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MATHEMATICAL AND SIMULINK MODEL OF THE PNEUMATIC SYSTEM WITH BRIDGING OF THE DUAL ACTION CYLINDER CHAMBERS

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Abstract. *This paper describes a detailed mathematical and simulink model of the pneumatic system with interconnected chambers developed to meet the needs for designing energy-efficient controls. The effects of nonlinear flow, air- compressibility under pressure in the cylinder chambers, time delay and the like are most carefully taken into consideration.*

Key words: *Dynamic Model, Pneumatic System, Bridging, Energy Efficiency*

1. INTRODUCTION

Pneumatic cylinders can offer better alternatives to electrical or hydraulic actuators for great many applications. The use of pneumatic actuators gives good system qualities at low cost [1][2]. They are likewise suitable since they are safer and easier to work with just as they suit clean environments. The energy used in the pneumatic system is that of compressed air which is obtained in special devices called compressors. Mostly, the costs of the electric power used for feeding the compressors in the production of compressed air amount to around 20% of the overall costs of electrical power for some factory. For this and many other reasons it is very important to carefully handle the use of compressed air [3].

The aim of the paper is to design a mathematical model of the pneumatic system which provides for one way of reducing the spending of compressed air. More exactly, the goal is to make the mathematical model itself a good basis for designing energy-efficient controls.

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2. Development of the Mathematical Model

For analysis and design of an automatic control system it is of utmost importance to previously create a dynamic mathematical model of the control object in question. The formation of the mathematical model of a system is probably the most important phase in designing any form of control. A good mathematical model provides the designer with all the needed information about the dynamics of the system in question which, in its turn, ensures ensuing design as well as the application of an adequate, that is, optimal control algorithm and then the control system itself. The most important thing is that the designed control itself can be checked on the model itself before being applied to the real system.

The mathematical model of the pneumatic system with bridging is a typically nonlinear model [2] due to the compressibility of the working medium, the work medium flow and various resistances and attenuation.

For the mathematical model to represent as best as possible a real physical one, it must comprise all the models of the system components. Accordingly, the model of the entire pneumatic system with bridging comprises:

- Stick-slip friction model,
- Nonlinear model of control valve,
- Nonlinear model of valve for bridging,
- Nonlinear model of cylinder dynamics,
- Nonlinear model of charging and discharging of the cylinder chambers with the working medium, and,
- Model of the working medium flow through the tubing.

A schematic representation of this system with all the necessary parameters is given in Fig. 1.

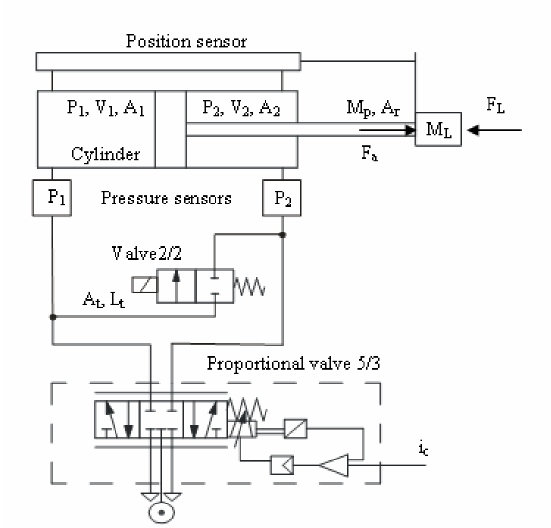

Fig. 1 View of the Pneumatic System with Bridging

By interconnecting the cylinder chambers, the possibility arises for using compressed air from the used cylinder chamber; in this way, the overall use of air is reduced. The pneumatic system with bridging [3] consists of:

- Pneumatic double-action cylinder,
- control manifold for charging,
- valve for bridging
- tubing, and giver (sensor) of stroke and pressure.

2.1. Dynamics of the Pneumatic System with Bridging of the Double-action Cylinder Chamber

The dynamic equation can be best represented by the motion equation (1):

$$(M_L + M_P)\ddot{x} + \beta\dot{x} + F_f + F_L = P_1 A_1 - P_2 A_2 - P_a A_r \tag{1}$$

where: M_L-the mass of external parts connected to the cylinder rod, M_P- the piston and rod assembly mass, x- the rod position, β- the viscous friction coefficient, F_f- the Coulomb friction force, F_L-the external force, P_1 and P_2- the absolute pressures in the cylinder's chambers, P_a- the absolute ambient pressure, A_1 and A_2- the piston effective areas, A_r-the rod cross sectional area.

