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METHODOLOGY OF WORKSPACE ANALYSIS OF MACHINE TOOLS BASED ON PARALLEL MECHANISMS

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Abstract: *The contemporary production using the processing method of removing materials has since recently implied the application of new concepts of machine tools. Among them, the significant place belongs to parallel mechanism-based machine tools. The paper describes the methodology of analysis of parallel mechanisms' workspace aiming at establishing the basis for automated design of parallel mechanism-based machine tools.*

Key Words: *Parallel Mechanism, Workspace, Analysis, Discretisation*

1. INTRODUCTION

Since its inception, starting mid-20th century till now, parallel mechanisms have been in wide-ranging professional literature referred to as a possible alternative to the conventional (serial) structure of the machine tools and robots. On the other hand, in addition to numerous advantages that this form of structure has in relation to the existing ones, its wider application was only possible due to intensive development of technology in the late 20th century. First of all, this refers to improvements made in the fields of the concept [3], control systems [6], formation of assemblies [2], [7], [8].

Further development of parallel mechanisms is, among other things, subject to research of their mechanical characteristics thereby making improvements to specific components and structure. Accordingly, the issues of description and systematization of the workspace, as well as their role in the design process of machine tools take a significant place.

As a result, a series of research projects have been made recently with the purpose to analyze workspace characteristics and their application in the process of machine tools design. This is mainly due to the fact that the workspace of such devised machine tools does not represent a regular geometric shape as it is the case with the conventionally designed machine tools with the serial kinematics. Application of parallel mechanisms in the mechanical structure of machine tools results in the workspace of the complex geometric form and reduced volume in relation to the conventional machine tools. This has initiated a greater number of research studies in the field of the workspace analysis, with a goal to

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establish the applicability of parallel mechanisms in machine tools from this perspective. As a result, and in order to overcome specific shortcomings of parallel mechanisms, a larger group of mechanisms with the hybrid, serial-parallel kinematics have recently appeared.

Research conducted in this field [4], [9], [11] aim at defining dimensions and topological characteristics of the workspace which is created by application of parallel mechanisms.

Application of *analytical methods* in the process of defining the workspace is particularly significant in designing and optimization of machine tools and manipulators of this type. A number of methods based on the principles of analytical mathematics belong to more significant research studies in the field of defining the parallel-mechanisms' workspace [5], [1] and [11].

The application of these methods results in exact boundaries of the mechanism workspace, whereas only the method of defining the maximal workspace requires the complex mathematical apparatus. Merlet [4] describes the implementation of specific methods in the program package aiming to visualize the workspace of the six-axis parallel mechanism based on Stewart's concept.

Accordingly, the specialized program system for the workspace analysis of the parallel-kinematics mechanisms named "workspace" has been developed at the Department of Production Mechanical Engineering of the Faculty of Technical Sciences in Novi Sad. The paper illustrates the structure and a part of results obtained by application of this program system in the analysis of several types of parallel mechanisms with the aim of its comparison with other types of parallel mechanisms.

2. Concept of the Program System for the Workspace Analysis

Setting the criteria for selection and design of machine tools based on parallel mechanisms represents a complex issue involving a series of analyses of their characteristics and impacts created by specific elements within the workspace of mechanisms. The basic pur-

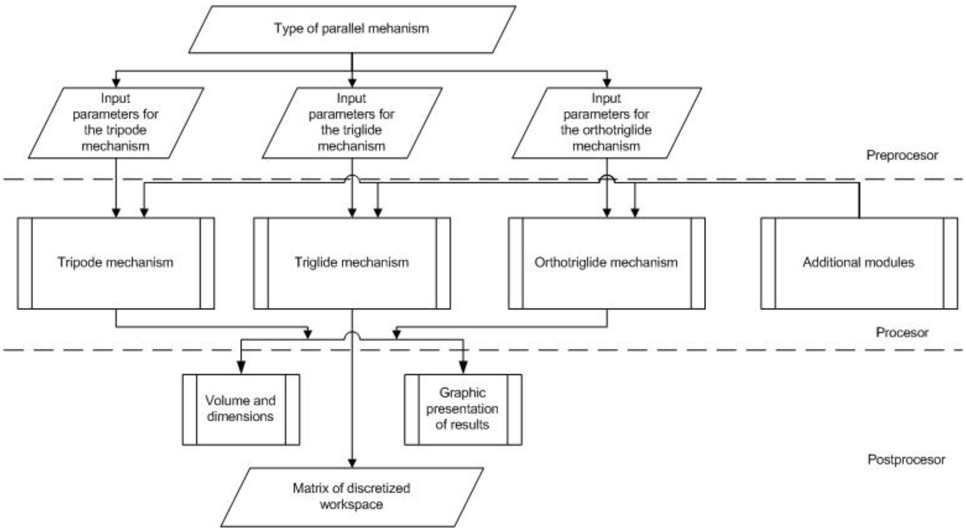


Fig. 1 Model of the parallel mechanism program system [10]

