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IMPACT OF ROLLING BEARING STRUCTURAL PARAMETERS ON THE BALANCED RIGID ROTOR'S OSCILLATION FREQUENCY

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Abstract. *Rolling bearings represent some of the most important uproar and vibration generators in mechanical structures. Accordingly, it is very important to know the impact of rolling bearing structure on the main parameters which determine the level of uproar and vibrations produced inside machines. This paper presents a mathematical model of the dynamic behavior of rigid rotor inside the rolling bearing and also a theoretical analysis of the rolling bearing structure with respect to the frequency of calibrated rigid rotor oscillation inside the rolling bearing.*

Key words: *Rolling Bearing, Vibrations, Construction, Specific Frequency*

1. INTRODUCTION

Requirements for uproar and vibration reduction in contemporary mechanical industry products are increasing every day. The need to meet these requirements is imposed, on one hand, by a more and more demanding and refined market, and, on the other hand, by a set of positive laws and regulations, which have lately been regulating ever rising standards with respect to the permitted level of uproar and vibrations generated by these products. One of the main uproar and vibrations generators in mechanical structures is built within rolling bearings. Because of that, it is of special importance to know the rolling bearing construction parameters' impact on the uproar and vibration level these products generate.

When it comes to the rolling bearings, the terms 'uproar' and 'vibrations' often describe similar and connected phenomena. The noise made by the machine's working is the result of vibrations generated by its rolling bearings. The noise, that is, vibrations, as physic phenomena, are determined with respect to two dimensions: amplitude and frequency. Amplitude is characteristic of the vibration signal intensity, while frequency is the

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iterating vibration speed in a time unit. In this paper, the analysis of rolling bearings structure's impact on the vibrations frequency generated by the rolling bearing is given.

The need to know the frequency of vibrations generated by rolling bearings could be generally divided into three categories.

The first category includes the applications where machines, because of high frequencies, are producing very harsh vibrations and noise. This is particularly the case with widely-consumed devices such as household appliances or office equipment, where noise or vibrations made by these devices could be very irritating. The vibrations and sounds get even higher when the given device is accumulating speed to get to its working one or when it is slowing down. This fact indicates that the device is, throughout its process of acceleration or slowing down, passing through the domain of resonance. Knowledge of frequencies and vibrations induced by the rolling bearing can contribute to solving these problems, reducing them or avoiding them altogether.

The second case embraces the machines which, for the sake of their proper operation, must have very high characteristics of working and positional accuracy. These are, for example, various instrumental machines, tin metal, paper and chemical films rolling mills, etc. Computer compact discs are an example where continuous bearing accuracy is needed, within the limits of 0,25 to 0,5 μm [1]. Rubbing machines axles often have to meet very high requirements at the point of geometric accuracy and surface roughness, with deviations more or less than 1 μm. Vibrations could largely contribute to deviations at the point of geometric accuracy and surface roughness and could produce percussive impulses which could cause permanent metallurgic damage to flame - hardened steel elements.

The third category comprises these cases where working accuracy is not so important as working availability, reliability or safety. Machines of this category often produce and emit large power; they have huge rotational components and are working at high working speeds. In the main, these are compressors, pumps, turbines, etc. In order to increase availability, operational safety and machines' working life, it is very important to identify, beforehand, changes of machine conditions of the mean of battering, deformations, unbalanced work and other kind of difficulties, in order to prevent excessive damages and their consequences. Rolling bearings progressively decay so that their vibration measurement is used for longest period of time to detect any problems; thus they have become cheaper and more reliable in last few years. For a correct analysis, the condition to know the frequency generated by the roller bearings is of vital importance.

Because of that, as for these applications as for solving problems from the first and second categories, it is very important to know the rolling bearing structure's impact on the dynamic behavior and vibration frequencies of rotating machines. An analysis of the main rolling bearing structural parameters impact on the oscillating frequency of the rotating system based on the rolling bearing is done in this paper.

