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MATHEMATICAL MODELING OF CHANGING OF DYNAMIC VISCOSITY, AS A FUNCTION OF TEMPERATURE AND PRESSURE, OF MINERAL OILS FOR HYDRAULIC SYSTEMS

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Abstract. *Viscosity is the most important characteristic of hydraulic fluid. The value and changing of viscosity have essential significance for all events in the hydraulic system. The viscosity of hydraulic fluid is changed with temperature, pressure and rate of shear. However, in analyzing the hydraulic system operation it is usual to use only temperature dependence of viscosity, while influence of pressure and shear rate are neglected.*
In this paper a mathematical model of changing the dynamic viscosity of mineral hydraulic oil (type HM and HV) with changing of temperature and pressure is given. It is proved that pressure dependence of viscosity cannot be neglected in an analysis of hydraulic system.

Key Words: *Hydraulic Oil, Viscosity, Hydraulic System*

1. INTRODUCTION

The most important physical property of hydraulic oil, which has fundamental importance for studying of hydraulic components and systems efficiency, is viscosity.

The optimal viscosity of hydraulic oil for a hydraulic system is a compromise between the lubrication requirements and the mechanical and volumetric efficiency [1], [2].

The usage of thinner hydraulic oils, as long as the fluid flow is laminar, will achieve higher hydraulic-mechanics efficiency ratings. However, the usage of thinner hydraulic oils will incur increased leakage within the hydraulic components (past working clearance in pumps, valves, etc.), thus reducing the volumetric efficiency of the system. In addition, the lubricating properties of hydraulic oil may become marginal as the viscosity is reduced to an excessively low value.

Conversely, the application of a more viscous hydraulic oil will increase the volumetric efficiency of the given hydraulic system, but at the same time energy losses will in-

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crease due to higher hydraulic oil friction (in piping, filters, pumps and valves). The suction ability of pumps is one of several limiting factors which define the maximum value of viscosity tolerated by the hydraulic system. If friction losses exceed the suction ability of the pump, the hydraulic oil will not completely fill the suction inlet (in which vacuum will appear) and this will result in cavitation.

Another important limiting factor, for allowed increasing of hydraulic oil viscosity in hydraulic system, is the loss of lubricating function. It should be emphasized that reasons for the loss of lubricating function at low and high values of viscosity are completely different.

In addition to the three previously mentioned requirements that define hydraulic oil viscosity (suction ability of pumps, lubricating and losses of energy) in the hydraulic system, the application of servo and proportional hydraulic systems introduce the fourth requirement: to keep value of viscosity in constant amount [1].

From this short review of problems and requirements, which appear in the hydraulic system, it can be seen that viscosity of hydraulic oil is a very significant construction characteristic.

The changing of viscosity with temperature and pressure is important not only for the sake of theoretical analysis but also regarding the practical application to the real hydraulic systems in which temperature and pressure are changing continuously.

2. Temperature Dependence of Viscosity

The viscosity of hydraulic and lubricating oil is extremely sensitive to the operating temperature. With increasing temperature the viscosity of oils falls rapidly.

The oil viscosity at a specific temperature can be either calculated from the viscosity-temperature equations or obtained from the viscosity-temperature ASTM chart.

Mineral base oils for hydraulic fluids are normally composed of complex hydrocarbon molecules. According to dominant presence specific hydrocarbon in crude oil, mineral oils are divided into: paraffinic, naphthenic and mixed oils. Hydraulic oils, almost on the whole, are based on highly refined paraffinic oils. Naphthenic oils are very rarely used as hydraulic oils because of their reduced availability [3].

As paraffinic mineral oils are the most widely spread hydraulic oils (about 90% used hydraulic fluids), the following analysis of influence of temperature and pressure on dynamic viscosity is applied to this type of oil.

2.1 Viscosity-Temperature Equations

There are several viscosity-temperature equations. Some of them are purely empirical whereas others are derived from theoretical models. The most commonly used equations are given in Table 2.1 [4].

Among them the most accurate is the Vogel equation. Three viscosity measurements at different temperatures for specific oil are needed in order to determine the three constants in this equation.

Apart from being very accurate the Vogel equations is useful in numerical analysis [4].

Table 2.1 Viscosity-temperature equations

Name	Equation	Comments
Reynolds	$\mu_0 = be^{-aT_A}$	Early equation; accurate only for a very limited temperature range
Slotte	$\mu_0 = \dfrac{a}{(b+T_A)^c}$	Reasonable; useful in numerical analysis
Walther	$(\nu_0 + a) = bd^{\frac{1}{T_A^c}}$	Forms the basis of the ASTM viscosity-temperature chart
Vogel	$\mu_0 = ae^{\frac{b}{(T_A - c)}}$	Most accurate; very useful in engineering calculations

where:

a, b, c, d – are constants;

ν_0 – is the kinematic viscosity at the atmospheric pressure [m^2/s];

μ_0 – is the dynamic viscosity at the atmospheric pressure [Pas];

T_A – is the absolute temperature [K].

