b of the structure.

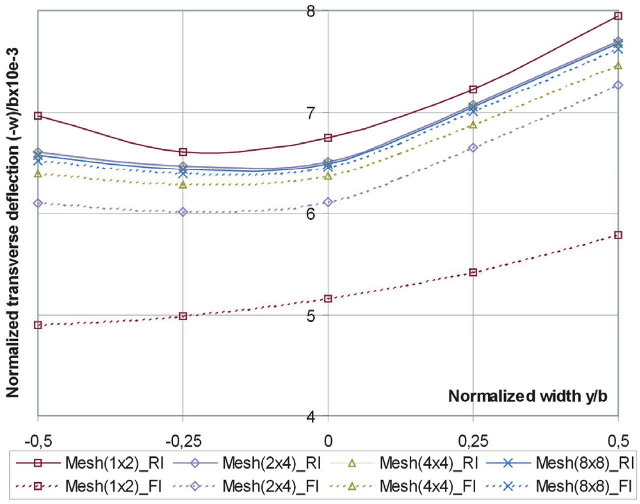

Fig. 4 Free edge transverse deflection – convergence of the full integrated (FI) and reduced integrated (RI) degenrate shell element

As the diagram shows, the refinement of the mesh with the fully integrated element (dotted lines in the diagram, curves denoted by FI in the legend) leads to a converged solution. However, a much faster convergence rate may be observed with the shell element implementing uniformly reduced integration (solid lines in the diagram, curves denoted by RI in the legend). This points out that the reduced integrated element is free of locking effects. In both cases, the convergence is monotonic and leads toward the same result, but while it proceeds from below with the fully integrated element, it is the other way around with the reduced integrated element.

3.1. Simply supported composite arch

In the second example, a simply supported cylindrical arch with radius R=100mm and width b=60mm is considered. The stacking sequence is in this case $[ISOL/45/-45/0]_s$. The thickness of each composite layer is 0.12 mm and of each isotropic layer 0.24 mm. The stacking sequence is balanced here, thus resulting in orthotropic material properties overall. Again, the load is uniformly distributed vertical force $q=2\times10^{-2}$ N/mm (Fig. 5). The structure is discretized with 3 meshes, each having 4 elements across the width, while the number of elements in the circumferential direction goes from 10 over 20 to 40.

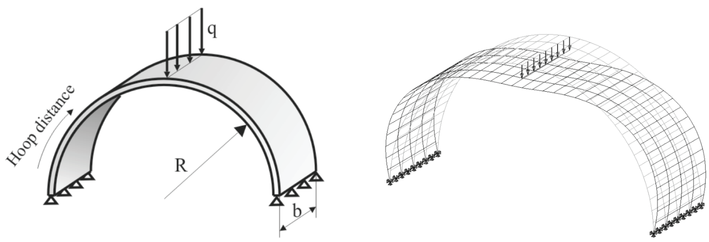

Fig. 5 Simply supported arch subjected to uniformly distributed vertical force

The case is computed with the originally developed degenerate shell element by using full and reduced integration, but also with the 8-node Semi-Loof element [1], which is based on the discrete Kirchhoff theory, thus not accounting for transverse shear strains, and is coded so that only full integration is offered. The results are given in diagrams in Figs. 6 and 7 following the same pattern of denotation used in the previous example.