2.2. Dynamics of the Pressure Change in the Pneumatic Cylinder Chambers

The equation describing the model of the pressure change in the pneumatic cylinder chambers is the following,

$$\dot{P}_i = \frac{RT}{V_{0i} + A_i\left(\frac{1}{2}L \pm x\right)}(\alpha_{ul}\dot{m}_{ul} - \alpha_{iz}\dot{m}_{iz}) \mp \alpha \frac{PA_i}{V_{0i} + A_i\left(\frac{1}{2}L \pm x\right)}\dot{x} \tag{2}$$

where: R-the ideal gas constant, T- temperatures, $\dot{m}_{ul}$ and $\dot{m}_{iz}$ - the mass flows entering and leaving the chamber, respectively, α, α_{ul}, α_{iz}- the coefficients taking the values between 1 and κ, depending on the heat transfer process, L- the piston stroke, i=1, 2 the cylinder chambers index, V_{0i}- the inactive cylinder volumes, A_i- the piston effective areas. In this form the pressure equation change accounts for the different heat transfer processes of the charging and discharging processes, air compression or expansion during the piston motion, the difference in effective areas on the opposite sides of the piston and the inactive volumes at the ends of stroke. The first term in equation (2) represents the effect on pressure of the gas flow in or out of the chamber while the second term of the equation accounts for the effect of piston motion [4][5].

2.3. Mathematical Model of Mass Flow of Compressed Air through the Tubes

The tubes connecting the control manifold for charging and the pneumatic cylinder have two effects on the system response. The first is that the pressure drop along the tube induces a decrease in the air flow at the stationary state of the valve. The second is that the signal will be delayed during its travel through the tube. These effects of the pressure drop and time delay have been studied by many authors [1]. Most of them have assumed a fully laminar flow through the tube.

The mass flow of compressed air at the tube outlet is:

$$\dot{m}_t(L_t,t)=\begin{cases} 0 & t<L_t/c, \\ e^{-\frac{R_t R T L_t}{2Pc}}\, h(t-\frac{L_t}{c}) & t>L_t/c. \end{cases} \tag{3}$$

where : c- the sound speed, R_t the flow resistance. L_t- the tube length.

Equation (3) describes in a simple form the mass flow at the tube outlet for any inlet flow. It shows that the flow at the outlet of the tube is attenuated due to attenuation during its travel through the tube; that is why it will be delayed for time L_t / c, which represents the time necessary for the input wave to travel through the entire length of the tube.

Flow resistance R_t can be obtained from the expression for the pressure drop along the tube:

$$\Delta P = f\frac{L_t}{D}\frac{\rho u^2}{2} = R_t u L_t \tag{4}$$

where f - the attenuation factor, D- the inner diameter of the tube.

For fully developed laminar flow $f = 64 / Re$, where Re – the Reynolds number, resistance R_t is:

$$R_t = \frac{32\mu}{D^2} \tag{5}$$

where μ -the dynamic viscosity of air.

For wholly turbulent flow $f = 0{,}316/Re^{1/4}$, resistance R_t is:

$$R_t = 0{,}158\frac{\mu}{D^2}\mathrm{Re}^{-1/4} \tag{6}$$

2.4. Mathematical Model of the Control Proportional Valve 5/3 with Electromagnetic Activation

The control manifold for charging is a critical component of each pneumatic system. It is the command device and should be able to provide fast and precisely controlled air flows in and out of the pneumatic cylinder chambers. There are many available designs for pneumatic valves, which differ by geometry of the active orifice, type of flow regulation, number of positions and ports, ways of activation, etc. For the case of such pneumatic systems in which the path control is important, proportional valves with electromagnetic activation are used as control manifolds. Such valves offer several advantages [4]:

- quasi-linear flow characteristic,
- small time constant,
- small internal leakage,
- ability to adjust both chambers pressure using one control signal,
- very small hysteresis, and,
- small internal friction.

The proportional valve used in this control of the double action pneumatic cylinder with the purpose to reduce expenditure of compressed air is of type 5/3, which means that it has two working, two reciprocal and one tube for charging as well as three positions. In the normal position, when there is no activation, the valve keeps all the tubes closed.

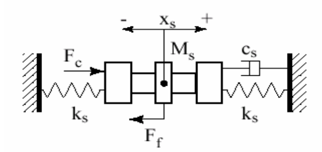

Fig. 2 Scheme of Proportional Valve 5/3

Fig. 2 shows a simplified scheme of the proportional valve 5/3 at the equilibrium position. If the valve piston is displaced to one or the other side, it would cause the compressed air flow inward, into one chamber ($\dot{m}_{ul} > 0,\ \dot{m}_{iz} = 0$) while from the other chamber the air will flow outward ($\dot{m}_{ul} = 0,\ \dot{m}_{iz} > 0$).

A fully developed mathematical model of the valve suitable for successful control design consists of valve dynamics as well as the model of the mass flow through the valve's orifice.