pose of the developed program solution is to define the matrix interpretation and to establish the space model of the discretized workspace for a number of typical types of parallel mechanisms. Introduction of the continuum discretization theory into the analysis issues of complex mechanisms, as it is required by the solution of inverse kinematics of parallel mechanisms, includes mathematical processing of a great number of non-linear equations. In the present circumstances, these assignments are solved by means of specialized program applications, such as Matlab, Mathcad or Mathematica, which enable implementation of theoretical analyses with simplification of descriptions and processing of more complex mathematical problems.

Consequently, the program system "workspace" was implemented as a modular program solution consisting of the three units: preprocessor, processor and postprocessor. Fig. 1 shows the model of the developed program system.

The preprocessor and postprocessor subsystem of the program system "workspace" has been implemented with the maximal use of resources provided by the Matlab program application. This includes the application of the graphical user interface and the support for manipulation and visualization of the spatial forms. The results of the analysis obtained after processing in the postprocessor subsystem include, in addition to the workspace volume and dimension values, its spatial presentation, as well as the matrix of coordinates enabling post-analysis so as to determine mathematical regularities arising from the analysis. Besides, as shown in Fig. 1, the processor subsystem is developed from a number of modules, of which the three crucial ones deal with the kinematics regularities of typical types of parallel mechanisms: tripod, triglide and orthotriglide. This concept of defining the program system processor leaves the possibility for its upgrade, in line with the development trends in the field of parallel mechanisms. Fig. 2 shows visual computer models of parallel mechanisms which may be analyzed in the existing version in the program system *"parallel mechanism"*.

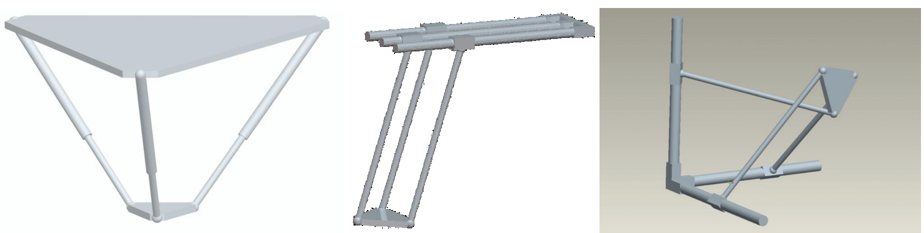


Fig. 2 Typical types of parallel mechanisms

As seen in Fig. 1 the parallel mechanism of the tripod type differs in the structure of its elements from the mechanisms of the triglide and orthotriglide type, which suggests that the considerable differences between the program modules constitute the program system processor.

3. BASIC CHARACTERISTICS OF PARALLEL MECHANISMS

3.1. Tripod mechanism

The tripod mechanism (Fig. 3) is in theory considered the simplest type of parallel mechanisms in terms of its construction, which was developed with minor modifications from the original Stewart's mechanism in 1965 [1]. Structure of this mechanism consists of two platforms (the immobile one, called the base, and the mobile one) which are interconnected by struts of variable length. Spherical or universal (cardan) joints are used for the connection between struts and platforms, while the complex spatial movement of the mobile platform is achieved by adding up the single axial movements of mechanism struts. The workspace analysis of the tripod mechanism implies a separate consideration of the vector polygon comprising the base, one of the mechanism struts and mobile platform. The polygon of the vector comprising one of the struts, as shown in Fig. 2, requires equation solving (1), (2) or (3):