2. ROLLING BEARING BALANCED RIGID ROTOR DYNAMIC MODEL

The very first research studies of the rigid rotor sleeve in the rolling bearing motion were done by Krjuchkov in 1959. [4]. He deduced that the centre of cross section trajectory of rolling bearing rotor has an ideal geometry, with inner radial irradiance, so that it could be presented by absolute sinusoid equation, according to:

$$y = \Delta \left| \sin \frac{\pi \cdot s}{\lambda} \right| \tag{1}$$

Here is:

Δ – top-top (pp) rotor oscillating amplitude [6];

λ – line distance between the bearings's rolling bodies;

s – curvilinear coordinate of bearing's rolling bodies centers of gravity (*Fig. 1*).

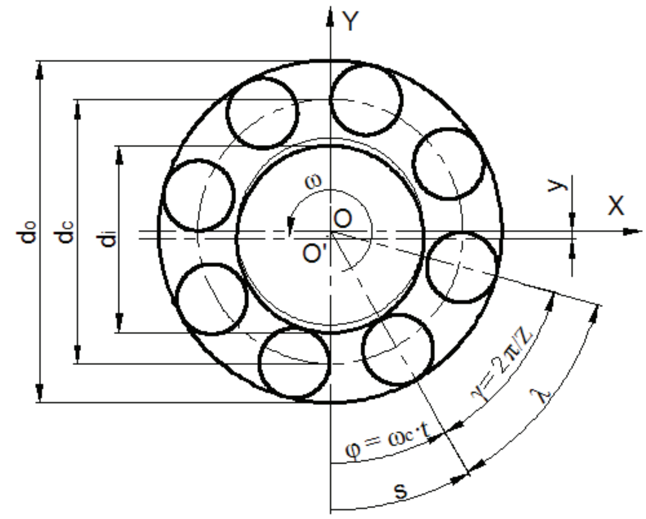

Fig. 1 Schematic Presentation of Rolling Bearing Kinematics

The path of the bearing's rolling bodies centers of gravity is divisional axis, i.e., the centers of the rolling bodies are moving along the divisional circumference of the cage. Divisional circumference diameter of the cage (d_c) is equal to arithmetical mean between inner (d_i) and external rolling path diameter (d_o):

$$d_c = \frac{d_i + d_o}{2} \tag{2}$$

Accordingly, we can get the line distance between bearing's rolling bodies (λ) if we divide the length of divisional circumference of the cage with the total number of rolling bodies (z), according to equation:

$$\lambda = \frac{\pi \cdot d_c}{z} \tag{3}$$

Curvilinear coordinate of bearing's rolling bodies centers of gravity (s) could be taken as a product of the cage divisional circumference radius and angular coordinate $\varphi = \varphi(t)$ of rolling bodies centers of gravity (Fig. 1), according to:

$$s = r_c \varphi = \frac{1}{2} d_c \varphi \tag{4}$$

Angular coordinate (φ) of bearing rolling bodies centers of gravity could be taken as function of time (t) and raw speed of the cage (ω_c), according to equation:

$$\varphi = \omega_c \cdot t = \frac{\omega}{2} \cdot \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha\right) \cdot t \tag{5}$$

Here it is:

ω – rotor angular speed,
d_b – diameter of rolling bodies,
α – working contact angle of bearing,
t – time.

Expression for raw speed of cage ω_c is managed under ordinal number [7]. Now expression (4) for curvilinear coordinate (s) could be written as:

$$s = \frac{1}{2} d_c \cdot \omega_c \cdot t = \frac{\omega}{2} \cdot \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha\right) \cdot \frac{d_c \cdot t}{2} \tag{6}$$

If we put expression (6) for curvilinear coordinate (s) and expression (3) for line distance between the bearings's rolling bodies (λ) in equation (1) we get:

$$y = \Delta \left| \sin\left(\frac{1}{2} z \cdot \omega_c \cdot t\right) \right| = \Delta \left| \sin\left[\frac{1}{2} \cdot \frac{z}{2} \cdot \omega \cdot \left(1 - \frac{d_b}{d_c} \cdot \cos\alpha\right) \cdot t\right] \right| \tag{7}$$

With relation to:

$$y = \Delta \left| \sin\left(\frac{f_o}{2} \cdot t\right) \right| \tag{8}$$

Where is:

$$f_o = \frac{z}{2} \cdot \omega \cdot \left(1 - \frac{d_b}{d_c} \cos\alpha\right) \tag{9}$$

Dimension f_o in expression (9) is called *characterized frequency of external bearing ring* and it is the speed which the rolling bodies have while passing through some fixed point on the external bearing ring. This frequency is often called *frequency of rotor rolling across bearing rolling bodies* [7].