In this paper, for calculation of numerical values of dynamic viscosity, at specific temperature, Vogel equation is used.

Measured experimental data of the kinematic viscosity for mineral hydraulic oil, from ISO VG 32 to ISO VG 68, at three different temperatures, and density at the temperature 15^{o}C, are given in Table 2.2 [5].

Table 2.2 Kinematic viscosity of testing hydraulic oil, at the three different temperatures and atmospheric pressure, and density of oil at the temperature of 15^{o}C

	HM 32	HM 46	HM 68	HVL 46
ρ(15^{o}C) [g/cm^3]	0.879	0,883	0,887	0,879
ν_0(20,5^{o}C) [mm^2/s]	80,13	146,92	217,37	123,88
ν_0(40^{o}C) [mm^2/s]	30,32	48,5	71,53	47,26
ν_0(100^{o}C) [mm^2/s]	5,24	6,89	8,8	8,24
IV	101	96	94	149

For known values of kinematic viscosity and density, at specific temperature, the value of dynamic viscosity is given by the equation

$$\mu_0 = \nu_0 \rho \,. \tag{2.1}$$

Density of oil, at specific temperature and atmospheric pressure, can be calculated by using experimentally-measured value of density at the temperature of 15^{o}C and volume-temperature expansion coefficient, α_p, for the same temperature

$$\rho = \rho_{15} - \rho_{15}\alpha_{p15}(T-15) = \rho_{15}[1-0.0007(T-15)] \,. \tag{2.2}$$

Table 2.3 Dynamic viscosity of testing hydraulic oil, at the three different temperatures and atmospheric pressure, calculated by using equation (2.1)

	HM 32	HM 46	HM 68	HVL 46
$\mu_{0(20.5^o C)} \cdot 10^2$ [Pas]	7,016	12,57	19,28	10,8469
$\mu_{0(40^o C)} \cdot 10^2$ [Pas]	2,62	4,21	6,234	4,0814
$\mu_{0(100^o C)} \cdot 10^2$ [Pas]	0,433	0,572	0,734	0,6812

Based on data from Table 2.3, constants *a*, *b*, *c*, of testing hydraulic oil, from Vogel equation, are calculated and given in Table 2.4.

Table 2.4 Constants from Vogel equation for testing hydraulic oil [5]

	HM 32	HM 46	HM 68	HVL 46
a	0,0000736317	0,0000633361	0,0000389689	0.000116198
b	797.7122	879.7742	1083.913	799.7249
c	177.3562	177.7865	166.2304	176.7128

3. Pressure dependence of viscosity

Viscosity of oil increases with growth of pressure. Chemical composition greatly influences the viscosity-pressure characteristic of a hydraulic fluid.

The best known equation, which describes viscosity-pressure behavior of hydraulic fluids, is Barus equation [3]:

$$\mu = \mu_0 e^{\alpha p} , \tag{3.1}$$

where:

μ – is the dynamic viscosity at the pressure 'p' [Pas];

μ_0 – is the dynamic viscosity at the atmospheric pressure [Pas];

α - pressure-viscosity coefficient that depends on pressure and temperature [1/Pas].

To adopt experimental data by a mathematical model, the so-called "Modulus Equation" was used. "Modulus Equation" is based on the Barus equation. The model comprises pressure, p [bar], and temperature, T [^{o}C], dependence of the dynamic viscosity [6].

$$\mu(p,T) = \mu_0 e^{\left[\frac{p}{a_1 + a_2 T + (b_1 + b_2 T)p}\right]} . \tag{3.2}$$

Dependence pressure-viscosity coefficient, α, of pressure and temperature is given by equation

$$\alpha(p,T) = \frac{\ln\mu - \ln\mu_0}{p - p_a} = \frac{1}{a_1 + a_2 T + (b_1 + b_2 T)p} . \tag{3.3}$$

Parameters a_1, a_2, b_1, b_2 represent the oil behavior and have to be calculated from experimental data. In accordance with the data given by the hydraulic oil producers ([7]), by using method of identification unknown parameters of mathematical model, constants from equation (3.3) are calculated.