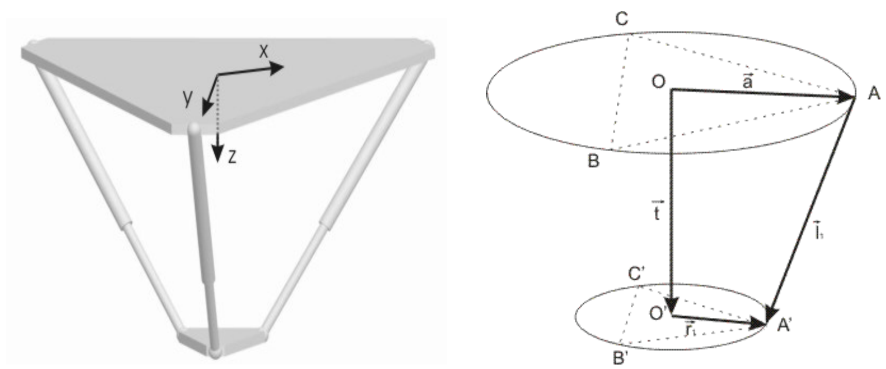


Fig. 3 Tripod mechanism and illustration of the vector polygon for the tripod mechanism

$$\vec{a} + \vec{l}_1 = \vec{t} + \vec{r}_1 \tag{1}$$

$$\vec{b} + \vec{l}_2 = \vec{t} + \vec{r}_2 \tag{2}$$

$$\vec{c} + \vec{l}_3 = \vec{t} + \vec{r}_3 \tag{3}$$

With the aim of completing the theoretical analysis, in addition to movement regularities, we should also include the restrictions resulting from the structural characteristics of specific mechanism elements which are provided in the form of in equations. These are as follows:

- marginal dimension of struts (minimal and maximal):

$$L_{\min} \leq |\vec{l}_i| \leq L_{\max}\text{, for } i = 1, 2, 3 \tag{4}$$

- marginal rotating angles in specific joints:

$$\theta_{\min} \leq \theta_{ibaze} \leq \theta_{\max}\text{, for } i = 1, 2, 3 \tag{5}$$

$$\theta_{\min} \leq \theta_{iplatforme} \leq \theta_{\max}\text{, for } i = 1, 2, 3 \tag{6}$$

- minimal distance between the struts (in order to prevent their collision):

$$r_{\min} \le r_{i,j}\ ,\ \text{for } i = 1, 2, 3, j = 1, 2, 3 \tag{7}$$

On the basis of equations system, the solutions of direct and inverse kinematics of the mechanism are defined ranging from (1) to (7), which are required to define the workspace shape of a specific parallel-mechanism-based machine tool and for its further analysis.

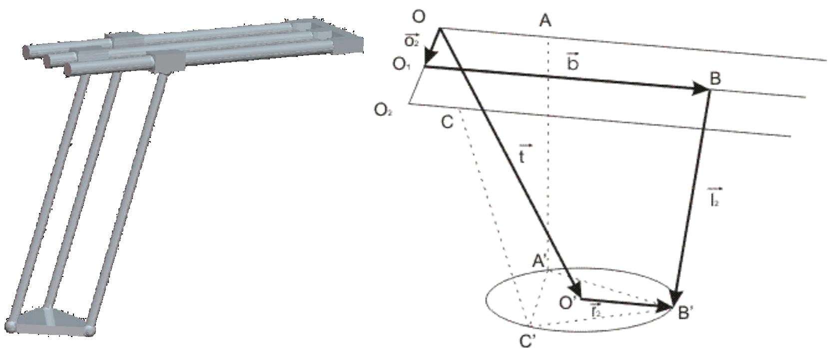
3.2 Triglide mechanism

The triglide mechanism is a more advanced type of parallel mechanisms developed in the late nineties of the twentieth century, with an intention to eliminate certain imperfections found in the classic Stewart's mechanism. This mainly refers to the workspace dimensions and shape as well as to the construction of telescopic struts.

Triglide mechanism differs from the previously analyzed tripod mechanism in two modifications including:

- Base construction, and,
- Construction of struts which are of the fixed length in this case.