Accordingly, the rotor cross-section center, with relation to the bearing housing orifice center, continuously oscillates during the bearing operation. These oscillations could be depicted with equation of absolute sinusoid (8). According to equation (8), the balanced rigid rotor oscillating frequency in the rolling bearing is equal to bearing external ring specific frequency. Dimension *pp*-amplitude of these vibrations is directly dependent on the dimension of bearing's rings relative motion. Relative motion of the bearing's rings is a direct index of dimension of radial working irradiance of the rolling bearing. According to that, the vibration of specific frequency of external bearing's ring and its harmonics ($k{\cdot}f_o$), is the best indicator of radial working irradiance dimension and it can be very an important diagnostic parameter of the rolling bearing working capacity and working correctness. In the technique of vibrations analysis, specific frequencies of rolling bearing vibrations are used for analyzing the vibration spectra for the detection of damage locations and various irregularities on the bearing.

3. Balanced Rigid Rotor Based on Rolling Bearings Oscillating Frequency Changes - Theoretical Analysis

Therefore, the rigid rotor oscillating frequency in an ideal rolling bearing with inner radial irradiance is equal to specific frequency of the bearing's external ring f_o, i.e. to the rolling bodies moving through some fixed point on the bearing external ring frequency. On the basis of equation (9) we see that this frequency is directly dependent on the rotor turnover speed and the bearing structure (number of rolling bodies z, diameter of rolling bodies d_b, divisional diameter of cage d_c and angle of contact α). Also, from this equation we could see that rolling bearing rotor oscillating frequency is not dependent on the inner radial irradiance dimension.

If we put in equation (9) that it is:

$$k_\omega = \frac{z}{2}\left(1 - \frac{d_b}{d_c}\cos\alpha\right), \tag{10}$$

we get that balanced rigid rotor oscillating frequency in an ideal rolling bearing has linear dependence of rotor turnover with linear coefficient k_ω, i.e.:

$$f_o = k_\omega \cdot \omega \, . \tag{11}$$

Factor k_ω is linear coefficient, whose dimension depends on the bearing structure. This coefficient is ratio between oscillating frequency and the balanced rigid rotor based on rolling bearings rotary frequency and it shows how many times the oscillating frequency is greater than its rotary frequency.

The factor shows what kind of impact the rolling bearing construction has on balanced rigid rotor oscillating frequency in the rolling bearing. According to equation (10) this factor depends on the bearing's rolling bodies number (z), ratio of diameter of rolling bodies and cage diameter (d_b/d_c) and bearing working contact angle (α).

By multiplying factor k_ω with rotary speed of shaft ω, it is not difficult to get rolling bearing balanced rigid rotor oscillating frequency. Because of that, great practical importance is to have diagrams for direct reading out of the value of factor k_ω, in dependence on the bearing construction.

Fig. 2 represents a diagram of dependence of factor k_ω on the bearing construction for rolling bearing with radial contact. With that diagram we could get value of factor k_ω in dependence on ratio d_b/d_c, for various values of rolling bodies number (z).

From the analysis shown in Fig. 2 we can conclude that factor k_ω linearly decreases with increasing ratio d_b/d_c while its value increases with the bearing rolling bodies' increase. Also, we see that, in depending on the bearing construction (number of rolling bodies and ratio d_b/d_c), the rotor oscillating frequency could be up to ten times greater than the rotary frequency.

4. IMPACT OF BEARING CONSTRUCTION AT BALANCED RIGID ROTOR AT ROLLING BEARINGS OSCILLATING FREQUENCY - THEORETICAL ANALYSIS

From expression (11) we can see that the rolling bearing rigid rotor oscillating frequency has linear dependence on the rotor rotary speed with linear coefficient k_ω. Value of linear coefficient k_ω depends on the bearing construction, with relation to bearing rolling bodies total number (z), then on ratio of diameter of rolling bodies and diameter of the cage (d_b/d_c) and working contact angle (α). For as bearing construction impact on oscillating frequency is shown only through linear coefficient k_ω, this means that the rolling bearing structural parameters frequency analysis is best to be done by analysis of changes of this coefficient.