Analyzing Fig. 2, the equation of motion can be written as:

$$M_s \ddot{x}_x = -c_s \dot{x}_s - F_f + k_s (x_{s0} - x_s) - k_s (x_{s0} - x_s) + F_i \tag{7}$$

where x_s-the valve piston displacement, x_{s0}-is the spring compression at the equilibrium, M_s-the piston mass, c_s- the viscous friction coefficient, F_f – the Coulomb friction force, k_s – the spring constant, F_i – the electromagnetic force produced by the coil.

As for the model of the mass flow through the valve's orifice, care must be taken that the pressure drop across the orifice is quite large so that the mass flow must be treated as compressible and turbulent. If the upstream to downstream pressure ratio is less than critical value (p_{cr}), the flow will attain acoustic velocity, that is, it will depend linearly on the upstream pressure. If this ratio is smaller than p_{cr}, the mass flow will depend nonlinearly on both pressures as can be seen in equation (8),:

$$\dot{m}_v = \begin{cases} c_f A_v C_1 \dfrac{P_g}{\sqrt{T}} \quad za \ \dfrac{P_d}{P_g} \le p_{cr}, \\ c_f A_v C_2 \dfrac{P_g}{\sqrt{T}} \left(\dfrac{P_d}{P_g}\right)^{1/\kappa} \sqrt{1 - \left(\dfrac{P_d}{P_g}\right)^{\frac{\kappa-1}{\kappa}}} \quad za \ \dfrac{P_d}{P_g} > p_{cr}, \end{cases} \tag{8}$$

where $\dot{m}_v$ – the mass flow through proportional valve orifice, c_f – nondimensional discharge coefficient, P_g and P_d – upstream and downstream pressure.

Parameters C_1, C_2 and p_{cr} are constants depending on a given fluid, in this case, air (κ=1.4),

$$C_1 = \sqrt{\frac{\kappa}{R}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}} = 0.040418,$$
$$C_2 = \sqrt{\frac{2\kappa}{R(\kappa-1)}} = 0.156174, \tag{9}$$
$$p_{cr} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} = 0.528.$$

The meaning of the upstream and downstream pressure in equation (8) is different for the charging and discharging process of the cylinder chambers. For the chamber's charging process, the charging pressure is regarded as the upstream pressure while the pressure in the cylinder chambers is the downstream one. For the discharging process, the pressure in the cylinder chambers is the upstream, while the ambient pressure is the downstream one.

In the equation of the mass flow across the orifice on the valve (8) there is parameter A_v, which represents active orifice area. This parameter is calculated separately both for the input and for the exhaust path through the valve (10) and (11):

$$A_{vul} = \begin{cases} 0 & x_s \le p_w - R_h; \\ n_h[2R_h^2 arctg\left(\sqrt{\dfrac{R_h - p_w + x_s}{R_h + p_w - x_s}}\right) - (p_w - x_s)\sqrt{R_h^2 - (p_w - x_s)^2}\,] & p_w - R_h < x_s < p_w + R_h; \\ \pi n_h R_h^2 & x_s \ge p_w + R_h; \end{cases} \tag{10}$$

$$A_{viz} = \begin{cases} \pi n_h R_h^2 & x_s \le -p_w - R_h; \\ n_h[2R_h^2 arctg\left(\sqrt{\dfrac{R_h - p_w + |x_s|}{R_h + p_w - |x_s|}}\right) - (p_w - |x_s|)\sqrt{R_h^2 - (p_w - |x_s|)^2}\,] & -p_w - R_h < x_s < -p_w + R_h; \\ 0 & x_s \ge -p_w + R_h. \end{cases} \tag{11}$$

where: R_h-the diameter of orifice on the tube, nh-the number of orifices, pw-the constant.

2.5. Mathematical Model of Valve for Bridging

The bridging valve is a very important component in the system which aims at saving compressed air expenditure. This is a component, valve 2/2 with electromagnetic activation which must provide fast and precise control of air flow in the pneumatic cylinder chambers. Such valves offer several advantages:

- simple design,
- simple to build-in,
- flexible ports design,
- small internal leakage,
- fast response time up to 4 ms, and,
- small internal friction.

The valve which is used to meet the needs of bridging the cylinder chambers is of type 2/2 with electromagnetic activation and normally closed.

The valve itself works like ON/OFF valve. When an activating signal is brought to it, it lets off a certain amount of compressed air, that is, when there is no signal, the valve is in an inactive position.

When the response velocity of the valve is great, it is not necessary to consider the dynamics of mechanical and electro parts but, instead, only the dynamics of the mass compressed air flow across the valve's orifice.