Table 3.1 Parameter values for pressure-viscosity coefficient, α [5]

	a_1 [bar]	a_2 [bar/°C]	b_1	b_2 [1/°C]
Hydraulic oil of paraffinic base structure	334	3.2557	0.026266	0.000315

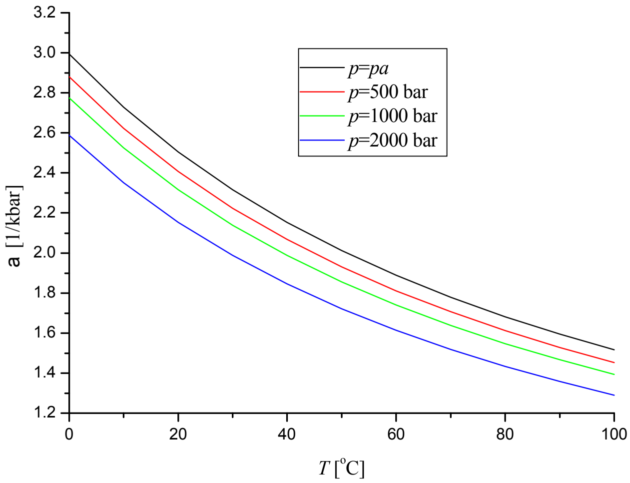

Figure 3.1 Pressure-viscosity coefficient hydraulic oil of paraffinic base structure

Example:

For the pressure of 350 bar and temperatures of 20.5 and 40°C, values of pressure-viscosity coefficient, are calculated by using of formula (3.3):

$$\alpha(p = 350\,\text{bar}, T = 20.5^{\text{o}}\,\text{C}) = 0.002426\,\text{bar}^{-1} \text{ and}$$
$$\alpha(p = 350\,\text{bar}, T = 40^{\text{o}}\,\text{C}) = 0.0020928\,\text{bar}^{-1},$$

and values of dynamic viscosity, are calculated by using of formula (3.2):

$$\mu(p = 350\,\text{bar}, T = 20.5^{\text{o}}\,\text{C}) = 2.3375 \cdot \mu_0 \text{ and}$$
$$\mu(p = 350\,\text{bar}, T = 40^{\text{o}}\,\text{C}) = 2.08 \cdot \mu_0,$$

Value of dynamic viscosity, at pressure of 350 bar and temperature of 20.5°C (40°C), is for 2.3375 (2.08) times higher than the value of dynamic viscosity at atmospheric pressure and at the same temperature.

4. Dynamic viscosity as a function of temperature and pressure for mineral hydraulic oils

After replacing μ_0 in equation (3.1), by the Vogel equation to describe temperature dependence at atmospheric pressure,

$$\mu(T) = \mu_0 = ae^{\left[\frac{b}{(T+273.15)-c}\right]}, \tag{4.1}$$

an equation is obtained with seven unknown parameters a, b, c, a_1, a_2, b_1, b_2,

$$\mu(p,T) = ae^{\left[\frac{b}{(T+273.15)-c}\right]}e^{\left[\frac{p}{a_1+a_2T+(b_1+b_2T)p}\right]}. \tag{4.2}$$

The pressures, in hydraulic systems, are usually smaller than 400 bars, and it results in $a_1 + a_2T >> (b_1 + b_2T)p$. Thus, equation (4.2) can be simplified and the number of unknown parameters can be reduced to five

$$\mu(p,T) = ae^{\left[\frac{b}{(T+273.15)-c}\right]}e^{\left[\frac{p}{a_1+a_2T}\right]}. \tag{4.3}$$

Therefore, for mineral hydraulic oils of paraffinic base structure, pressure viscosity coefficient, α, can be calculated by using the formula

$$\alpha(T) = \frac{1}{a_1 + a_2T} = \frac{1}{334 + 3.2557 \cdot T} \quad [\text{bar}^{-1}] \ . \tag{4.4}$$

The values of dynamic viscosity are calculated by using the data from Tables 2.4 and 3.1, and charts are given for mineral oil HM 46 and HM 32.

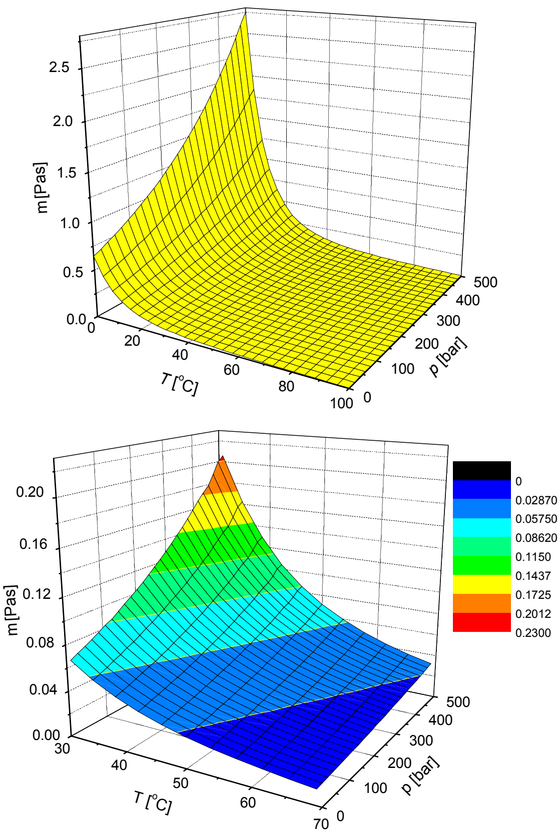

Figure 4.1 Changing of dynamic viscosity of mineral hydraulic oil HM 46 as a function of temperature and pressure