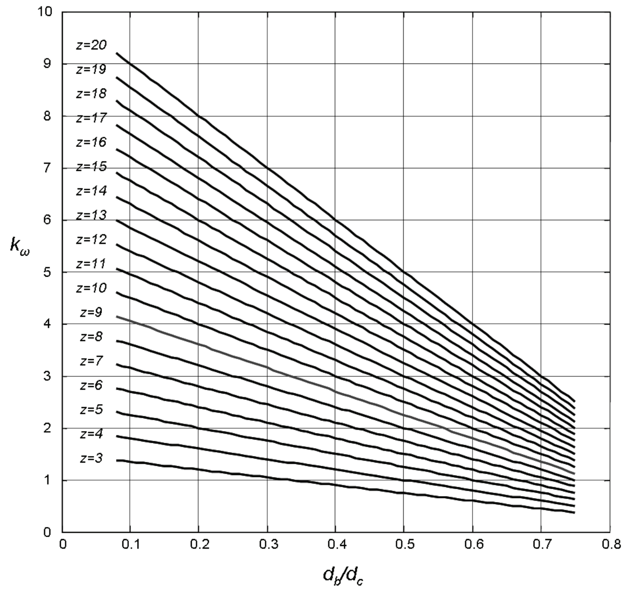

Fig 2 Factor k_ω Dependence on Ratio d_b/d_c and Rolling Bodies Number (z) for Single Breasted Round Bearing with Radial Contact

4.1. Impact of rolling bodies number on balanced rigid rotor based at rolling bearings oscillating frequency - theoretical analysis

To do a correct analysis of impact of rolling bodies number on balanced rigid rotor at rolling bearing oscillating frequency, it is necessary to exclude the impact of all other parameters on factor k_ω value. For that object, the scope of analysis rolling bearing with radial contact is taken which has working contact angle α equal to zero, i.e. $\cos\alpha = 1$. It means that during analysis the value of linear coefficient k_ω should vary in dependence on ratio d_b/d_c.

On the basis of these assumptions, the expression for linear coefficient k_ω value is getting on the following shape:

$$k_\omega = \frac{z}{2}\cdot\left(1-\frac{d_b}{d_c}\right) \tag{12}$$

Further, if we want to take into consideration angular speed impact on the rotor oscillating frequency, it is enough only to multiply the value from the expression above with angular speed of rotor – ω.

In Fig. 3 dependence of coefficient k_ω is given, and at the same time dependence of balanced rigid rotor oscillating frequency on the number of bearing with radial contact rolling bodies. Dependence is given for ratio of rolling bodies diameter and intermediate diameter of the cage which are within limits of d_b/d_c=0.05-0.4. Limits of ratio d_b/d_c are specified on the basis of recommended formulae for rolling bodies diameter estimation and intermediate diameter of the cage according to standard ISO 8826-2.

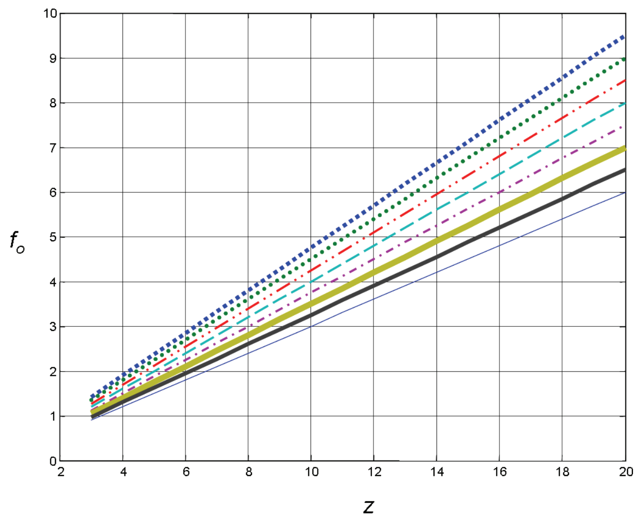

Fig 3 Dependence of Factor k_ω on the Number of Rolling Bodies for Various Values of Diameter of Rolling Bodies and Intermediate Diameter of the Cage Ratio (d_b/d_c)

On the basis of Fig. 3 we can see that with increasing of rolling bodies number in bearing, there is linear increasing of linear coefficient k_ω, which means that rotor oscillating frequency also increases, and that the lines with lower values of ratio of rolling bodies diameter and intermediate diameter of the cage have a steeper slope (d_b/d_c), i.e. increasing of rolling bodies number, with lower values of d_b/d_c ratio has a greater impact on the rolling bearing balanced rigid rotor oscillating frequency.