The equation describing the mass compressed air flow is identical to equations (8) and (9). The only difference is in parameter A_v which is in this case constant and equal to active area of the orifice on the given valve.

2.6. Mathematical Model of Friction

There are many models of friction, but in order to meet the needs of the pneumatic system described in this paper, not all of them but only quite frequent stick-slip model is analyzed.

The model is so conceived as to distinguish two types of friction, namely:

- static, and,
- sliding.

As for static friction in the model there is an area in which, when the piston displacement velocity is approximately equal to zero, the friction force is quite large and it opposes the piston motion. This opposition lasts until F_{stick} static friction force does not transform by the pressure force in the cylinder chambers. If the velocity of the cylinder piston displacement is different from zero, sliding friction force F_{slip} sets up whose values is less than the static force.

Stick-slip model of friction can be represented in a simplified way as in Fig. 3,

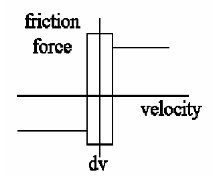

Fig. 3 Simplified Form of Stick-slip Model

where:ΔV - the area in which the cylinder piston displacement velocities are approximately zero.

Fig. 3 presents a simplified model of friction that makes it clear that, if the cylinder piston is to be moved, then so-called discontinuous velocity ΔV has to be attained. Once this velocity is attained, the friction force decreases and reaches value F_{slip}.

3. Development of the Simulink Model

For analysis and design of an automatic control system it is of utmost importance to previously create a dynamic mathematical model of the control object in question. The formation of the mathematical model of a system is probably the most important phase in designing any form of control. The complete mathematical model of the pneumatic system with bridging of double action cylinder chambers consists of the valve dynamics equation, two equations for chamber pressure time derivatives and the piston-load equation of motion. The most important thing is that the designed control itself can be checked on the model itself before being applied to the real system. The best way for checking the designed control is usage of simulink model in mathlab environment.

Fig. 4 shows the simulink model of pneumatic system with bridging of double action cylinder chambers. Characteristic dynamics of individual part of pneumatic system are presented by subsystems.

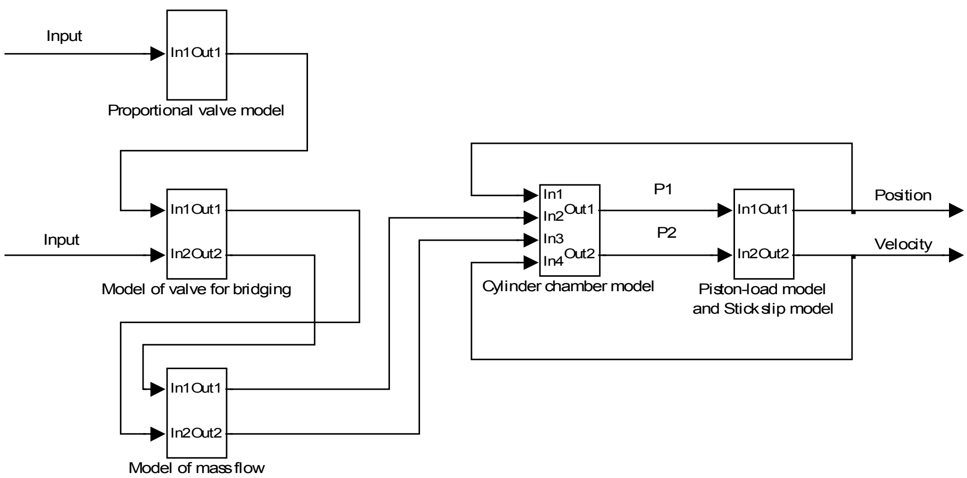

Fig. 4 Simulink Model of the Pneumatic System with Bridging of Double Action Cylinder Chambers

4. CONCLUSION

In this paper a mathematical and simulink model of the pneumatic system with bridging of double action cylinder chambers is described. As a control manifold, the proportional valve 5/3 is used while valve 2/2 is used for bridging. The model itself gives a good basis for further design of control aiming at further reduction of compressed air expenditure.

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MATEMATIČKI I SIMULINK MODEL PNEUMATSKOG SISTEMA SA PREMOŠĆAVANJEM KOMORA PNEUMATSKOG CILINDRA DVOSTRANOG DEJSTVA

Vladislav Blagojević, Miodrag Stojiljković

Ovaj rad detaljno opisuje matematički i simulink model pneumatskog sistema sa premošćavanjem komora razvijen za potrebe projektovanja energetski efikasnih upravljanja. Efekti nelinearnog protoka, stišljivosti vazduha pod pritiskom u komorama cilindra, i kašnjenja signala pažljivo su razmatrani.

Ključne reči: *Dinamički model, Pneumatski sistem, Premošćavanje, Energetska efikasnost*