The base structure of the triglide mechanism comprises three separate and parallel guides. The sliders attached to the struts slide along the guides, whereas the struts are affixed to the mobile platform from the other side. Spherical or universal joints are used as connecting elements between struts and the base, that is, the mobile platform. Fig. 4 shows the arrangement of vectors circumscribing the position of the mobile platform attached to one of the mechanism struts.



a) b)

Fig 4 Triglide mechanism and presentation of the triglide mechanism vector polygon

On the basis of such defined vector polygon, the following equations are obtained for the three sliders:

$$\vec{OA} + \vec{l}_1 = \vec{t} + \vec{r}^{R}{}_1 \tag{8}$$

$$\vec{o}_2 + O_1\vec{B} + \vec{l}_2 = \vec{t} + \vec{r}^{R}{}_2 \tag{9}$$

$$\vec{o}_3 + O_1\vec{C} + \vec{l}_3 = \vec{t} + \vec{r}^{R}{}_3 \tag{10}$$

In the general form, these equations may be presented as follows:

$$\vec{o}_i + \vec{b}_i + \vec{l}_i = \vec{t} + \vec{r}^{R}{}_i \tag{11}$$

Vector $\vec{o}_i$ presents the complementary base vector typical of the triglide mechanism with the following properties:

$$\vec{o}_1 = \vec{0}\,,\ \vec{o}_2 = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix},\ \vec{o}_3 = \begin{pmatrix} 0 \\ 2y \\ 0 \end{pmatrix} \tag{12}$$

Value y in vector $\vec{o}$ presents the distance between the guides of the triglide mechanism. After the introduction of rotation matrix (12), the general equation transforms into:

$$\vec{o}_i + \vec{b}_i + \vec{l}_i = \vec{t} + R_{\theta\psi\phi}\vec{r}_i\,,\ \text{for } i = 1, 2, 3 \tag{13}$$

For further analysis of triglide mechanism kinematics, it is also necessary to take into consideration the physical restrictions in the triglide mechanism. The following restrictions are found in the triglide mechanism:

▪ *Maximum travel of sliders*

The maximum travel of sliders of the triglide mechanism is subject to the guide's lengths, which is expressed in vector equations.

$$0 \le |\,\vec{OA}\,|,|\,\vec{OB}\,|,|\,\vec{OC}\,| \le S_{\max} \tag{14}$$

▪ *Joints deflection angles*

The values on the sliders and the mobile platforms have to be, as in the case of the tripod mechanism, within the limits determined by physical characteristics of joints.

$$\theta_{\min} \le \theta_{ibaze} \le \theta_{\max} \tag{15}$$

$$\theta_{\min} \le \theta_{iplatforme} \le \theta_{\max} \tag{16}$$

▪ *Distance between the struts*

As is the case with all mechanisms making the complex movement in space, the distance between the struts has to be above the marginal value at any point of the platform movement. This restriction is introduced in order to eliminate the possibility of the struts' collision during the mechanism movement.

$$r_{\min} \le r_{i,j} \tag{17}$$

3.3 Orthotriglide

Conceptually, the orthotriglide mechanism represents a modified version of the triglide mechanism, the base of which consists of three orthogonal guides. This type of parallel mechanism is developed to provide a complex movement of the mobile platform in space by linear movement of sliders in three orthogonal directions. This enables upgrading of conventional machine tools with parallel mechanisms as well as their further research.

This type of parallel mechanisms consists of three orthogonal guides along which the sliders linked by the fixed-length struts are sliding, fitting together with the mobile platform. The base and struts, as well as the struts and the mobile platform are connected by means of spherical or universal joints. Fig. 5 shows the arrangement of vectors on the orthotriglide mechanism, which defines the position of the mobile platform. Visual model of the orthotriglide mechanism, as well as the arrangement of vectors for elements position for the orthotriglide mechanism is shown in Fig. 5.

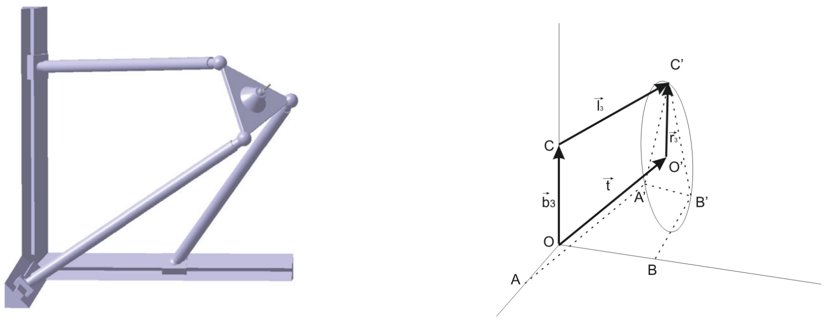


Fig. 5 Ortotriglide mechanism

On the basis of the vector polygon shown in Fig. 5, the following regularities defining the position of the mobile platform in space may be established:

For points OAA'O' $$O\vec{A} + \vec{l}_1 = \vec{t} + \vec{r}^{R}{}_1 \quad (18)$$

For points OBB'O' $$O\vec{B} + \vec{l}_2 = \vec{t} + \vec{r}^{R}{}_2 \quad (19)$$

For points OCC'O' $$O\vec{C} + \vec{l}_3 = \vec{t} + \vec{r}^{R}{}_3 \quad (20)$$

The general form of these equations is:

$$\vec{b}_i + \vec{l}_i = \vec{t} + \vec{r}^{R}{}_i \text{, for } i = 1, 2, 3 \quad (21)$$

In equation (21) in which $\vec{b}_i$ presents the base vector consisting of orthogonal sliders vectors (22).

$$b_1 = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix},\ b_2 = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix},\ b_3 = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \quad (22)$$

On the basis of the base vector definition, it is evident that there is a considerable difference between the base vector of the tripod and orthotriglide mechanism, which affects resolving of kinematic problems.

After adding of the rotation matrix, expression (21) reads:

$$\vec{b}_i + \vec{l}_i = \vec{t} + R_{\theta\psi\phi}\vec{r}_i \text{, for } i = 1, 2, 3 \quad (23)$$

Physical restrictions of the orthotriglide mechanism concept considerably influence the kinematics of the mechanism. These include as follows:

▪ *Maximum travel of the sliders*

This restriction of the orthotriglide mechanism depends on the length of guides, which in vector equation is expressed as follows:

$$0 \leq |\vec{OA}|, |\vec{OB}|, |\vec{OC}| \leq S_{\max} \tag{24}$$

▪ *Joints deflection angles*

The restriction in the struts movement enabled by the joints is defined as follows (25) and (26):

$$\theta_{\min} \leq \theta_{ibaze} \leq \theta_{\max} \tag{25}$$

$$\theta_{\min} \leq \theta_{iplatforme} \leq \theta_{\max} \tag{26}$$

▪ *Distance between the struts*

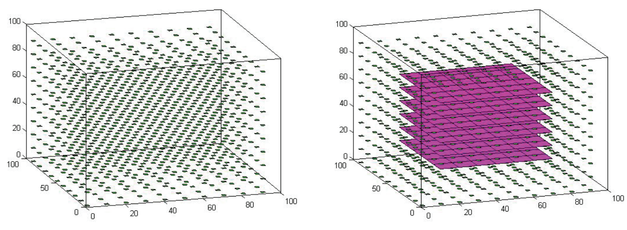
In order to prevent the collision of struts, the distance between them has to be above the marginal one.

$$r_{\min} \leq r_{i,j} \tag{27}$$

4. Methodology of Workspace Analysis of Machine Tools Based on Parallel Mechanisms

A need for further improvements of parallel mechanisms influenced the research in the direction of defining and systematization of forms, dimensions, as well as other geometric characteristics of their workspace. With that in mind, for the purpose of the "workspace" program system, the methodology for defining workspaces of specific types of parallel mechanisms was introduced, as well as the criteria for their further analysis. This includes determination of the spatial computer model of the workspace, its visualization and maximum level of flexibility in the process of further analysis.

These requirements resulted in defining of the processor of the "workspace" program system by means of implementing the workspace defining procedure based on the continuum discretization (Fig. 6).



a) b)

Fig. 6 Workspace defining procedure

The application of this analysis method and defining of the workspace raises two very important questions to be considered. These include:

- the accuracy of the performed analysis and
- he possibility of subsequent analyses of the volume and related parameters.

The analysis error is the factor which can be completely influenced by correction of the analyzed point's density. Consequently, we can significantly influence the time required for the workspace formation. This factor has the constant value and is equal to half a number of the search steps during the analysis.

The application of the discrete analysis of the workspace is particularly favorable due to its space description method which is suitable for presentation of the results in the form of the matrix of characteristic points' coordinates. As a result, this considerably facilitates calculation of the workspace dimensions, that is, the volume of the workspace of parallel and conventional mechanisms, etc.

Accordingly, it is possible to establish the workspace model with the desired level of accuracy in the form of "the cloud" of discrete coordinates which allows of further visualization and numeric processing of obtained results.