4.2. Impact of ratio of diameter of rolling bodies and intermediate diameter of the cage (d_b/d_c) on rolling bearing balanced rigid rotor oscillating frequency - theoretical analysis

Impact of ratio of diameter of rolling bodies and intermediate diameter of the cage (d_b/d_c) on rolling bearing balanced rigid rotor oscillating frequency is shown in Figs. 1, 4 and 5. Fig. 1 gives dependence of coefficient k_ω on ratio d_b/d_c values for various values of rolling bodies number. Fig. 4 gives dependence of k_ω/z ratio on value of d_b/d_c ratio for various values of contact angle α. Fig. 5 gives three dimensional dependence of k_ω/z ratio on contact angle α values and d_b/d_c ratio.

It is obvious from these diagrams that the rotor oscillating frequency has a linear decreasing trend with increasing of ratio of rolling bodies diameter and intermediate diameter of the cage (d_b/d_c). This decreasing trend is more sought for if the bearing has more rolling bodies, while change of contact angle value has lesser impact on that dependence. Also, by analyzing the displayed diagrams we can see that the lines of dependence are most indented on the diagram in Fig. 2, while on the other diagrams they are closer. This fact tells us that the total number of rolling bodies z has the greatest impact on oscillating frequency of all the bearing structure parameters.

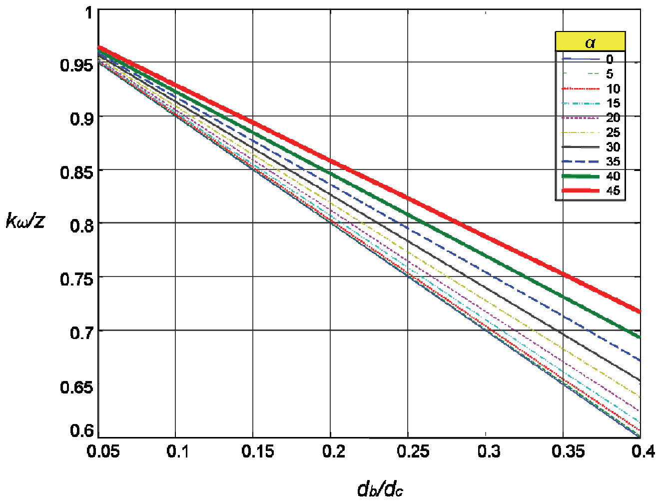

Fig. 4 Dependence of k_ω/z Ratio on the Ratio of Rolling Bodies Diameter and Intermediate Diameter of the Cage (d_b/d_c) for Various Values of Contact Angle α

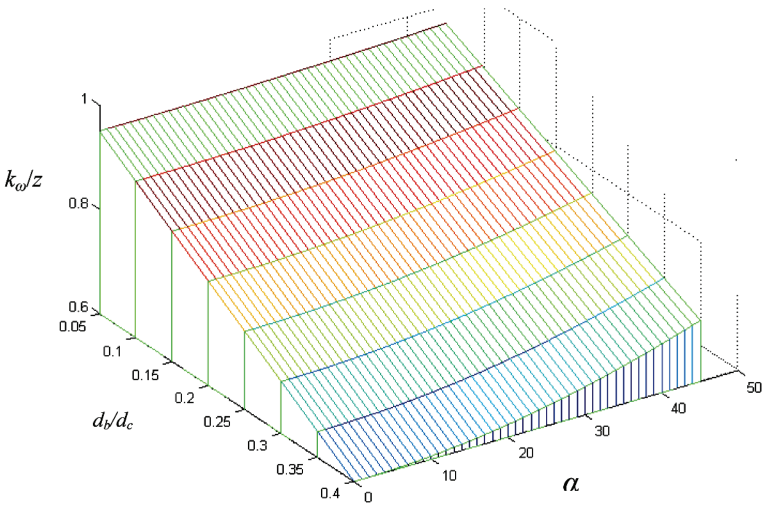

Fig. 5 Three Dimensional Dependence of k_ω/z Ratio on the Ratio of Rolling Bodies Diameter and Intermediate Diameter of the Cage (d_b/d_c) and Contact Angle α