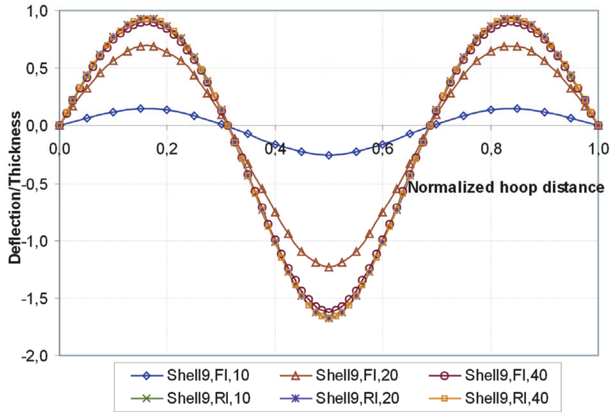

Fig. 6 Radial deflection of the arch – convergence of the 9-node degenerate shell element

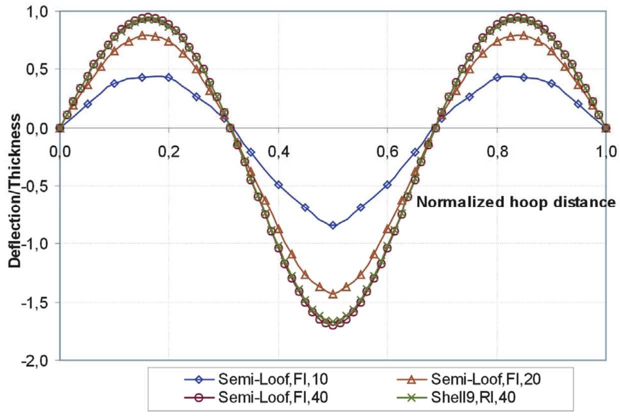

Fig. 7 Radial deflection of the arch – convergence of the 8-node Semi-Loof element

The example confirms the convergence behavior observed in the first example. The 9-node degenerate shell element with reduced integration demonstrates excellent convergence behavior. Both the degenerate shell element and the Semi-Loof element with full integration converge to the same result, however significantly slower compared to the degenerate shell element implementing uniformly reduced integration.

4. Conclusions

Structural analysis of modern thin-walled structures involving lightweight materials, such as fiber-reinforced laminates, requires very efficient numerical tools for successful modeling and simulation of their behavior. Within the framework of FEM, this is accomplished by means of shell elements, which are, however, susceptible to shear and membrane locking effects. The paper considers the aspect of reduced integration as a possible solution to the problem. The efficiency of the method has been successfully demonstrated. As a disadvantage, the technique may give rise to spurious zero-energy modes, which are, however, typically not invoked in an assembly of elements [3]. This fact provides the reliability of the approach. In using the approach, one should bear in mind the fact that the convergence of the result does not necessarily proceed from below.

The considered technique has a significant influence on simulations efficiency. Reduced integration, as the name itself suggests, requires fewer integration points to integrate the element stiffness matrix. In geometrically nonlinear computations, which require re-assemblage of the stiffness matrix in each increment and iteration, this reduces the required CPU-time. Even more important is the fact that converged solution is reached with significantly fewer degrees of freedom. This has a great positive impact on simulation efficiency of transient dynamic analyses.

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O EFIKASNOSTI FE-LJUSKE SA EKVIVALENTNIM SLOJEM ZA KOMPOZITNE LAMINARNE KONSTRUKCIJE

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Naglašena potreba za lakim konstrukcijama zahteva razvoj novih materijala od kojih se očekuje da ponude brojne prednosti u odnosu na konvencionalne konstruktivne materijale. Kompozitni laminati sa ojačanjima u vidu vlakana spadaju u najsavremenije konstruktivne materijale. Efektivna simulacija ponašanja konstrukcija koje uključuju ove materijale je osnova njihovog lakšeg, bržeg i jeftinijeg konstruisanja. U tu svrhu autori koriste nedavno razvijeni konačni element tipa ljuske koji koristi pristup ekvivalentnog sloja materijala. Rad istržuje uticaj reda numeričke integracije na efikasnost i tačnost razvijenog numeričkog alata. Uniformno redukovana integracija je upoređena sa punom integracijom na setu primera tankozidih konstrukcija od kompozitnih laminata sa ojačanjima u vidu vlakana.

Ključne reči: *kompozitni laminati, tankozide konstrukcije, FE-ljuska, redukovana integracija*