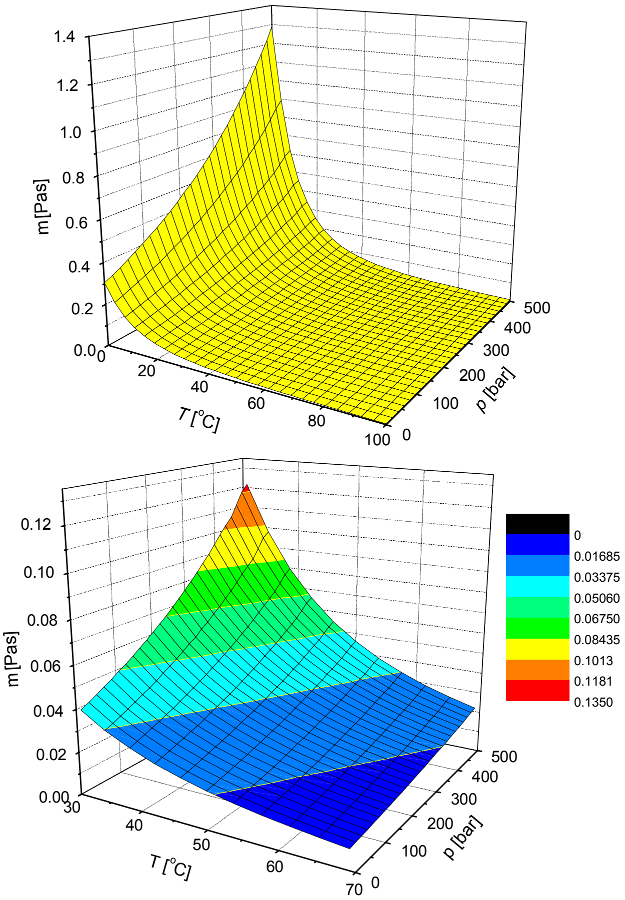

Figure 4.2 Changing of dynamic viscosity of mineral hydraulic oil HM 32
as a function of temperature and pressure

5. Conclusions

The demand for highly efficient hydraulic systems is permanently increasing. Because of that, the need for accurately mathematical modeling of the fluid behavior is also increasing.

In this paper the mathematical model for calculation of dynamic viscosity is given as a function of temperature and pressure for mineral hydraulic oils. It is shown that neglect of influence of working pressure can lead to significant mistakes at calculating of value of dynamic viscosity of hydraulic oils (example, part 3). The error value increases with the growth of pressure and decrease of temperature.

Model (4.2), or (4.3), is very useful for analyzing the flow of hydraulic oil through clearances into hydraulic components, especially the hydraulic components of automation control (servovalve, LS compensator, etc.) [5].

The parameters in model (4.2), or (4.3), represent fluid behavior and have to be calculated from experimental data for each of the used oils separately. However, for engineering applications and ISO viscosity gradations and types (HM and HV), the previously calculated values of the parameters can be used with very satisfactory accuracy.

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MATEMATIČKO MODELOVANJE PROMENE DINAMIČKOG VISKOZITETA, U FUNKCIJI TEMPERATURE I PRITISKA, MINERALNIH ULJA ZA HIDRAULIČNE SISTEME

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Viskozitet je najvažnija karakteristika hidrauličnog fluida. Vrednost i promena viskoziteta imaju fundamentalni značaj za sva dešavanja u hidrauličnom sistemu. Viskozitet hidrauličnog fluida se menja sa promenom temperature, pritiska i brzine deformacije. Međutim, za analizu rada hidrauličnog sistema obično se uzima u obzir samo promena viskoziteta sa promenom temperature, dok se uticaj pritiska i brzine deformacije zanemaruje.

U ovom radu je dat matematički model promene dinamičkog viskoziteta mineralnog hidrauličnog ulja (tip HM i HV) sa promenom temperature i pritiska, i pokazano je da se uticaj pritiska na promenu viskoziteta ne može zanemariti pri analizi hidrauličnog sistema.

Ključne reči: *hidraulično ulje, viskozitet, hidraulični sistem.*