Fig. 7 shows the discretized workspace model and the diagram of its defining procedure.

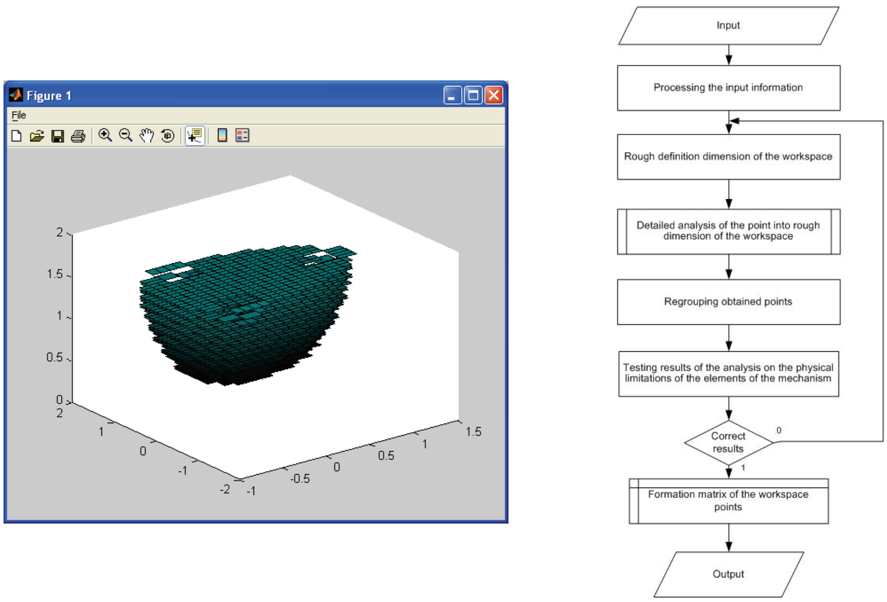


Fig. 7 Discretized workspace of the parallel mechanism of the tripod type and diagram of its defining procedure

In addition to visual workspace presentation, the criteria applied to compare parallel mechanisms have major importance for the assessment of their utilization level. These criteria result from:

- Actual workspace dimensions and their interrelation
- Workspace volume of the parallel mechanism
- Ratio between the ideal and actual workspace volume

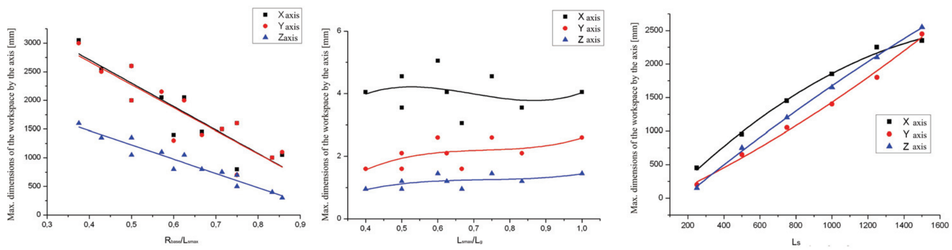
Subject to the research requirements, the individual or all the above-mentioned criteria are incorporated in the postprocessor program system for the workspace analysis. The postprocessor program system module "workspace" comprises units which are used for calculation of all above-mentioned criteria.

5. RESULTS

One of the most important tasks of the program system "workspace" is to create the base for analyzing a number of parallel mechanism types, as well as systematization of geometric characteristics of their workspace. Accordingly, this enables determination of criteria necessary in the process of machine tools design. With that in mind, analysis of several dozens of parallel mechanisms of the tripod, triglide and orthotriglide type has been undertaken. The obtained results have been classified into four categories which allow for the setting of numeric criteria. These include as follows:

- Analysis of overall dimensions of the mechanism workspace
- Workspace volume
- Ratio between the workspace volumes of the machine tools with parallel and conventional kinematics
- Workspace shape

Analysis of actual overall dimensions of the workspace results in the functional regularities on the basis of which the workspace dimensions are envisaged in all three axes. Fig. 8 illustrates the results obtained for tripod, triglide and orthotriglide mechanism.



a) tripod b) triglide c) orthotriglide

Fig. 8 Analysis of overall workspace dimensions

The second analysis parameter, the workspace volume, enables the analysis and determination of functional dependencies of the workspace volume on the mechanism dimensions. Fig. 9 shows this result for the tripod mechanism.