4.3. Impact of contact angle *α* value on rolling bearing balanced rigid rotor oscillating frequency - theoretical analysis

Contact angle α has great importance for constructive performance and working characteristics of the rolling bearing. This is an angle between radial direction and direction defining the points at which the rolling bodies touch with the bearing rings. Impact of contact angle α on rolling bearing balanced rigid rotor oscillating frequency is shown in Figs. 5 and 6. It is obvious in Fig. 6 that the contact angle value increases thus causing the increase of the rolling bearing rotor oscillating frequency, and that this increasing is more sought for with higher values of d_b/d_c ratio. If we have smaller values of d_b/d_c ratio, the contact angle has almost no impact on the rolling bearing balanced rigid rotor oscillating frequency.

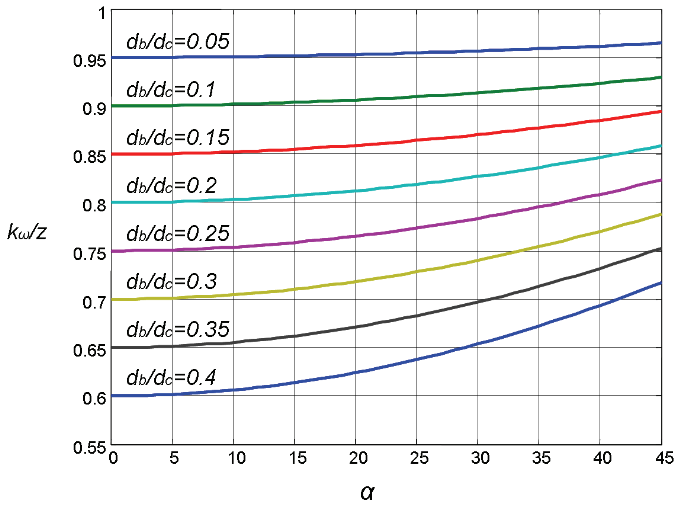

Fig. 6 Dependence of k_ω/z Ratio on Contact Angle α for Various Values of Rolling Bodies Diameter and Intermediate Diameter of the Cage (d_b/d_c)

5. Conclusion

On the basis of the above-given facts, the following can be concluded:

1. Balanced rigid rotor based on rolling bearings cross section center continuously oscillate during the bearing's operation. Frequency of these oscillations is equal to specific vibration frequency of the external bearing ring, i.e. to frequency of transition of the rolling bodies across some fixed point at the external bearing ring.
2. Specific frequency of external bearing ring has linear dependence on rotor turnover speed with linear coefficient k_ω which is dependent on bearing construction (rolling bodies number z, rolling bodies diameter d_b, cage diameter d_c and contact angle α).
3. If we multiply linear coefficient k_ω with angular speed of rotor rotation, we directly get rolling bearing balanced rigid rotor oscillating frequency; diagrams which can

help us to directly determine coefficient k_ω depending on bearing construction parameters can be very helpful.

4. According to these diagrams, the total number of bearing rolling bodies has the most impact of all constructive bearing parameters, while impact of working contact angle value on oscillating frequency has practical significance only at higher values of rolling bodies diameter and intermediate diameter of the cage ratio (d_b/d_c).

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UTICAJ KONSTRUKCIONIH PARAMETARA KOTRLJAJNOG LEŽAJA NA FREKVENCIJU OSCILOVANJA BALANSIRANOG KRUTOG ROTORA

Radoslav Tomović, Vojislav Miltenović

Kotrljajni ležajevi su jedni od najznačajnijih generatora buke i vibracija u mašinskim konstrukcijama. Zbog toga je od velikog značaja poznavanje uticaja konstrukcije kotrljajnog ležaja na osnovne parametre koji određuju nivo buke i vibracija generisan u mašinama. U ovom radu je predstavljen matematički model dinamičkog ponašanja krutog rotora u kotrljajnom ležaju i data je teoretska analiza uticaja konstrukcije kotrljajnog ležaja na frekvenciju oscilovanja balansiranog krutog rotora u kotrljajnom ležaju.

Ključne reči: *Kotrljajni ležaj, vibracije, konstrukcija, specifična frekvencija*