As one of the most significant criteria for comparison of the machine tools with serial and parallel kinematics is the ratio between their workspace volumes. This factor is most frequently called the factor of the space coverage. It considerably justifies the application of this concept in the machine tools construction. Fig. 10 shows this dependence for triglide mechanisms.

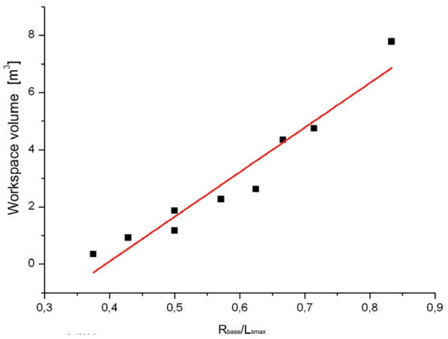


Fig. 9 Analysis of functional dependencies of the workspace volume on the mechanism dimensions

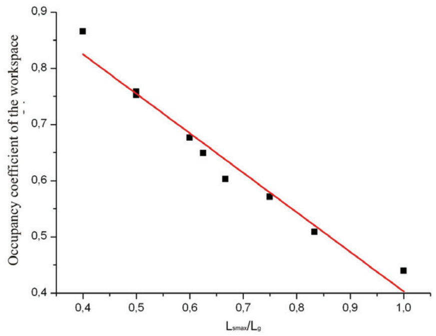


Fig. 10 Ratio between the workspace volumes of mechanisms with parallel and serial kinematics

Apart from the illustrated dependence of the overall workspace dimensions and volume size on dimensions and the mechanism dimensions' ratio in parallel mechanisms, it is also evident that they affect the shape of the workspace. Fig. 11 shows some of the parallel-mechanisms' workspace shapes of the tripod type.

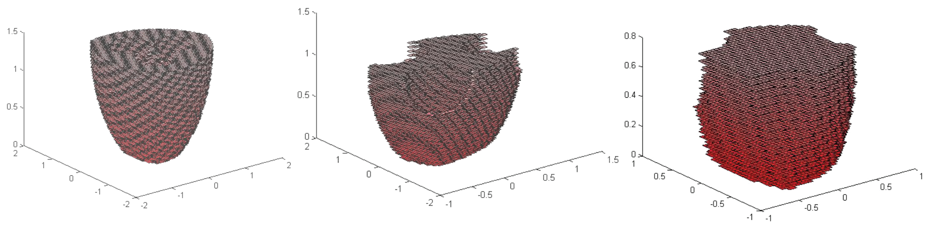


Fig. 11 Parallel-mechanisms' workspace shapes of the tripod type

6. Conclusion

The research studies, of which only one segment is presented in this paper, only attempt to consolidate scientific activities with a view to improving parallel mechanisms and applying the obtained results in the process of automated machine tool design. The results of these research studies have been integrated; they signify a step forward in the process of development of parallel-mechanism-based machine tools. Furthermore, it has to be emphasized that integration of these systems by application of specialized program applications considerably simplifies the analysis processes. Results of the workspace analysis obtained by application of the program system *"parallel mechanism"* enabled setting of the criteria for selection of parallel mechanism elements and their optimization. This is an inevitable link in the process of development of the program systems for automated design of machine tools based on these mechanisms. In further research, a very important feature of the program system is a modular concept which provides considerable possibilities for extension and improvement of the system by upgrading it with new types of mechanisms.

Further research in the field of the workspace analysis for parallel-mechanisms or hybrid parallel-serial mechanisms provides for setting up of the knowledge base on potential applicability of each, so-far implemented, type of mechanisms in the field of design and development of machine tools.

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METODOLOGIJA ANALIZE RADNOG PROSTORA MAŠINA ALATKI NA BAZI PARALELNIH MEHANIZAMA

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Savremena proizvodnja primenom metoda obrade skidanjem materijala poslednjih godina podrazumeva primenu novih koncepcija mašina alatki. Među njima značajno mesto pripada mašinama alatkama na bazi paralelnih mehanizama. U radu je opisana metodologija analize radog prostora paralelnih mehanizama u cilju stvaranja podloga za automatizovano projektovanje mašina alatki baziranih na njima.

Ključne reči: *paralelni mehanizam, radni prostor, analiza, diskretizacija modela*